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Why Access to Mathematics Matters and How it Can be Measured

This chapter discusses the importance of mathematics knowledge for acquiring numeracy skills and developing problem-solving abilities. It presents the concept of "opportunity to learn" and argues that measuring opportunity to learn is of critical importance for international comparisons of curricula and student performance. An overview of the data on opportunity to learn in PISA 2012 shows that education systems differ greatly in the degree to which students are exposed to mathematics concepts and also in the way mathematics problems are formulated and presented to students.

The teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students with routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking (Polya, 1973).

Countries repeatedly reform their mathematics curricula to make sure they are relevant to students and societies (Cai and Ni, 2011; Usiskin and Willmore, 2008). Over time, reforms have been based on various factors, including on two observations: both national and international assessments proved that too many students were completing compulsory schooling without being able to use basic mathematics; and the evidence often showed that disadvantaged students were relegated to mathematics courses that were poorer in content and quality – a violation of the principle that all students should be exposed to high-quality instruction.

What the data tell us

- Numeracy skills are used daily in many jobs and are important for a wide range of outcomes in adult life, from successful employment to good health and civic participation.
- In 2012, the average 15-year-old student in an OECD country spent 3 hours and 32 minutes per week in regular mathematics lessons at school; 13 minutes more per week than the average student did in 2003.
- On average across OECD countries, less than 30% of students reported to know well the concept of arithmetic mean; less than 50% of students reported to know well the concepts of polygon and divisor.
- There are large international differences in students' average familiarity with algebraic and geometric concepts. Students in Macao-China reported the most familiarity with algebraic concepts, while students in Shanghai-China had the most familiarity with geometric concepts.
- There is only a weak correlation between students' exposure to applied mathematics and to pure mathematics at the system level, suggesting that the two methods of instruction rarely complement each other.

International data on students' classroom experiences with mathematics are illuminating because they show that policy makers and experts in charge of reform tend to think about mathematics differently than students do (Schoenfeld, 1983; Brown et al., 2008). For the skilled mathematician, solving a mathematics problem is an exciting process of discovery and mental training; for many students towards the end of compulsory education, mathematics is a well-defined set of facts that must be rehearsed until it is learned (Echazarra et al., 2016).

Notwithstanding the good intentions of mathematics teachers, weaker students who are underexposed to the practice of mathematics problem-solving – in many cases, these are students from disadvantaged families – never get an opportunity to develop a "taste for, and some means

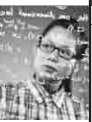
of, independent thinking" (Polya, 1973). Given the importance of mathematics reasoning for life, mathematics curricula need to be enriching and challenging also for those students who do not plan to continue their formal education after compulsory schooling and for those who have fallen behind, in knowledge and self-confidence, since primary school.

What these results mean for policy

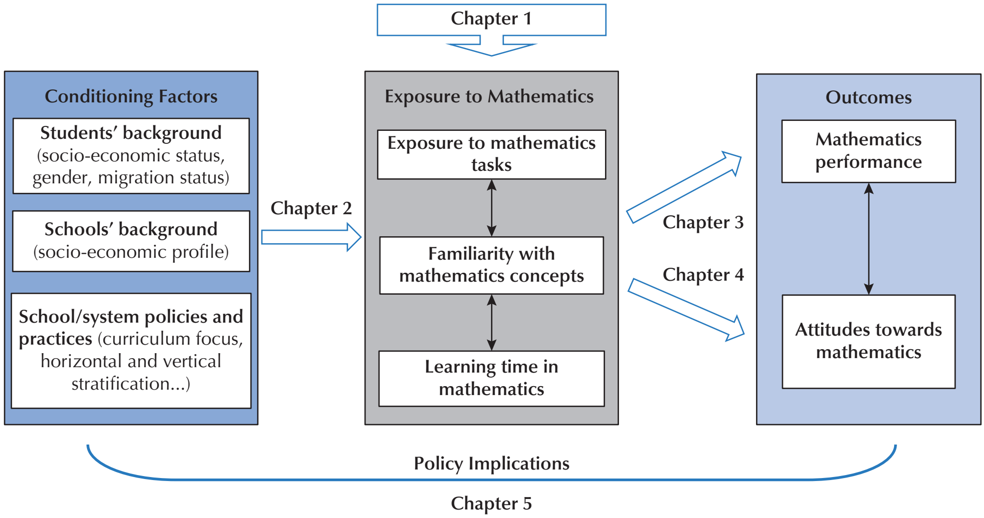
- All students need mathematics for their adult life. Reducing socio-economic inequalities in access to mathematics content is thus an important policy lever for increasing social mobility.
- In many countries, the small share of students who reported that they know well and understand basic concepts signals the need to increase the effectiveness of mathematics teaching by focusing on key mathematics ideas and making more connections across topics.
- The large differences between the intended, the implemented and the achieved curriculum suggest the importance of regularly collecting data on students' exposure to mathematics content.
- International comparisons of curriculum standards, frameworks and teaching material can help countries to design reforms that increase the coherence of the mathematics curriculum.

Achieving equitable opportunities to learn involves not only the content and flexibility of the curriculum, but also how students from different socio-economic backgrounds progress through the system, how well learning materials match students' skills, and how teachers understand and manage the learning needs of diverse students. No matter how detailed and flexible the curriculum might be, mathematics teachers need to make difficult trade-offs to design mathematics lessons that are both accessible to weak students and challenging to bright ones.

This report uses data from PISA 2012 to describe students' opportunity to learn mathematics, including mathematics instruction time and the mathematics content to which students are exposed. It illustrates how students', schools' and systems' characteristics interact in affecting students' capacity to use the mathematics knowledge they acquire at school to solve real-world problems. Figure 1.1 shows the analytical framework of the report. This chapter introduces the concept of opportunity to learn, describes the metrics on content coverage and exposure developed for PISA 2012, and discusses how these metrics capture international differences in mathematics curricula. The second chapter takes one step back to examine student-, school- and system-level variables that can explain how these differences arise. The third chapter looks at how time spent on pure and applied mathematics tasks affects student performance in PISA, while the fourth chapter focuses on the relationship between content exposure and students' attitudes towards mathematics, such as mathematics self-concept and anxiety, which are closely related to mathematics performance. The fifth chapter discusses the policy implications of the preceding analyses.



■ Figure 1.1 ■
The analytical framework

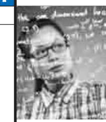


THE IMPORTANCE OF MATHEMATICS SKILLS IN EVERYDAY LIFE

Mathematics teachers are accustomed to answering questions about the usefulness of what they teach. Not only students, but also parents and policy makers often worry about a mismatch between what is taught at school and the quantitative skills needed in everyday life. While it might be difficult to explain why students spend so much time learning algebra and geometry, mathematics is a core part of the curriculum for virtually every secondary student in the world. Is this justified? Should all students learn a significant amount of mathematics beyond what is needed to make simple calculations?

One of the rationales used to explain the central role of mathematics in global education curricula is the idea, dating back to Plato, that mathematics education enhances higher-order thinking skills. Those who are good at mathematics tend to be good thinkers, and those who are trained in mathematics learn to be good thinkers. According to this view, mathematics should be taught for its own sake, rather than to serve more concrete and practical aims.

Beyond the effects of mathematics training on some abstract mental faculties, there is a more intuitive and practical benefit from mastering mathematics at a reasonably good level: mathematics is a gatekeeper. The mathematics studied at school is the main entry point to quantitative literacy, and without solid quantitative skills a person cannot do many jobs. Exam scores in mathematics are, in fact, important factors in determining acceptance into higher education programmes leading to scientific and professional careers.



The demand for STEM (science, technology, engineering and mathematics) professionals has been continuously rising over recent years. For example, employment of STEM professionals across the European Union was approximately 12% higher in 2013 than it was in 2000, notwithstanding the effects of the economic crisis (European Parliament, 2015). Moreover, organisations compete for talent, and many of them now use rigorous quantitative assessments that test both verbal and mathematical ability when selecting employees (Schmitt, 2013).

The value of having quantitative skills has risen over recent years. Our societies are "drenched with data" (Steen, 2001), and the level of number skills needed to carry on daily life activities has increased. Understanding concepts such as "exponential growth" or "line of best fit", assessing the rate at which a variable is changing or knowing what to expect from the flip of a coin have become important for making informed judgements and choices. Computers have reduced the need for mechanical calculations, but the importance of understanding numbers has become even greater in the digital age. In fact, the more people can do with information technology in mathematics, the greater the need for their understanding of and their ability to critically analyse what they are doing (OECD, 2015).

Data from the Survey of Adult Skills, a product of the OECD Programme for the International Assessment of Adult Competencies (PIAAC), provide some tools for assessing the value of quantitative skills at work and in everyday life (Box 1.1). The survey assesses numeracy skills, defined as "the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life" (OECD, 2013a). Numeracy thus refers not just to the ability to perform basic calculations, but to a wide range of skills, such as being able to measure, use and interpret statistical information; understand and use shape, design, location and direction; and think critically about quantitative and mathematical information (Gal and Tout, 2014). The survey measures numeracy from adults' answers to a set of carefully designed and contextualised problems.

Box 1.1. **The Survey of Adult Skills (PIAAC)**

The Survey of Adult Skills is an international survey conducted as part of the Programme for the International Assessment of Adult Competencies (PIAAC). It measures the key cognitive and workplace skills needed for individuals to participate in society and for economies to prosper. The survey is conducted among adults aged 16 to 65. It assesses their literacy and numeracy skills, as well as their ability to solve problems in technology-rich environments, and collects a broad range of information, including how skills are used at work, at home and in the community.

The first round was conducted in 2011-2012 in 24 countries and subnational regions. Results of the second round, released in June 2016, include 9 additional countries.

Source:
http://www.oecd.org/site/piaac/

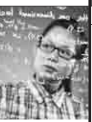
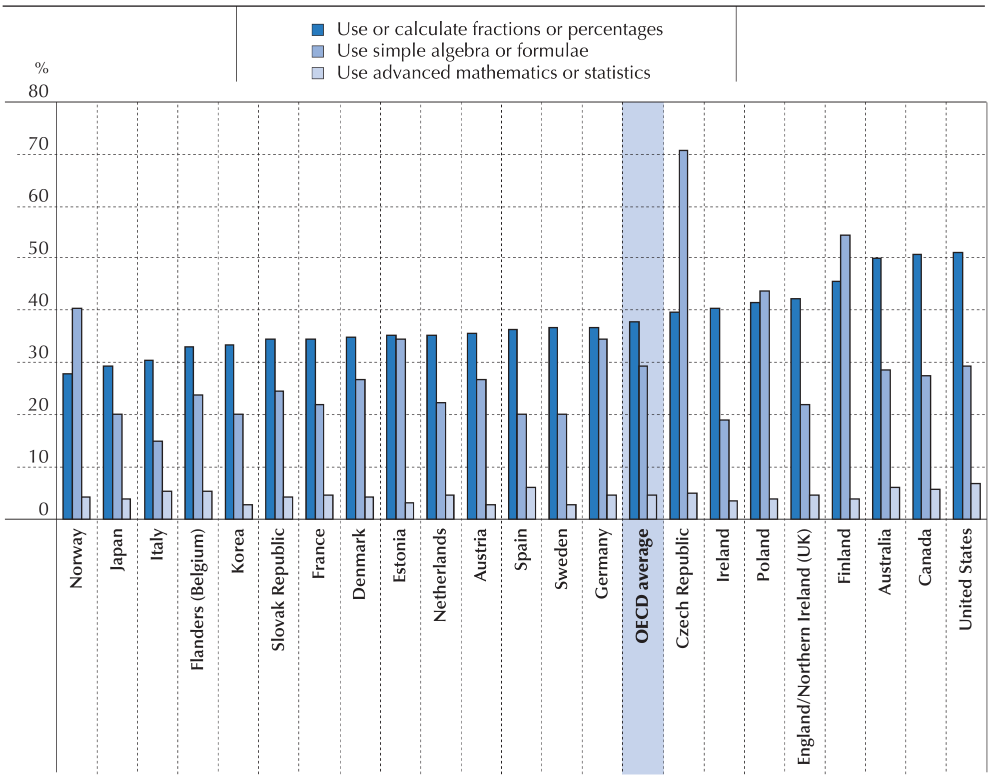


Figure 1.2 shows the extent to which various numeracy skills are used at work, as assessed in the Survey of Adult Skills. On average across participating OECD countries, 38% of workers aged 16 to 65 use or calculate fractions, decimals or percentages, 29% use simple algebra or formulas, and 4% use advanced mathematics at work at least once a week. More than one in three workers in Estonia, Germany, Norway and Poland use algebra at work weekly or daily, as do more than one in two workers in the Czech Republic and Finland. The use of mathematics at work is not limited to the top-paying occupations. On average across OECD countries, 36% of workers in the highest earnings quartile use algebra at work, compared to 18% of workers in the bottom earnings quartile (Table 1.1b).

■ Figure 1.2 ■

Numeracy skills used at work

Percentage of workers who reported that they use these numerical skills at work at least once a week



Note: The OECD countries included in the analysis are: Australia, Austria, Flanders (Belgium), Canada, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Japan, Korea, the Netherlands, Norway, Poland, the Slovak Republic, Spain, Sweden, England/Northern Ireland (UK) and the United States.

Countries and economies are ranked in ascending order of the percentage of workers who reported that they use or calculate fractions or percentages at work.

Source: OECD, Survey of Adult Skills (PIAAC) (2012), Table 1.1a.

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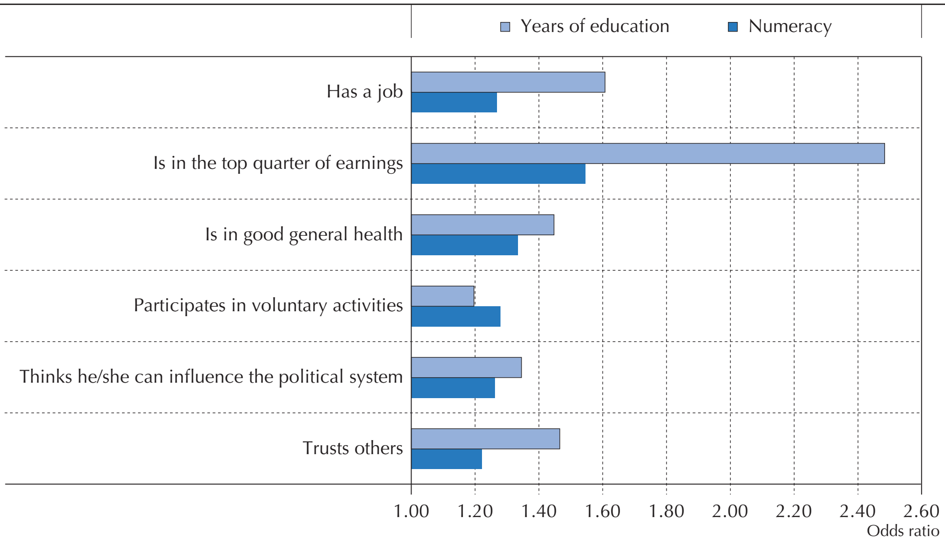
Important decisions in one's personal life, on the job, and in matters of public interest call for sophisticated quantitative reasoning (Schoenfeld, 2002). For example, perceptions about the levels of health risks are less accurate among individuals with low numeracy (Carman and Kooreman, 2014), and low numeracy constrains informed patient choice, reduces medication compliance and limits access to treatments (Nelson et al., 2008). Data from the Survey of Adult Skills show that higher numeracy skills are strongly correlated with other outcomes, such as participation in the labour market, income, good health, participation in volunteer activities, feeling that one has an influence on political life, and the level of trust in others (Figure 1.3). Adults performing 50 points higher than the mean on the survey's numeracy scale are 27% more likely to have a job and 55% more likely to earn high wages than adults performing at the mean. A numeracy score 50 points above the mean raises the odds of being employed to the same level as completing two additional years of education would do.

■ Figure 1.3 ■

Relationship between years of education and numeracy, and economic and social outcomes

Increase in the likelihood of the outcome related to an increase of one standard deviation in years of education or in numeracy; OECD average (22 countries)



How to read the chart: An odds ratio of 1.27 corresponding to the outcome "has a job" and "numeracy" means that an individual who scored one standard deviation higher than another on the Survey of Adult Skills (PIAAC) numeracy scale is 27% more likely to be employed.

Notes: "Years of education" has an average standard deviation of 3.7 years; "numeracy" has an average standard deviation of 51 points.

The OECD countries included in the analyses are: Australia, Austria, Flanders (Belgium), Canada, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Japan, Korea, the Netherlands, Norway, Poland, the Slovak Republic, Spain, Sweden, England/Northern Ireland (UK), and the United States.

Source: OECD, Survey of Adult Skills (PIAAC) (2012), Table 1.2.

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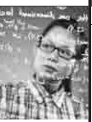


These results from the Survey of Adult Skills show relationships, and cannot be interpreted as the causal effect of mathematics instruction on life outcome. However, the findings are consistent with a large literature showing that attending more advanced mathematics courses has an impact on labour market outcomes (Joensen and Nielsen, 2009; Levine and Zimmerman, 1995). In a study on students' earnings a decade after graduation in the United States, Rose and Betts (2004) find that the math curriculum is responsible for around 27% of the earnings gap experienced by students from lowest-income families relative to middle-income families.

THE RELATIONSHIP BETWEEN MATHEMATICS KNOWLEDGE AND MATHEMATICAL LITERACY

Many argue that the traditional mathematics curriculum fails students because it emphasises a type of mathematics that is radically different from the one used at the workplace (Steen, 2001). Problem solving at work is characterised by pragmatic approaches and techniques that are quick and efficient for specific types of tasks, while the formal mathematics taught at school strives for consistency and generality (Hoyles et al., 2010). This argument has gained popularity because it is not easy to define which mathematics content in the curriculum is most likely to help develop numeracy. Workplace mathematics is also a moving target: changes in society, in technology and in the practice of mathematics also shift the priorities among the many mathematics topics that can be useful for solving problems at work.

Are the differences between school mathematics and the numeracy skills used in life really so large? A look at the PISA performance of students with different levels of exposure to mathematics at school can help to answer this question. PISA assesses the mathematical literacy of students. Mathematical literacy is closely related to the concept of numeracy used in the Survey of Adult Skills,[1] even if it has a stronger connection with the mathematics knowledge acquired at school.

The mathematics framework of PISA defines mathematical literacy as:

"an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens" (OECD, 2013b).

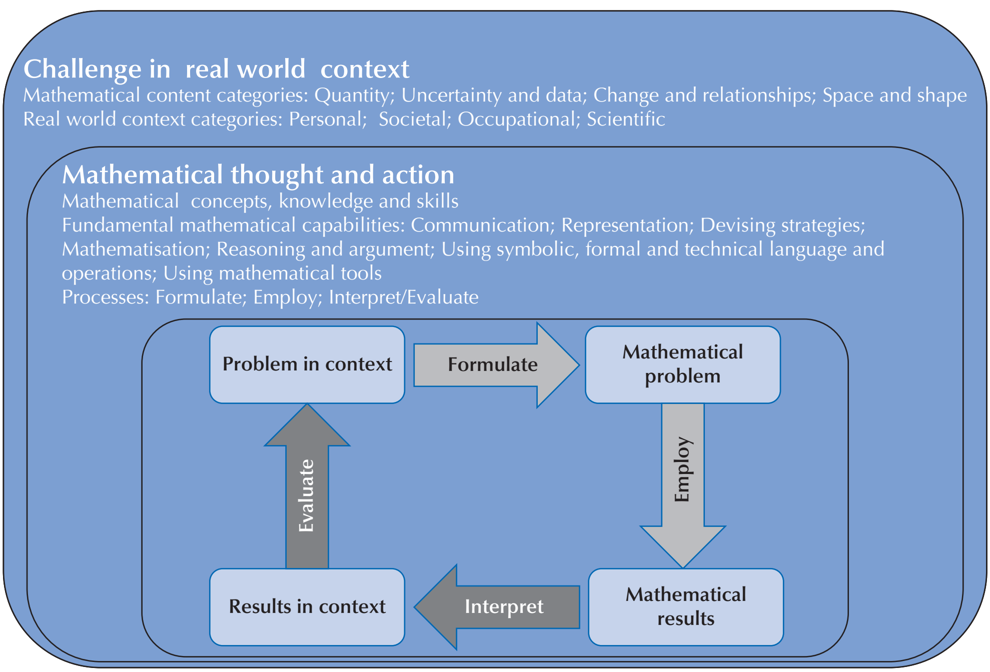
The focus of PISA has been less about what students know after studying a particular curriculum, and more on students' ability to use what they have learned at school to address authentic, real-life challenges and problems (OECD, 2013b; Cogan and Schmidt, 2015). In the description of what students should know and be able to do at different levels of achievement, the PISA mathematics framework refers to "big ideas" (core concepts); it does not specify algebra or geometry or any other specific facet of mathematics. But this does not mean that the structure of the mathematics curriculum, the mastery of concepts and the time spent on mathematics exercises do not matter for developing students' mathematical literacy.

Mathematical literacy and mathematics knowledge – defined as familiarity with mathematics concepts and procedures – are, in fact, not separate but intertwined. Mathematics content areas and concepts have been developed over time as a means to understand and interpret natural and social phenomena (OECD, 2013b). Exposure to this codified content helps students to understand the underlying structure of real problems, shaping what they see and how they behave when they encounter new situations related to those they have previously abstracted and codified (Roterham and Willingham, 2010).

Figure 1.4 shows a simplified version of the stages through which students use the mathematics they learn at school to solve real-life problems. In the first stage, the student takes advantage of his or her knowledge of mathematics first to recognise the mathematical nature of a problem and then to formulate the problem in mathematical terms. The downward-pointing arrow in Figure 1.4 depicts the work undertaken as the problem-solver uses mathematical concepts, procedures, facts and tools to obtain the results. This stage typically involves mathematical reasoning, manipulation, transformation and computation. Next, the results need to be interpreted in terms of the original problem. These processes of formulating, employing and interpreting mathematics draw on the problem-solver's knowledge about individual topics and on a range of fundamental mathematics capabilities.

■ Figure 1.4 ■

The PISA model of mathematical literacy



Exposure to mathematics content helps students to navigate through the processes of formulating, employing and interpreting mathematics. However, becoming mathematically literate requires more than just acquiring knowledge and practicing. Students have to learn to recognise how mathematics can help them deal with situations, solve problems and make sound judgements. The challenge for schools, beyond selecting *which* fundamentals to teach, is *how* to teach these fundamentals in a way that improves students' problem-solving abilities. Teachers not only have to carefully select the content of their lessons, but they also have to tailor the delivery of this content to suit the different capacities of students.

PISA 2012 included detailed information on the types of mathematics students had the opportunity to learn. In an assessment focusing on mathematics skills for life, this information provides a unique opportunity to better understand the relationship between the mathematics taught in school and that used outside of school.

THE CONCEPT OF OPPORTUNITY TO LEARN

The opportunity to learn (OTL) concept refers to the notion that what a student learns at school is related to the content taught in the classroom and the time a student spends learning this content (Cogan and Schmidt, 2015; Schmidt and Maier, 2009). The most quoted definition of OTL comes from Husen's report of the 1964 First International Mathematics Study (FIMS): "whether or not students have had the opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test" (Husen, 1967, pp. 162-163, cited in Burstein, 1993). Research on opportunity to learn started as an afterthought in FIMS when analysts became concerned that not all the tested students had the same opportunities to study a particular topic or to learn how to solve a particular type of problem presented in the assessment (Floden, 2002).

Carroll's (1963) model of school learning provides a strong theoretical basis for the analysis of OTL. The model expresses key factors of learning, including aptitude and ability, in the metric of time, so that the crucial question is no longer "*What* can this student learn?" but "*How long* will it take this student to learn?". The following relationship describes the elements of the model:

$$\text{Learning} = f\left[\frac{\begin{pmatrix}\textbf{\textit{Opportunity to learn}}\textit{ or}\\ \textit{Time allocated for learning}\end{pmatrix} * \begin{pmatrix}\textbf{\textit{Perseverance}}\textit{ or}\\ \textit{Percentage of time actually spent engaged in learning}\end{pmatrix}}{\begin{pmatrix}\textbf{\textit{Aptitude}}\textit{ or}\\ \textit{Time actually needed to learn}\end{pmatrix} * \begin{pmatrix}\textbf{\textit{Quality of instruction}} * \textbf{\textit{Ability}}\textit{ to understand}\end{pmatrix}}\right]$$

Aptitude, ability and perseverance are student characteristics, while opportunity to learn and the quality of instruction are mainly controlled by teachers within the conditions established by the education system. After Carroll, several authoritative reviews of research concluded that time spent on content and the way in which time is organised are primary factors influencing student achievement (Carroll, 1989; Scheerens and Bosker, 1997; Marzano, 2003). Within a short period of time, OTL had a profound impact on the thinking of researchers and practitioners alike (Marzano, 2003).

The school curriculum defines the intended objectives of the education system in terms of content coverage and time allocated to topics. Beyond the *intended* curriculum, what matters for students' learning is the *implemented* curriculum, or the content actually delivered by the teachers. The existence of a single coherent mathematics curriculum delivered by all teachers is nothing more than a myth: discrepancies between the intended and the delivered curriculum exist across all education systems (Floden, 2002; Schmidt et al., 1997; Schmidt et al., 2001). Even when highly structured textbooks are used, teachers make independent choices regarding which topics will be covered and to what extent (Doyle, 1992; Valverde et al., 2002). Teachers might depart from the intended curriculum because some of their students are not sufficiently prepared to absorb the content of overly ambitious and lengthy textbooks, or because the curriculum itself dissuades teachers from sticking closely to its plans. For example, teachers might omit some material because they know that the students' future teachers will have to cover the same material again. Starting from what is taught in classrooms and how it is taught, the *achieved* curriculum – what students actually learn – is, in turn, related to students' ability, aptitude and attitudes towards learning.

Students' opportunity to learn depends on both the intended and the implemented curriculum. Students may not be exposed to certain mathematics concepts because these concepts are not included in the curriculum or because teachers may not cover them. Data collected as part of the 2011 Trends in International Mathematics and Science Study (TIMSS) (Mullis et al., 2012) show that a core set of topics is covered in the intended curriculum of most countries. However, large differences across countries exist in the allocation of these topics to different grades, and in the percentage of teachers who actually teach the topic in each grade (Table 1.3). The percentage of students who are taught basic topics, like fractions, in grade 8 is relatively low (less than 50% in most participating countries), consistent with the fact that fractions are supposed to be covered in the early grades (in most countries, fractions are not expected to be covered after grade 7). In contrast, linear equations and formulas for perimeters, areas and volumes are expected to be covered in the eighth grade in almost all participating countries. But in Hong Kong-China, Japan, Norway, Slovenia, Sweden, Chinese Taipei and Ukraine – where linear equations are part of the eighth-grade curriculum – less than 50% of students in grade 8 are taught them. Teachers may decide not to cover a certain topic with some students or to cover it in earlier or later grades, especially when the curriculum allows for such flexibility.

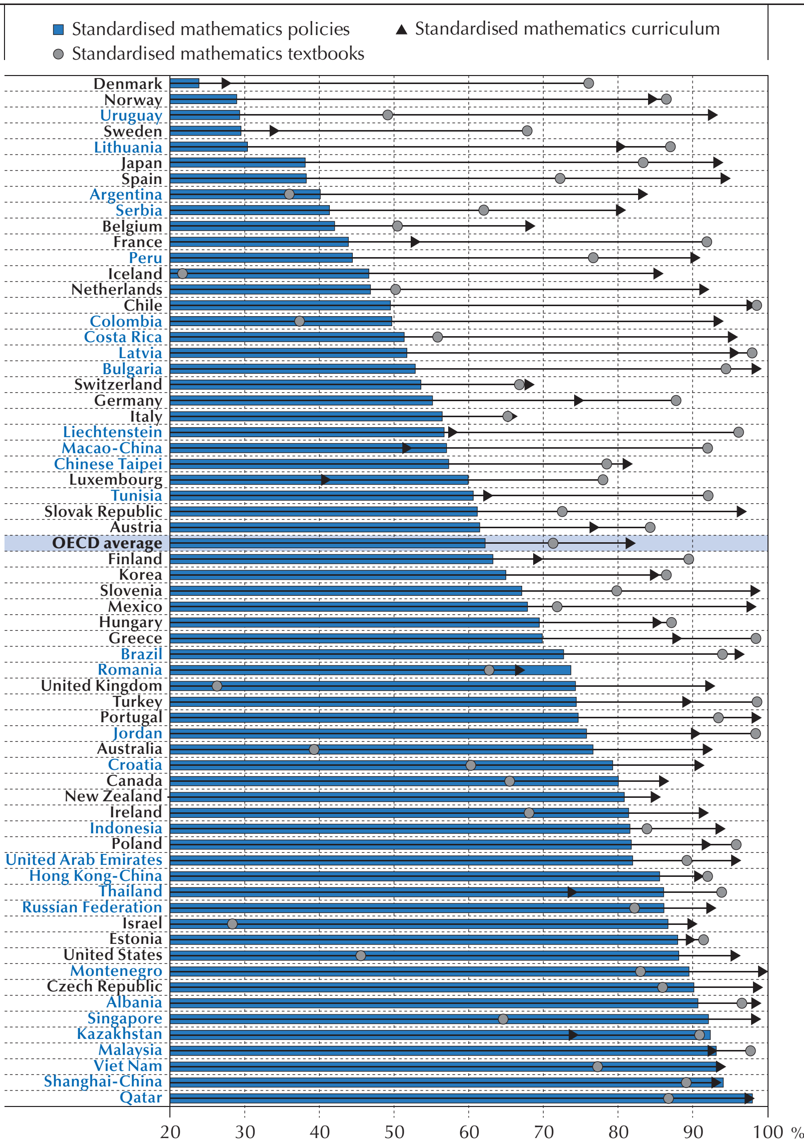
Standardisation policies – such as using a common curriculum across all classes in a school – can limit the freedom of teachers to define the content of their instruction. Figure 1.5 shows that there are large differences across countries in the level of standardisation of mathematics teaching. Around 60% of students in OECD countries are in schools that adopt standardised mathematics policies with shared instructional materials accompanied by staff development and training. These policies are relatively rare in the Nordic countries, but relatively common in several Asian countries and economies. In all countries and economies but Denmark, Luxembourg and Sweden, the majority of students attends schools where teachers are required to follow a mathematics curriculum that specifies the content to be covered each month.

Textbooks are a key link between the intended and the implemented curriculum. Textbooks influence which topics are likely to be covered by teachers, in which order and through

■ Figure 1.5 ■

Use of standardised practices for curriculum and teaching

Percentage of students in schools that practice standardised policies for mathematics teaching, curriculum and textbooks



Note: A standardised policy for mathematics consists of a school curriculum with shared instructional materials accompanied by staff development and training. A standardised curriculum specifies content that mathematics teachers should follow at least monthly. All measures are reported by the school's principal.

Countries and economies are ranked in ascending order of the percentage of students in schools that use standardised mathematics policies.

Source: OECD, PISA 2012 Database, Table 1.5.

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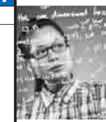


which pedagogical strategies (Freeman and Porter, 1989; Grossman and Thompson, 2008; Johansson, 2005; Reys et al., 2003; Stathopoulou, Gana and Chaviaris, 2012). Adopting a single textbook for all mathematics classes in a school is a common practice in most countries (Figure 1.5), but at the same time teachers have a major role in selecting textbooks. On average across OECD countries, 77% of students attend schools where teachers choose textbooks, while principals and school governing boards are less involved (Table 1.4). Only in Greece, Jordan, Luxembourg and Malaysia over 80% of students attend schools where the national education authority chooses which textbooks are used in the school.

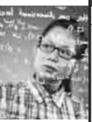
MEASURING OPPORTUNITY TO LEARN IN PISA

For international comparisons, measures of OTL are relevant in two ways: as a possible factor leading to international differences in achievement, and as indicators of cross-national and within-countries differences in the implemented curriculum. If OTL is not taken into account in cross-national comparisons, its effects might be mistakenly attributed to other characteristics of students or education systems (Schmidt et al., 2014). A clear international picture of the similarities and differences in the content students are given the opportunity to learn provides each country with a context for considering curriculum reforms and evaluating equity in access to learning opportunities.

There are two main approaches to measuring OTL. The first, adopted in early studies, such as the First International Mathematics Study (FIMS), measures students' exposure to content at the classroom level through a teacher survey. The second, used in PISA 2012, presents exemplar problems to test-takers, asking them whether they have seen anything similar during their school lessons. Both approaches have advantages and shortcomings. Teachers' reports are generally more accurate descriptions of the delivered curriculum. Students' reports can provide more reliable measures of the time students are actually engaged in learning the topic, under the assumption that students can objectively establish the similarity between what they do in class and what they see in the problems presented in the questionnaire.

The student questionnaire in PISA 2012 included several questions on the degree to which students encounter various types of mathematics problems in their courses, how familiar they are with certain formal mathematics content, and how frequently they are taught to solve specific mathematics tasks. Responses to these questions were used to construct a number of OTL measures and indices, as detailed in Box 1.2.

Based on students' self-reports, the data show substantial variation across education systems in students' exposure to mathematics content. These international differences emerge clearly from the simplest measure of OTL in PISA – the time students reported spending in mathematics classes each week. In 2012, the average 15-year-old student in an OECD country spent 3 hours and 32 minutes per week in mathematics lessons (Figure 1.6). However, behind this average lie great variations among school systems. While 15-year-old students in Canada spent more than 5 hours per week in mathematics lessons, students in Hungary spent 2 hours and 30 minutes per week.



Box 1.2. **Measures of Opportunity to Learn in PISA 2012**

PISA 2012 assessed Opportunity to Learn mathematics through a number of measures:

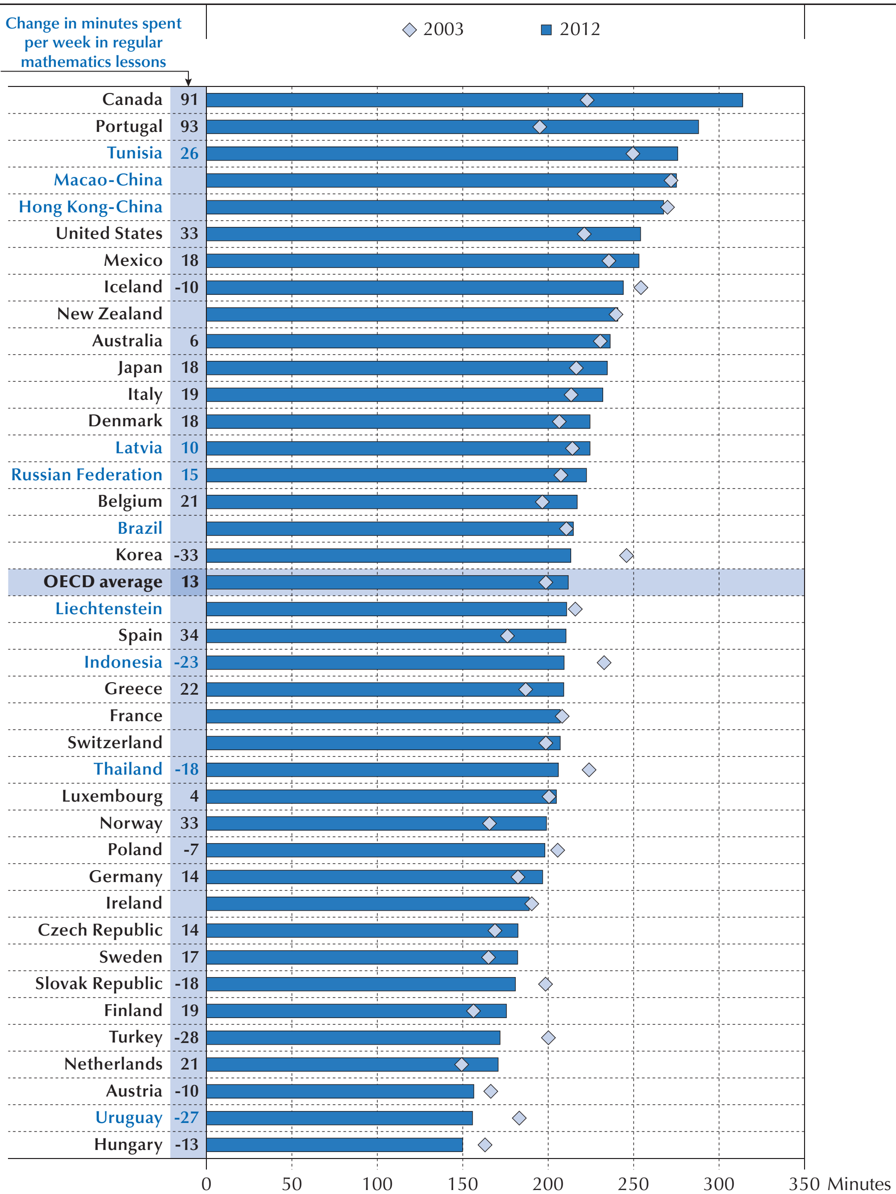
- **Time** spent per week in regular mathematics lessons, in minutes.
- **Exposure** to different types of mathematics tasks during time in school (Question 1 at the end of this chapter), which was scaled to derive two indices (both indices are normalised to have an OECD average of 0 and a standard deviation of 1):
  - The *index of exposure to applied mathematics* refers to student-reported experience with applied tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.
  - The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school that require knowledge of algebra (linear and quadratic equations).
- **Familiarity with mathematics concepts:** Question 2 (reproduced at the end of this chapter) asked students to judge how familiar they were with 13 mathematics concepts. Replies were used to create the *index of familiarity with mathematics,* which was normalised to have an OECD average of 0 and a standard deviation of 1. Part of the analysis contained in this report looks at familiarity with:
  - Algebra, measured as the average student's familiarity with the concepts of exponential function, quadratic function and linear equation.
  - Geometry, measured as the average student's familiarity with the concepts of vector, polygon, congruent figure and cosine.

The question about familiarity also included three foils, i.e. non-existing pseudo-concepts. Responses indicating that students heard of these concepts or knew them well were considered to indicate overclaiming. The *index of familiarity with mathematics* used in this report is corrected for overclaiming.

- Frequency of experience with **specific mathematics tasks** in mathematics lessons and in tests, including the following:
  - Algebraic word problems (Question 3a, reported at the end of this chapter), such as: "Ann is two years older than Betty and Betty is four years older than Sam. When Betty is 30, how old is Sam?".
  - Procedural tasks (Question 3b), such as solving a linear equation or finding the volume of a box.
  - Pure mathematics problems (Question 3c), such as determining the height of a pyramid using geometrical theorems, and solving a problem with prime numbers.
  - Contextualised mathematics problems (Question 3d), such as interpreting a trend in a chart.

■ Figure 1.6 ■

Change between 2003 and 2012 in the time spent per week in mathematics classes



Notes: Statistically significant changes between 2003 and 2012 in the time spent per week in regular mathematics lessons are shown next to the country/economy name.
Only countries with comparable data for both PISA 2003 and PISA 2012 are included.
Countries and economies are ranked in descending order of the time spent in mathematics classes per week in 2012.
Source: OECD, PISA 2012 Database, Table 1.6.
StatLink *http://dx.doi.org/10.1787/888933376897*

Time spent in mathematics classes has increased over the past decade. Across OECD countries, students in 2012 spent an average of 13 minutes more per week in mathematics classes than their counterparts did in 2003. In some countries, the average time spent in regular mathematics classes increased much more than that. In Canada and Portugal, for example, students in 2012 spent 1.5 hours more in mathematics classes than their counterparts in 2003 did, and students in Norway, Spain and the United States spent at least 30 minutes more. The amount of time students spent in mathematics lessons increased by more than 15 minutes in another 11 countries and economies. Only in Korea, which had the fifth longest mathematics class time in 2003, did that class time shrink over the period – by more than 30 minutes.

Education systems differ substantially not only in the time allocated to mathematics teaching, but also in how this time is allocated to different topics. PISA asked students how familiar they are with certain formal mathematics content, including such topics as quadratic functions, radicals and the cosine of an angle (see Box 1.2 for a description of the *index of familiarity with mathematics*). On average across OECD countries, less than 30% of 15-year-old students reported to know well and understand the concept of arithmetic mean; less than 50% of students reported to know well and understand the concepts of divisor and polygon (Table 1.7).

Students in Hong Kong-China, Japan, Korea, Macao-China, Shanghai-China, Spain and Chinese Taipei are most familiar with mathematics concepts in general (Table 1.8). More specifically, Figure 1.7 shows that students in Japan, Macao-China and Singapore reported greater familiarity with the algebraic concepts of linear equation, quadratic function and exponential function. Most students in Shanghai-China reported frequent exposure to the geometric concepts of vector, polygon, congruent figure and cosine. The high levels of exposure to advanced mathematics concepts among Asian students is partly due to the academically oriented mathematics curricula in those countries/economies (Morris and Williamson, 2000), to the emphasis on advanced mathematics courses in teacher-training programmes (Ding et al., 2013), and to a culture of high-stakes examinations that requires teachers to cover all the topics students will need to know for their future tests (Yang, 2014).

At the other end of the spectrum, the majority of students in Sweden reported that they had either never encountered or had encountered only once or twice these algebraic and geometric concepts. In several countries, students reported greater familiarity with algebra than with geometry, or vice versa. For example, while 15-year-old students in Greece were among the most frequently exposed to geometry, they lagged behind the OECD average in exposure to algebra.

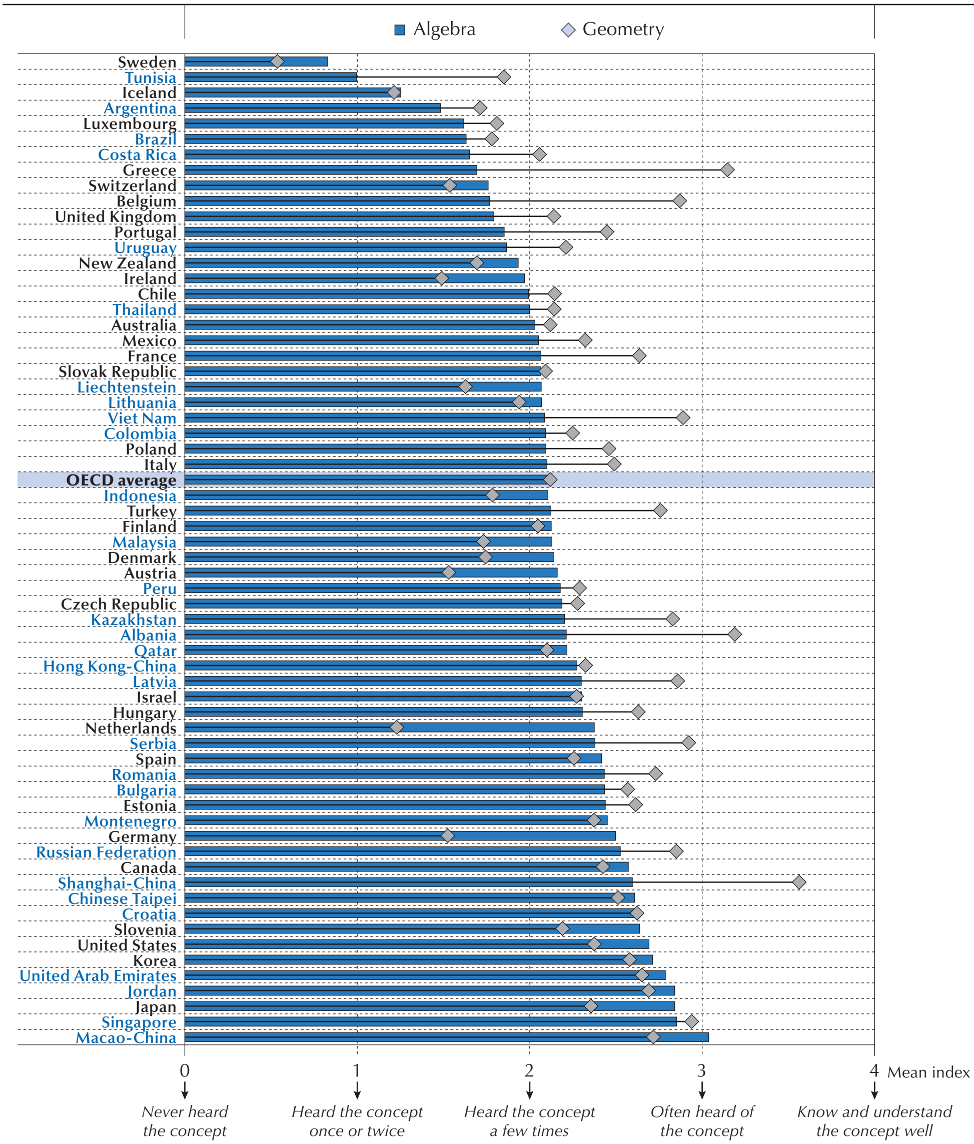
Another set of questions in PISA 2012 was intended to determine whether the teaching of mathematics was more oriented towards pure or applied mathematics (see Question 1 at the end of this chapter). Students' responses to these questions were used to derive the two indices of exposure to pure mathematics and exposure to applied mathematics (Box 1.2).

Students in Korea, the Russian Federation, Singapore and Spain reported the most frequent exposure to pure mathematics at school. Students in Kazakhstan, Korea, Poland and Thailand reported the greatest exposure to applied mathematics (Figure 1.8). Across education systems, there is only a weak relationship between average exposure to applied mathematics and average

■ Figure 1.7 ■

Students' familiarity with algebra and geometry

Self-reported knowledge of mathematics concepts



Notes: Familiarity with geometry is measured as the average student's familiarity with the concepts of vector, polygon, congruent figure and cosine. Familiarity with algebra is measured as the average student's familiarity with the concepts of exponential function, quadratic function and linear equation.

Countries and economies are ranked in ascending order of average familiarity with algebra.

Source: OECD, PISA 2012 Database, Table 1.8.

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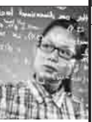
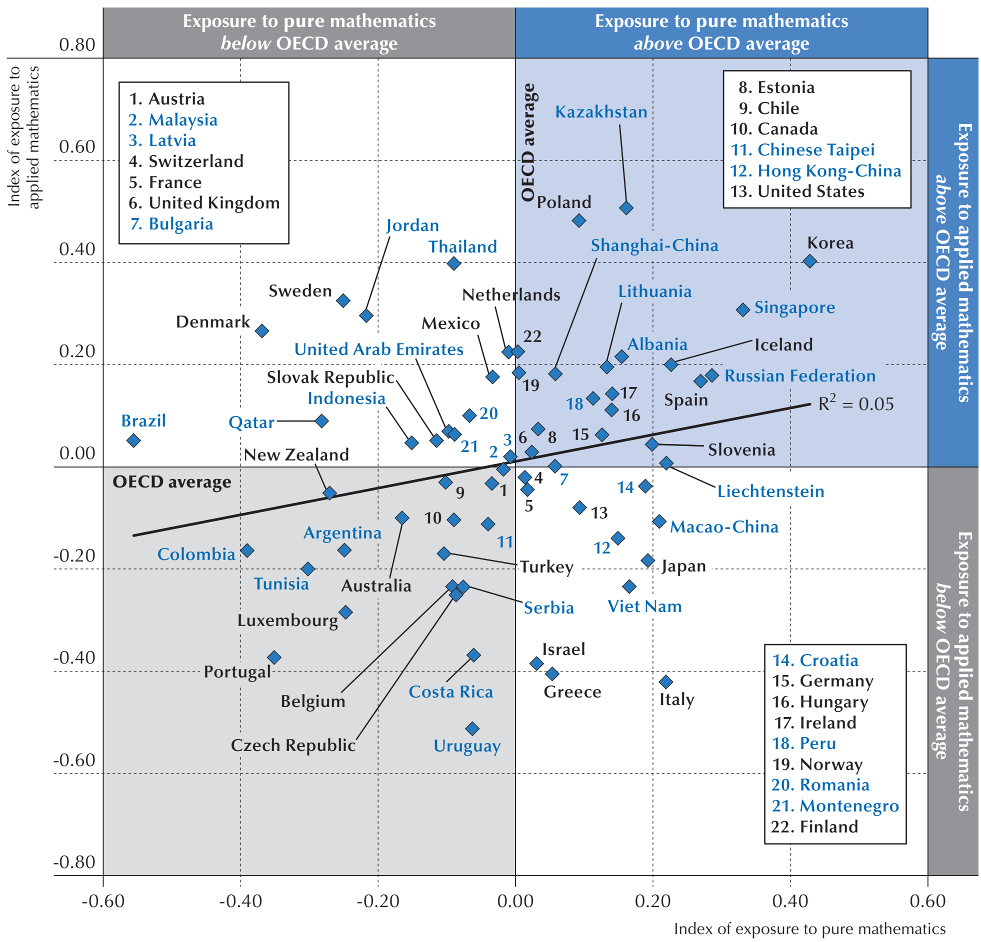


■ Figure 1.8 ■

Relationship between exposure to applied mathematics and exposure to pure mathematics



Notes: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks requiring knowledge of algebra (linear and quadratic equations).
The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.
Source: OECD, PISA 2012 Database, Table 1.9a.
StatLink http://dx.doi.org/10.1787/888933376914

exposure to pure mathematics. Several education systems, including those in Greece, Hong Kong-China, Italy, Japan, Macao-China, the United States and Viet Nam, devote more time to pure mathematics problems than to applied problems, while the opposite is observed in Brazil, Denmark, Jordan, Mexico, Montenegro, Qatar, Romania, the Slovak Republic, Sweden, Thailand and the United Arab Emirates (Table 1.9a).

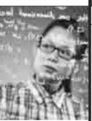
A suitable balance between formal and applied content has been one of the most contentious issues in the public debate on mathematics education. “Maths wars” have raged between those who think that “underlying ideas must be elevated above the examples that illustrate them” (Munson, 2010) and those who believe that “algorithms are harmful” and children should be left free “to invent their own arithmetic without the instruction they are now receiving from textbooks and workbooks” (Kamii and Dominick, 1998: 132). This debate has focused on the structure, presentation and type of problems included in mathematics textbooks; on the extent to which all students should learn mathematics, the type of mathematics they should learn, and the types of problems that are suitable for them to work on as they learn it; and on the type of representations emphasised for student learning and problem solving (Goldin, 2008; Schoenfeld, 2004).

The alternating fortunes of the advocates of “traditional” and “reform” mathematics have influenced curriculum changes, the direction of pedagogical innovations and the content of in-service or pre-service teacher training (Klein, 2003; Schoenfeld, 2004). Some mathematics curricula have tried to reach a middle ground between the two extremes, emphasising the importance of both a high level of mathematics rigour and of opportunities to use mathematics in real-life contexts. In Germany, for instance, the ability to construct models to interpret and understand real problems is one of six compulsory competencies in the new national “Educational Standards” for mathematics (OECD, 2011).

Exposure to mathematics tends to increase as students move to higher grades in schools, but this progression varies across different mathematics content (Figure 1.9). The indices of exposure to pure mathematics and familiarity with mathematics show clear progressions as students advance through the school system. The progression is steeper for familiarity with mathematics because the 13 mathematics concepts included in the measure cover an exhaustive range of material at different levels of difficulty, while the *index of exposure to pure mathematics* is based on a set of algebraic concepts (linear and quadratic equation) of average difficulty for 15-year-old students.

By contrast, students in lower and higher grades reported similar levels of exposure to applied mathematics. This may be because the *index of applied mathematics* in PISA is based on students’ reports of exposure to relatively simple contextualised tasks that require basic numeric skills. Different patterns of exposure to applied mathematics, depending on the students’ grade level, are observed across countries. Students in the Netherlands are relatively frequently exposed to applied mathematics, and this exposure increases among older students. By contrast, in the Czech Republic and the Slovak Republic, teachers in higher grades tend to focus less on the types of applied mathematics tasks that are presented in the PISA questionnaire (Table 1.10).

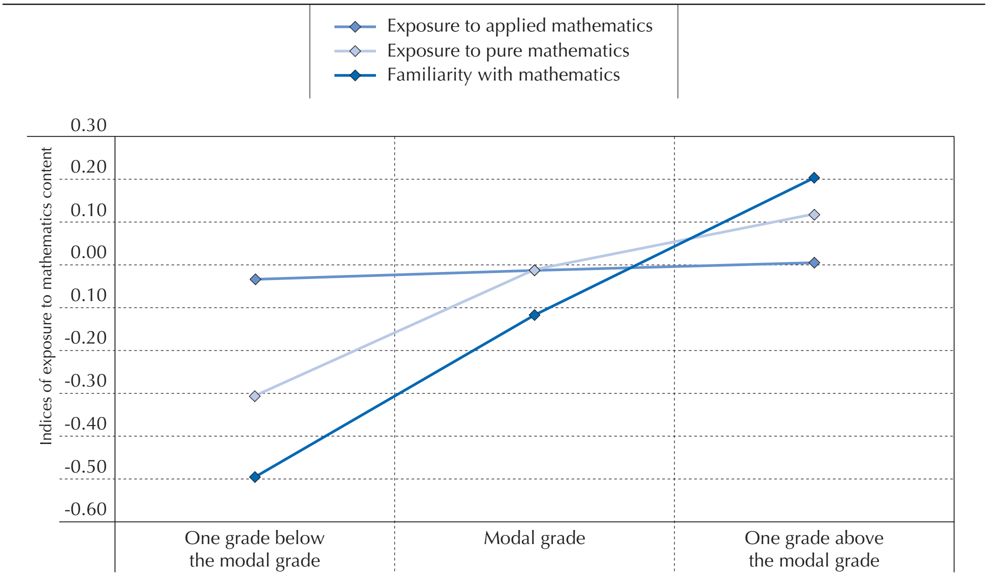
It is difficult to teach mathematics as both general and concrete. Research suggests that, to achieve this, several different representations (e.g. numerical, verbal, symbolic and graphical) of concepts and phenomena are essential, as are the links and transitions between these representations (e.g. Janvier, 1987). The questions on opportunity to learn in PISA 2012 tried to illustrate international differences in the way mathematics problems are presented to students (Box 1.2).



■ Figure 1.9 ■

Exposure to mathematics content in class, by grade

OECD average (23 countries)



Notes: The modal grade is defined in each country as the grade with the largest number of students tested in PISA.
The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school that require knowledge of algebra (linear and quadratic equations).
The *index of exposure to applied mathematics* measures student-reported experience with applied mathematics tasks, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.
The *index of familiarity with mathematics* is based on students' responses to 13 items measuring students' self-reported familiarity with mathematics concepts (such as exponential function, divisor, quadratic function, etc.).
The OECD average for exposure to pure, applied and familiarity with mathematics is calculated only for countries with a valid number of students in the three grades (one grade below the modal grade, the modal grade and one grade above the modal grade).
Source: OECD, PISA 2012 Database, Table 1.10.
StatLink http://dx.doi.org/10.1787/888933376925

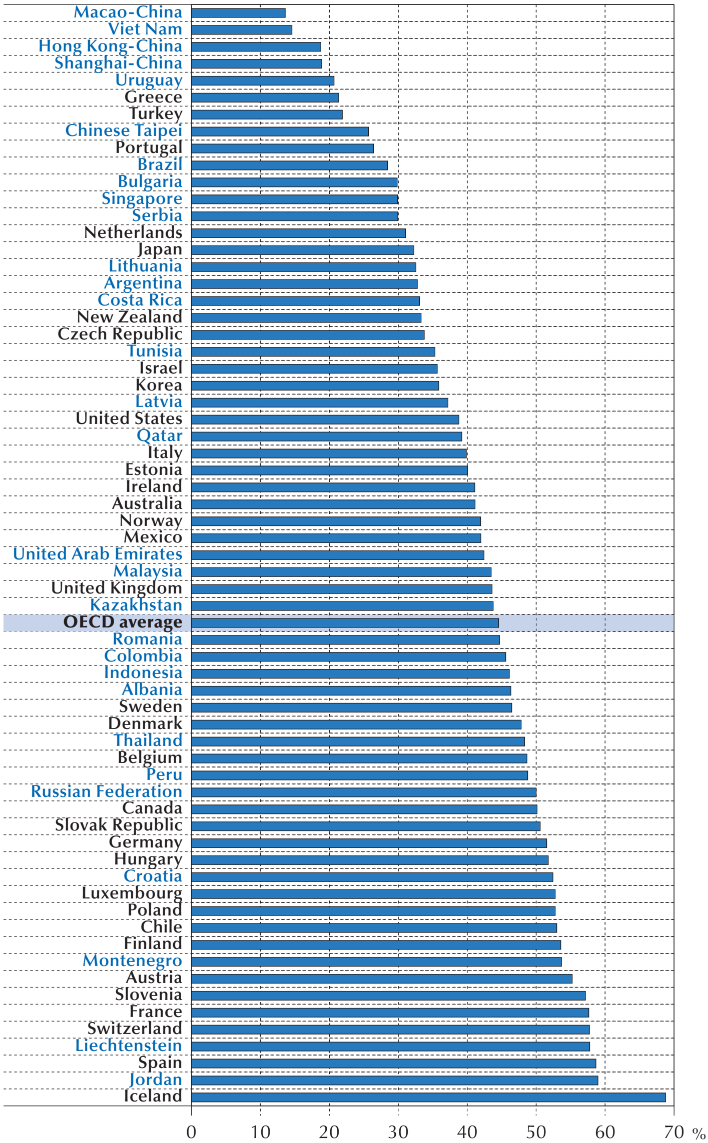
Word problems are used consistently throughout the mathematics curriculum. They are often developed by teachers who wish to connect the mathematics tasks to students' experiences more directly, and to provide contexts to which students can more easily relate (see Question 3a at the end of the chapter). On average across OECD countries, 87% of students see this type of problem at least sometimes in their mathematics lessons, and 79% see these problems at least sometimes in their assessments (Table 1.11a). In the East Asian economies of Hong Kong-China, Macao-China, Shanghai-China and Viet Nam, less than 20% of students are frequently exposed to algebraic word problems (Figure 1.10a).

Another question asked students to describe the extent to which they encounter contextualised mathematical problems, similar to those used in PISA, during their mathematics lessons and assessments (see Question 3d at the end of this chapter). This type of problem requires students to apply mathematics knowledge to find a solution to a problem that arises in everyday life or

■ Figure 1.10a ■

Exposure to algebraic word problems during mathematics lessons

Percentage of students who reported that they are frequently exposed to algebraic word problems during their mathematics lessons



Note: An example of an algebraic word problem is the following: "Ann is two years older than Betty and Betty is four years older than Sam. When Betty is 30, how old is Sam?"

Countries and economies are ranked in ascending order of the percentage of students who reported that they are frequently exposed to algebraic word problems during mathematics lessons.

Source: OECD, PISA 2012 Database, Table 1.11a.

StatLink *http://dx.doi.org/10.1787/888933376935*

work, such as interpreting a trend in a chart. Most mathematics teachers make limited use of PISA-type mathematics problems in their lessons. Only 21% of students reported seeing this type of problem frequently at school (Figure 1.10b), and 45% reported seeing these problems only sometimes (Table 1.11b). Applied mathematics problems requiring interpretation and reasoning in a real-life context are even more rarely used in assessments.

PISA-type mathematics problems often require a skill in "mathematics modelling" – making connections between the real world and mathematics. Mathematics modelling has been discussed and recommended most intensely during the past few decades (Blum and Borromeo Ferri, 2009); however, it is rarely applied in everyday school practice, possibly because it is more difficult both for students and teachers than the replication of routine exercises. Several high-performing countries and economies are among those where students are less likely to report exposure to the kinds of contextualised mathematics problems like those included in the PISA test (Figure 1.10b). This does not mean that exposure to contextualised tasks has a negative effect on performance: rather, it is more likely that contextualisation is used to facilitate access to complex mathematics concepts of students with a weaker knowledge base (see also Table 3.8b on the relationship between exposure to contextualised tasks and performance within countries). Teaching mathematics is complex, and there are other factors that influence performance more than the amount of real-life connections students make during a task (Mosvold, 2008). Moreover, effectively applying contextualised problems in the classroom significantly depends on teachers' ability to support students' capacity to transfer what they learned in a specific context to similar problems in different contexts (see Box 1.3).

Formalised tasks that require applying procedural knowledge (such as those presented in Question 3b at the end of this chapter) are most commonly used in mathematics instruction. PISA shows that around 68% of students in OECD countries see this type of problem frequently in their mathematics lessons (Figure 1.10c), and another 25% of students are sometimes exposed to these problems (Table 1.11c). Almost 90% of students reported solving these problems as part of their assessments at least sometimes (Table 1.11c). At the system level, countries and economies whose students reported frequent exposure to procedural mathematics problems also frequently use algebraic word problems in mathematics classes (Table 1.12).

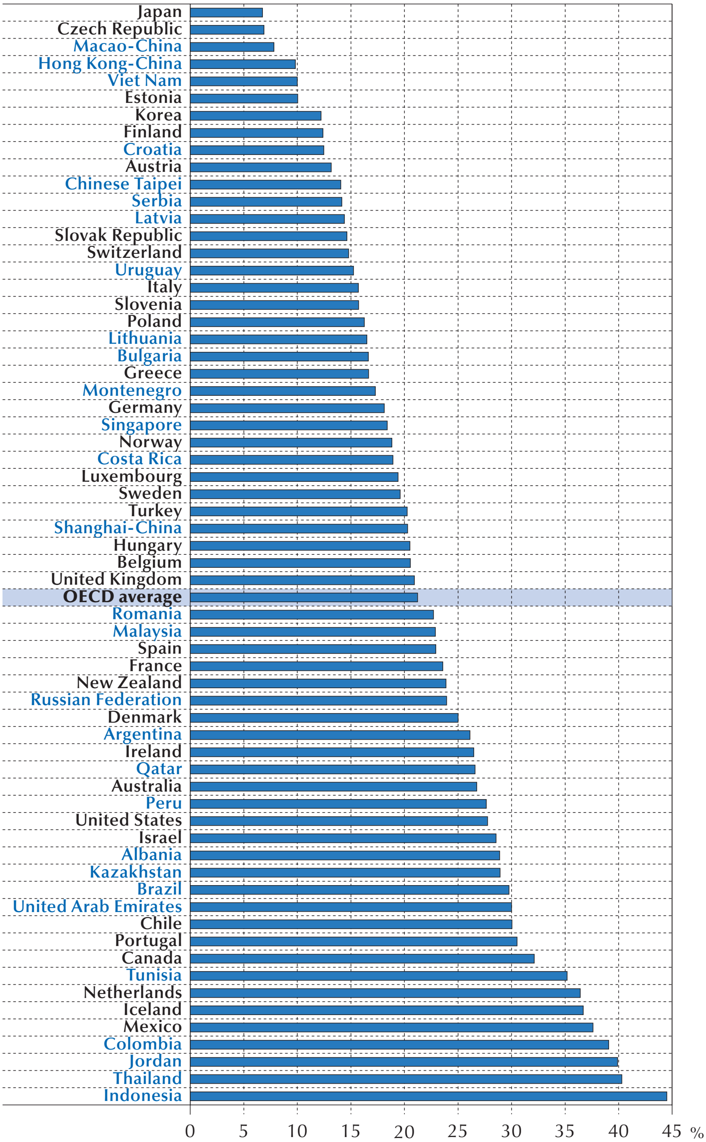
The dominance of procedural mathematics compared with modelling is problematic if students fail to establish the connection between procedures and concepts. For example, students often look at the operational side of equations arriving at the solution with no real understanding of the concept of the equation (Niss, 1987). In the long-standing debate about the relationship between procedural and conceptual knowledge, there is a prevalent view that instruction should develop conceptual knowledge before focusing on procedural knowledge (Grouws and Cebulla, 2000; NCTM, 2000, 2014). A recent analysis of the evidence further suggests that conceptual understanding and procedural fluency are equally important as interdependent strands of mathematical proficiency (Rittle-Johnson et al., 2015). Both contribute to the long-term development of problem-solving skills.

Pure mathematics problems are also examined in the PISA student questionnaire (see Question 3c at the end of this chapter). These problems require a foundation of conceptual knowledge

■ Figure 1.10b ■

Exposure to contextualised mathematics problems during mathematics lessons

Percentage of students who reported that they are frequently exposed to contextualised mathematics problems during their mathematics lessons



Note: Contextualised mathematics problems require the application of mathematics knowledge to find an answer to a problem that arises in everyday life or work, such as interpreting a trend in a chart.

Countries and economies are ranked in ascending order of the percentage of students who reported that they are frequently exposed to contextualised mathematics problems during mathematics lessons.

Source: OECD, PISA 2012 Database, Table 1.11b.

StatLink http://dx.doi.org/10.1787/888933376946

Box 1.3. **Advantages and possible costs of contextualised mathematics**

Mathematics and science teachers at all levels are frequently encouraged to incorporate concrete, meaningful, real-world examples into their lessons when teaching new material (Rivet and Krajcik, 2008). First, concrete examples are easier to process than more abstract representations and connect the learner's existing knowledge with new, to-be-learned knowledge. For instance, a mathematics instructor teaching simple probability theory may present probabilities by rolling a six-sided die. Second, tasks embedded in real-life contexts have high motivational power; students are most easily engaged with problems that are taken from their everyday lives (Hiebert et al., 1996). Well-designed real-life tasks can also encourage the idea that mathematics is a useful discipline (Trafton et al., 2001).

Despite these advantages, research suggests that concrete examples may also come with a cost. For example, any information presented that is not essential tends to distract learners from the relevant content, leading to poorer recall for that material (the "seductive details effect"; Day et al., 2015; Harp and Mayer, 1998). Grounding mathematics using concrete contexts can thus potentially limit its applicability to similar situations in which just the surface details are changed, particularly for low-performers. In a series of experiments with undergraduate and high school students, Kaminski et al. (2008) found that learning one, two or three concrete examples resulted in little or no transfer, whereas learning one generic example resulted in significant transfer. On these grounds, the benefits of contextualised problems exceed their costs only if the tasks are very well designed (e.g. minimising unnecessary distractions) and if teachers address the transfer problem, for instance by presenting a concrete example and then a generic example for the same topic.

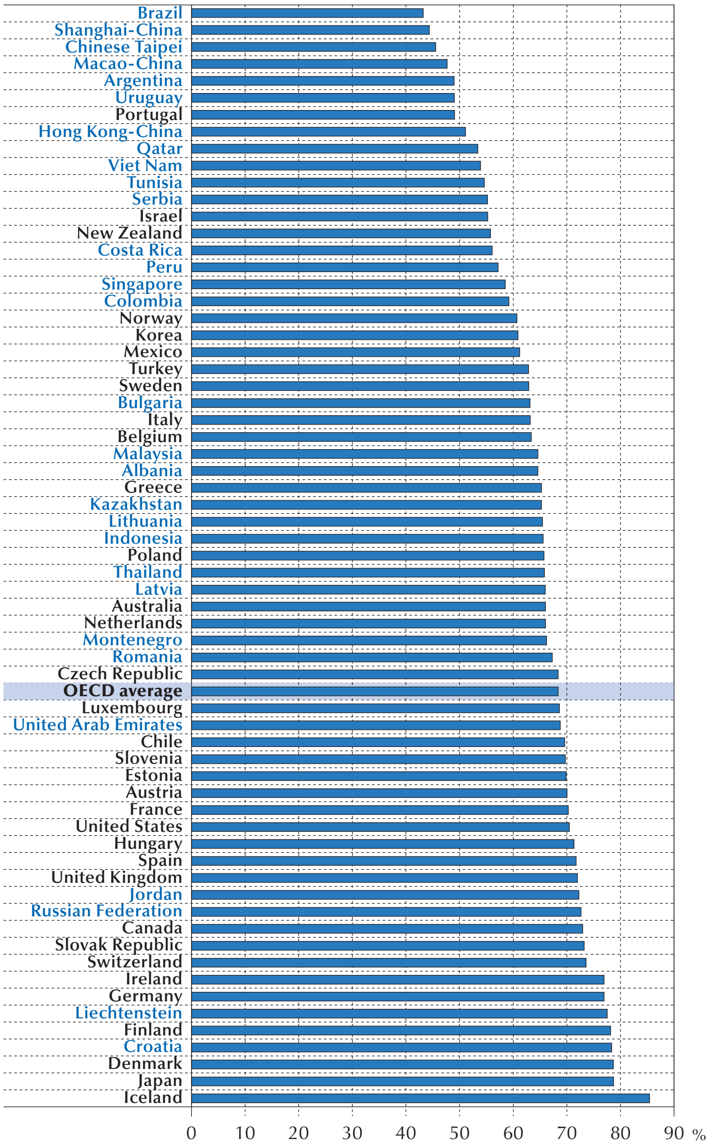
Mathematics teaching in the Netherlands has traditionally had the highest amount of connections to real-life (Hiebert et al., 2003); most textbook problems present some kind of real-life context (Mosvold, 2008). Several other countries have taken initiatives to increase the frequency and to improve the quality of real-life mathematics tasks students tackle in class. For example, the Singapore Mathematics Assessment and Pedagogy Project (SMAPP) developed a new assessment system that includes real-life mathematics, producing contextualised tasks that teachers can use in their lessons. According to the SMAPP framework, a good task should: include links to real life using relevant data; connect to the curriculum; assess multiple competencies and content knowledge; enrich student experiences; and include scaled levels of difficulty. The tasks were developed by a team of mathematicians, reviewed by teachers, and then revised after testing in real lessons. Japan recently revised its "Course of Study" and introduced mathematical activities with stronger connections with real-life problems.

and the use of procedures that are not automatised, but rather require conscious selection, reflection and sequencing of steps. Three out of four students across OECD countries see this type of problem either frequently or sometimes in their mathematics lessons (Table 1.11d), and two out of three students solve these problems at least sometimes in the tests they take at school. Students in Finland, Norway and Sweden are less exposed to this type of task than students in other countries and economies.

■ Figure 1.10c ■

Exposure to procedural mathematics tasks during mathematics lessons

Percentage of students who reported that they are frequently exposed to procedural mathematics tasks during their mathematics lessons

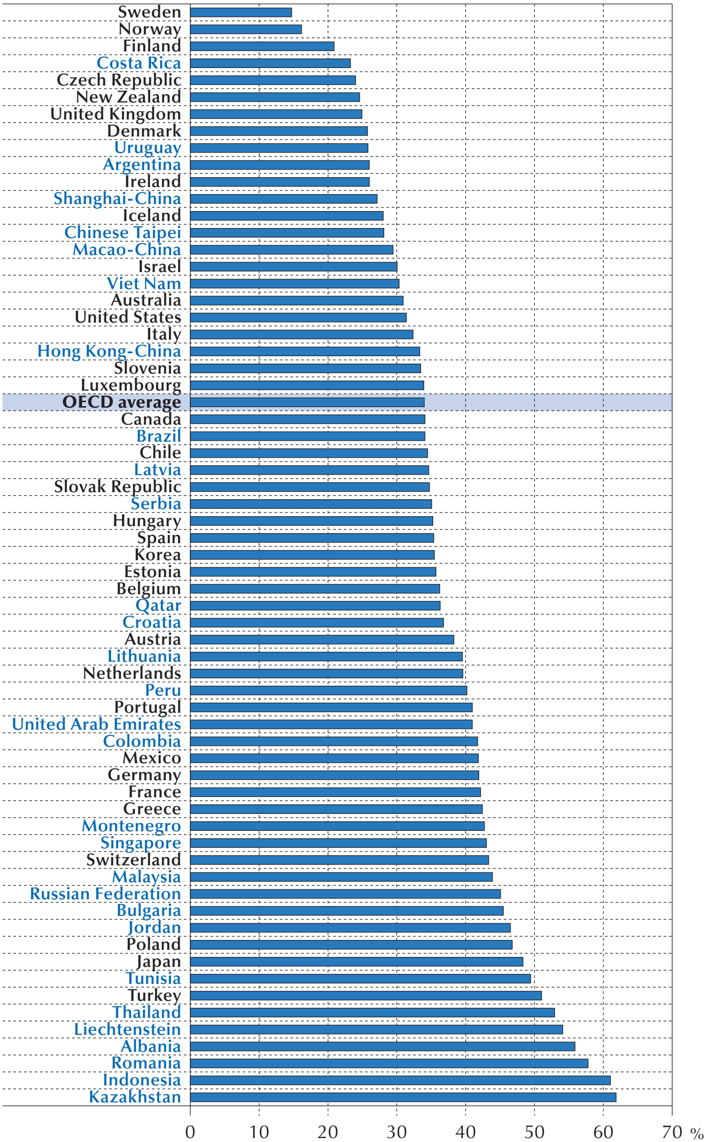


Note: Solving a linear equation or finding the volume of a box are examples of procedural mathematics tasks.
Countries and economies are ranked in ascending order of the percentage of students who reported that they are frequently exposed to procedural mathematics tasks during mathematics lessons.
Source: OECD, PISA 2012 Database, Table 1.11c.
StatLink http://dx.doi.org/10.1787/888933376953

■ Figure 1.10d ■

Exposure to pure mathematics problems during mathematics lessons

Percentage of students who reported that they are frequently exposed to pure mathematics problems during mathematics lessons

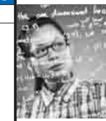


Note: Pure mathematics problems require the use of mathematics knowledge to draw conclusions, without referring to any practical application (e.g. determining the height of a pyramid using geometrical theorems).

Countries and economies are ranked in ascending order of the percentage of students who reported that they are frequently exposed to pure mathematics problems during mathematics lessons.

Source: OECD, PISA 2012 Database, Table 1.11d.

StatLink http://dx.doi.org/10.1787/888933376962



PISA provides substantial data on the international variation in the intensity, topic coverage and representation of mathematics instruction. These data show remarkable differences between education systems in the opportunity to learn mathematics. The value of these data for education policy emerges when they are used in combination with information on student performance on the PISA assessment of mathematics (Chapter 3), and when the analysis moves beyond country means to look at how opportunity to learn is distributed among students of different socio-economic status (Chapter 2). If a solid knowledge of mathematical concepts is necessary to solve non-routine mathematics problems and to apply mathematics in complex contexts outside the classroom, then socio-economic differences in access to mathematics knowledge will perpetuate differences in student performance – and in later social and economic outcomes – that are linked to socio-economic status.

QUESTIONS USED TO MEASURE OPPORTUNITY TO LEARN IN PISA 2012

The PISA 2012 student questionnaire contains six questions on opportunity to learn mathematics. Box 1.2 explains how responses were scaled and combined into several indices. These questions are shown below.

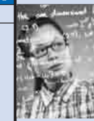
EXPOSURE TO PURE AND APPLIED MATHEMATICS

This question asks students to report on the frequency with which they have encountered specific applied and pure mathematics tasks during mathematics lessons. Students' responses to the items a) through f) in this question were scaled to produce the *index of exposure to applied mathematics* and responses to the items g) through i) were used for the *index of exposure to pure mathematics*.

Question 1

How often have you encountered the following types of mathematics tasks during your time at school?
(Please tick only one box in each row.)

		Frequently	Sometimes	Rarely	Never
	Applied mathematics tasks				
a)	Working out from a <train timetable> how long it would take to get from one place to another.	□1	□2	□3	□4
b)	Calculating how much more expensive a computer would be after adding tax.	□1	□2	□3	□4
c)	Calculating how many square metres of tiles you need to cover a floor.	□1	□2	□3	□4
d)	Understanding scientific tables presented in an article.	□1	□2	□3	□4
e)	Finding the actual distance between two places on a map with a 1:10,000 scale.	□1	□2	□3	□4
f)	Calculating the power consumption of an electronic appliance per week.	□1	□2	□3	□4
	Pure mathematics tasks				
g)	Solving an equation like: $6x^2 + 5 = 29$	□1	□2	□3	□4
h)	Solving an equation like: $2(x + 3) = (x + 3)(x - 3)$	□1	□2	□3	□4
i)	Solving an equation like: $3x + 5 = 17$	□1	□2	□3	□4



FAMILIARITY WITH MATHEMATICS

This question evaluates students' familiarity with different mathematical concepts covered in the mathematics curriculum.

Question 2

Thinking about mathematical concepts: how familiar are you with the following terms?
(Please tick only one box in each row.)

		Never heard of it	Heard of it once or twice	Heard of it a few times	Heard of it often	Know it well, understand the concept
a)	Exponential Function	☐1	☐2	☐3	☐4	☐5
b)	Divisor	☐1	☐2	☐3	☐4	☐5
c)	Quadratic Function	☐1	☐2	☐3	☐4	☐5
d)	Linear Equation	☐1	☐2	☐3	☐4	☐5
e)	Vectors	☐1	☐2	☐3	☐4	☐5
f)	Complex Number	☐1	☐2	☐3	☐4	☐5
g)	Rational Number	☐1	☐2	☐3	☐4	☐5
h)	Radicals	☐1	☐2	☐3	☐4	☐5
i)	Polygon	☐1	☐2	☐3	☐4	☐5
j)	Congruent Figure	☐1	☐2	☐3	☐4	☐5
k)	Cosine	☐1	☐2	☐3	☐4	☐5
l)	Arithmetic Mean	☐1	☐2	☐3	☐4	☐5
m)	Probability	☐1	☐2	☐3	☐4	☐5

STUDENTS' EXPOSURE TO DIFFERENT KINDS OF MATHEMATICS PROBLEMS

The following four questions explore students' experience with different types of mathematics problems at school. They include a brief description of the type of problem and two examples of mathematics problems for each type. The students had to read each problem but did not have to solve it.

Question 3a: Algebraic word problems

The box is a series of problems. Each requires you to understand a problem written in text and perform the appropriate calculations. Usually the problem talks about practical situations, but the numbers and people and places mentioned are made up. All the information you need is given. Here are two examples:

1. <Ann> is two years older than <Betty> and <Betty> is four times as old as <Sam>. When <Betty> is 30, how old is <Sam>?

2. Mr <Smith> bought a television and a bed. The television cost <$625> but he got a 10% discount. The bed cost <$200>. He paid <$20> for delivery. How much money did Mr <Smith> spend?

We want to know about your experience with these types of word problems at school. Do not solve them!

(Please tick only one box in each row.)

		Frequently	Sometimes	Rarely	Never
a)	How often have you encountered these types of problems in your **mathematics lessons**?	☐1	☐2	☐3	☐4
b)	How often have you encountered these types of problems in the **tests you have taken at school**?	☐1	☐2	☐3	☐4

Question 3b: Procedural mathematics problems

Below are examples of another set of mathematical skills.

1) Solve 2x + 3 = 7.

2) Find the volume of a box with sides 3m, 4m and 5m.

We want to know about your experience with these types of problems at school. Do not solve them!

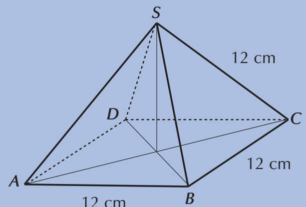
(Please tick only one box in each row.)

		Frequently	Sometimes	Rarely	Never
a)	How often have you encountered these types of problems in your **mathematics lessons**?	☐1	☐2	☐3	☐4
b)	How often have you encountered these types of problems in the **tests you have taken at school**?	☐1	☐2	☐3	☐4

Question 3c: Pure mathematics problems

In the next type of problem, you have to use mathematical knowledge and draw conclusions. There is no practical application provided. Here are two examples.

1) Here you need to use geometrical theorems:



Determine the height of the pyramid.

2) Here you have to know what a prime number is:

If n is any number: can $(n+1)^2$ be a prime number?

We want to know about your experience with these types of problems at school. Do not solve them!

(Please tick only one box in each row.)

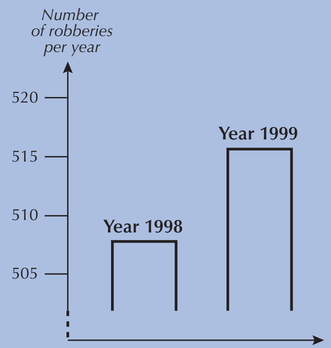
		Frequently	Sometimes	Rarely	Never
a)	How often have you encountered these types of problems in your **mathematics lessons?**	☐1	☐2	☐3	☐4
b)	How often have you encountered these types of problems in the **tests you have taken at school**?	☐1	☐2	☐3	☐4

Question 3d: Contextualised mathematics problems

In this type of problem, you have to apply suitable mathematical knowledge to find a useful answer to a problem that arises in everyday life or work. The data and information are about real situations. Here are two examples.

Example 1

A TV reporter says "This graph shows that there is a huge increase in the number of robberies from 1998 to 1999."



Do you consider the reporter's statement to be a reasonable interpretation of the graph?

Give an explanation to support your answer.

Example 2

For years the relationship between a person's recommended maximum heart rate and the person's age was described by the following formula:

Recommended maximum heart rate = 220 – age

Recent research showed that this formula should be modified slightly. The new formula is as follows:

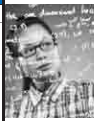
Recommended maximum heart rate = 208 – (0.7 × age)

From which age onwards does the recommended maximum heart rate increase as a result of the introduction of the new formula? Show your work.

We want to know about your experience with these types of problems at school. Do not solve them!

(Please check only one box in each row.)

		Frequently	Sometimes	Rarely	Never
a)	How often have you encountered these types of problems in your **mathematics lessons**?	☐1	☐2	☐3	☐4
b)	How often have you encountered these types of problems in the **tests you have taken at school**?	☐1	☐2	☐3	☐4



Note

1. The OECD Survey of Adult Skills defines numeracy as the ability to access, use, interpret and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life (OECD, 2012).

References

Blum, W. and **R. Borromeo Ferri** (2009), "Mathematical modelling: Can it be taught and learnt?", *Journal of Mathematical Modelling and Application*, Vol. 1/1, pp. 45-58.

Brown, M., P. Brown, and **T. Bibby** (2008), '"I Would Rather Die": Reasons given by 16-Year-Olds for Not Continuing Their Study of Mathematics'. *Research in Mathematics Education*, Vol. 10/1, pp. 3-18, http://dx.doi.org/10.1080/14794800801915814.

Burstein, L. (1993), "Prologue: Studying learning, growth, and instruction cross-nationally: Lessons learned about why and why not engage in cross-national studies", in L. Burstein (ed.), *The IEA Study of Mathematics III: Student Growth and Classroom Processes,* Pergamon Press, New York, NY, pp. 27-49.

Cai, J. and **Y. Ni** (2011), "Investigating curricular effect on the teaching and learning of mathematics in a cultural context: Theoretical and methodological considerations", *International Journal of Educational Research*, Vol. 50, pp. 65-70.

Carman, K.G. and **P. Kooreman** (2014), "Probability perceptions and preventive health care", *Journal of Risk and Uncertainty*, Vol. 49/1, pp. 43-71.

Carroll, J.B. (1963), "A model of school learning", *Teachers College Record*, Vol. 64, pp. 723-733.

Carroll, J.B. (1989), "The Carroll Model: A 25-year retrospective and prospective view, *Educational Researcher*, Vol. 18/1, pp. 26-31.

Cogan, L.S. and **W.H. Schmidt** (2015), "The concept of opportunity to learn (OTL) in international comparisons of education", in K. Stacey and R. Turner (eds.), *Assessing Mathematical Literacy*, Springer International Publishing, Cham, Switzerland, pp. 207-216.

Day, S.B., B.A. Motz, and **R.L. Goldstone** (2015), "The Cognitive Costs of Context: The Effects of Concreteness and Immersiveness in Instructional Examples", *Frontiers in Psychology*, 6 (December), http://dx.doi.org/10.3389/fpsyg.2015.01876.

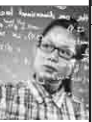
Ding, M., Y. Li, X. Li and **J. Gu** (2013), "Knowing and understanding instructional mathematics content through intensive studies of textbooks", in Y. Li and R. Huang (eds.), *How Chinese Teach Mathematics and Improve Teaching,* Routledge, New York, NY, pp. 66-82.

Doyle, W. (1992), "Curriculum and pedagogy", in P.W. Jackson (ed.), *Handbook of Research on Curriculum,* MacMillan, New York, NY, pp. 486-516.

Echazarra, A., et al. (2016), 'How Teachers Teach and Students Learn'. OECD Education Working Papers. Paris: Organisation for Economic Co-operation and Development. http://www.oecd-ilibrary.org/content/workingpaper/5jm29kpt0xxx-en.

European Parliament (2015), "Encouraging STEM studies for the Labour Market", Study for the EMPL Committee, Policy Department Economic and Scientific Policy, Brussels.

Floden, R.E. (2002), "The measurement of opportunity to learn", in A.C. Porter and A. Gamoran (eds.), *Methodological Advances in Cross-National Surveys of Educational Achievement*, Board on International Comparative Studies in Education, National Academy Press, Washington, DC, pp. 231-266.



Freeman, D.J. and **A.C. Porter** (1989), 'Do Textbooks Dictate the Content of Mathematics Instruction in Elementary Schools?' *American Educational Research Journal*, Vol. 26/3, pp. 403-21.

Gal, I. and **D. Tout** (2014), "Comparison of PIAAC and PISA frameworks for numeracy and mathematical literacy", *OECD Education Working Papers*, No. 102, OECD Publishing, Paris, http://dx.doi.org/10.1787/5jz3wl63cs6f-en.

Goldin, G.A. (2008), "Perspectives on representation in mathematical learning and problem solving", in L.D. English (ed.), *Handbook of International Research in Mathematics Education* (2nd ed.), Routledge, New York, NY, pp. 176-201.

Grossman, P. and **C. Thompson** (2008), 'Learning from Curriculum Materials: Scaffolds for New Teachers?' *Teaching and Teacher Education*, Vol. 24/8, pp. 2014-26, http://dx.doi.org/10.1016/j.tate.2008.05.002.

Grouws, D.A. and **K.J. Cebulla** (2000), *Improving Student Achievement in Mathematics*, International Bureau of Education (IBE), Geneva, Switzerland.

Harp, S.F. and **R.E. Mayer** (1998), "How Seductive Details Do Their Damage: A Theory of Cognitive Interest in Science Learning", *Journal of Educational Psychology*, Vol. 90/3, pp. 414-34, http://dx.doi.org/10.1037/0022-0663.90.3.414.

Hiebert, J., et al. (2003), "Teaching Mathematics in Seven Countries: Results from the TIMSS 1999 Video Study", March 26, https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2003013.

Hiebert, J., et al. (1996), "Problem Solving as a Basis for Reform in Curriculum and Instruction: The Case of Mathematics", *Educational Researcher*, Vol. 25/4, pp. 12-21, http://dx.doi.org/10.2307/1176776.

Houang, R.T., and **W.H. Schmidt** (2008), TIMSS international curriculum analysis and measuring educational opportunities, Retrieved from www.iea.nl/fileadmin/user_upload/IRC2008/Papers/TIMSS_Mathematics/Houang_Schmidt.pdf.

Hoyles, C., et al. (2010), *Improving mathematics at work: The need for techno-mathematical literacies*, Routledge, London.

Husen, T. (ed.) (1967), *International Study of Achievement in Mathematics: A Comparison of Twelve Countries* (Vol. I), Wiley and Sons, New York, NY.

Jacob, B. (1999), "Instructional materials for K-8 mathematics classrooms: The California adoption", in E. Gavosto, S. Krantz and W. McCallum (eds.), *Contemporary Issues in Mathematics Education*, Mathematical Sciences Research Institute Publications, Cambridge University Press, Cambridge, UK, pp. 109-122.

Janvier, C. (ed.) (1987), *Problems of Representations in the Teaching and Learning of Mathematics*, Lawrence Erlbaum Associates, Hillsdale, NJ.

Joensen, J.S. and **H.S. Nielsen** (2009). "Is there a Causal Effect of High School Math on Labor Market Outcomes?" *Journal of Human Resources*, Vol. 44/1, pp. 171-198.

Johansson, M. (2005), *Mathematics Textbooks-the Link between the Intended and the Implemented Curriculum, Paper Presented to 'the Mathematics Education into the 21st Century Project' Universiti Teknologi, Malaysia*.

Kamii, C. and **A. Dominick** (1998), "The harmful effects of algorithms in grades 1- 4", in L.J. Morrow and M.J. Kenney (eds.), *The Teaching and Learning of Algorithms in School Mathematics*, 1998 Yearbook, National Council of Teachers of Mathematics, Reston, VA, pp. 131-140.

Kaminski, J.A., V.M. Sloutsky, and **A.F. Heckler** (2008), 'The Advantage of Abstract Examples in Learning Math', *Science*, Vol. 320/5875, pp. 454-55, http://dx.doi.org/10.1126/science.1154659.

Klein, D. (2003), "A brief history of American K-12 mathematics education in the 20th century", in J.M. Royer (ed.), *Mathematical Cognition*, Information Age Publishing, Greenwich, CT, pp. 175-225.

Levine, P. B. and **D.J. Zimmerman** (1995), "The Benefit of Additional High-School Math and Science Classes for Young Men and Women", *Journal of Business and Economic Statistics*, Vol. 13/2, pp. 137-149.

Marzano, R. (2003), *What Works in Schools: Translating Research into Action*, Association for Supervision and Curriculum Development, Alexandria, VA.

Morris, P. and **J. Williamson** (eds.) (2000), *Teacher Education in the Asia-Pacific Region: A Comparative Study,* Falmer Press, New York, NY.

Mosvold, R. (2008), "Real-Life Connections in Japan and the Netherlands : National Teaching Patterns and Cultural Beliefs", pp. 1-18, https://brage.bibsys.no/xmlui/handle/11250/185486.

Mullis, I.V.S., et al. (2012), *TIMSS 2011 Encyclopedia: Education Policy and Curriculum in Mathematics and Science, Volumes 1 and 2*, Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College, http://timssandpirls.bc.edu/timss2011/encyclopedia-timss.html.

Munson, A. (2010), "Bourbaki at seventy-five: Its influence in France and beyond", *Journal of Mathematics Education at Teachers College*, Vol. 1/2, pp. 18-21.

NCTM (ed.) (2000), *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA.

NCTM (ed.) (2014), *Principles to Actions: Ensuring Mathematical Success for All*, National Council of Teachers of Mathematics, Reston, VA.

Nelson, W.L., et al. (2008), Clinical implications of numeracy: Theory and practice, *Annals of Behavioral Medicine,* Vol. 35, pp. 261-274.

Niss, M. (1987), "Application and modelling in mathematics curricula – state and trends", *International Journal for Mathematical Education in Science and Technology*, Vol. 18/4, pp. 487-505.

OECD (2015), *Students, Computers and Learning: Making the Connection,* PISA, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264239555-en.

OECD (2013a), *Technical Report of the Survey of Adult Skills*, OECD Publishing, Paris, http://www.oecd.org/site/piaac/_Technical%20Report_17OCT13.pdf.

OECD (2013b), *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy*, PISA, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264190511-en.

OECD (2012), *Literacy, Numeracy and Problem Solving in Technology-Rich Environments*. Paris: Organisation for Economic Co-operation and Development, http://www.oecd-ilibrary.org/content/book/9789264128859-en.

OECD (2011), *Lessons from PISA for the United States*: *Strong Performers and Successful Reformers in Education*, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264096660-en.

Polya, G. (1973), *How to solve it: A new aspect of mathematical method*, Princeton University Press, Princeton, New Jersey.

Reys, R., et al. (2003), 'Assessing the Impact of" Standards"-Based Middle Grades Mathematics Curriculum Materials on Student Achievement', *Journal for Research in Mathematics Education*, pp. 74-95.

Rittle-Johnson, B., M. Schneider and **J.R. Star** (2015), "Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics", *Educational Psychology Review*, Vol. 27/4, pp. 587-597.

Rivet, A.E. and **J.S. Krajcik** (2008), "Contextualizing Instruction: Leveraging Students' Prior Knowledge and Experiences to Foster Understanding of Middle School Science", *Journal of Research in Science Teaching*, 45(1): 79-100, http://dx.doi.org/10.1002/tea.20203.

Rose, H. and **J.R. Betts** (2004), "The Effect of High School Courses on Earnings", *The Review of Economics and Statistics*, 86(2): 497-513.

Roterham, A.J. and **D.T. Willingham** (2010), "'21st-century' skills: Not new, but a worthy challenge", *American Educator*, Vol. 34/1, pp. 17-20.

Scheerens, J. and **R. Bosker** (1997), *The Foundations of Educational Effectiveness*, Elsevier, New York, NY.

Schmidt, W.H. and **A. Maier** (2009), "Opportunity to learn", in G. Sykes, B. Schneider and D.N. Plank (eds.), *Handbook of Education Policy Research*, Routledge, New York, NY.

Schmidt, W.H., C.C. McKnight, R.T. Houang, H. Wang, D.E. Wiley, L.S. Cogan, R.G. Wolfe (2001), *Why Schools Matter: A Cross-National Comparison of Curriculum and Learning*, Jossey-Bass, San Francisco, CA.

Schmidt, W.H., C.C. McKnight, G. Valverde, R.T. Houang and **D.E. Wiley** (eds.) (1997), *Many Visions, Many Aims: A Cross-National Investigation of Curricular Intentions in School Mathematics*, Springer, Berlin.

Schmidt,W., P. Zoido and **L. Cogan** (2014), "Schooling Matters: Opportunity to Learn in PISA 2012", *OECD Education Working Papers*, No. 95, OECD Publishing, Paris, http://dx.doi.org/10.1787/5k3v0hldmchl-en.

Schmitt, N. (ed) (2013), *The Oxford Handbook of Personnel Assessment and Selection,* Oxford University Press, Oxford, England.

Schoenfeld, A. (1983), Problem solving in the mathematics curriculum: A report, recommendations, and an annotated bibliography, Mathematical Association of America, Washington, DC.

Schoenfeld, A. (2002), "Making mathematics work for all children: Issues of standards, testing, and equity", *Educational Researcher*, Vol. 31/1, pp. 13-2.

Schoenfeld, A. (2004), "The math wars", *Educational Policy,* Vol. 18/1, pp. 253-286.

Stathopoulou, C., E. Gana, and **P. Chaviaris** (2012), "Curriculum, Textbooks, Mathematics Teaching: What Really Happens in the Classroom?", Quaderni di Ricerca in Didattica (Mathematics), Vol. 22/1, http://www.academia.edu/download/30869446/CIEAEM_63_Pproceedings_QRDM_Issue_22__Suppl.pdf#page=252.

Stedøy, I.M. (ed.) (2004), *Mathematics Education - The Nordic Way,* NTNU-trykk, Trondheim, Norway.

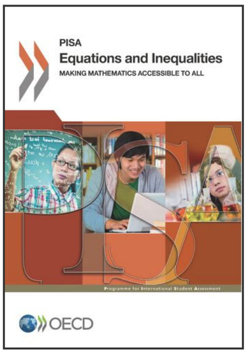
Steen, L.A. (ed.) (2001), *Mathematics and Democracy: The Case for Quantitative Literacy,* National Council on Education and the Disciplines, Princeton, NJ.

Trafton, P.R., B.J. Reys, and **D.G. Wasman** (2001), "Standards-Based Mathematics Curriculum Materials: A Phrase in Search of a Definition", *The Phi Delta Kappan*, Vol. 83/3, pp. 259-64.

Usiskin, Z. and **E. Willmore** (Eds.) (2008), *Mathematics curriculum in Pacific Rim countries — China, Japan, Korea, and Singapore*, Charlotte, NC: Information Age Publishing.

Valverde, G., L.J. Bianchi, R. Wolfe, W.H. Schmidt and **R.T. Houang** (2002), *According to the Book: Using TIMSS to Investigate the Translation of Policy Into Practice Through the World of Textbooks*, Springer, Berlin.

Yang, X. (2014), "Conception and characteristics of expert mathematics teachers in China"*, Perspektiven der Mathematikdidaktik Series*, Springer Spektrum, Wiesbaden, Germany.



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