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on International Area
Aggregates

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SOME OBSERVATIONS ON INTERNATIONAL AREA AGGREGATES

This paper deals with the construction of statistics for area or zone totals for groups of countries. It discusses various ways to construct volume and value series and reviews some of the implications for resulting indirect price indices. The paper then takes a more specific look at the Euro area and provides an empirical example concerning the aggregation of private final consumption for the 12 countries of the Euro area.

Cette étude examine la construction des statistiques pour des totaux des zones, c'est à dire, des agrégats à travers un groupe des pays. Elle discute différentes façons d'arriver à des séries en valeur et en volume et elle évalue quelques implications pour des indices de prix implicites qui pourrait en résulter. Ensuite, l'étude examine d'une manière plus détaillée la zone Euro et présente un exemple empirique qui traite l'agrégation de la consommation privée finale à travers les 12 pays de la zone Euro.

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Introduction

In January 2001, a small internal task force was formed at the OECD to discuss the implications of the introduction of the Euro into the statistical and analytical systems of the organisation. Two main issues arose in this context: (a) the presentation of national data of EMU member countries in OECD publications and databases; (b) the aggregation of national data to totals for the euro area, and in particular the treatment of historical data relating to periods prior to the introduction of the euro. The present paper deals with the second issue and provides the background to the conclusions and recommendations reached in the group's report.

This report starts out with a brief discussion of different possibilities to aggregate economic time series across countries. It examines first the aggregation of time series in current national prices and then goes on to the aggregation of volume series. This initial discussion is general in nature. More specific points relating to euro area aggregation are taken up in the following section. The note ends with an empirical example.

AGGREGATION ACROSS COUNTRIES

1. Value indices

Suppose one has a set of data for different countries ($i=1, 2, \dots, N$) that consists of observations on volumes or quantities (X_t^i), prices (P_t^i), exchange rates (ϵ_t^i), and purchasing power parities (ppp_t^i). Volume and price variables for each country are presented here as if they relate to single products, but they can easily be imagined as vectors of quantities and prices of many different products. Exchange rates express a country's currency in terms of the currency of a reference country (typically the USD for OECD purposes). Purchasing power parities (PPPs) are price relatives that express a country's set of prices relative to the set prices of a reference country or an average of other countries. Both exchange rates and PPPs constitute conversion factors from national to international currencies and prices but they command a different interpretation.

Without providing an in-depth discussion here, one notes that the main difference between exchange rates and PPPs is that the former are *price relatives of different currencies* whereas the latter constitute *price relatives of products in different countries*. In a very simple world where all products are internationally traded, and where the demand and supply for currencies is only governed by the money flows associated with imports and exports, and where competitive conditions prevail on all markets, one would expect exchange rates and PPPs to be similar. In a more complex world where many goods and services are not traded internationally and where relative interest rates and capital flows shape demand and supply for different currencies, exchange rates and PPPs differ. There is no clear-cut answer as to whether exchange rates or PPPs should be preferred in constructing area totals although some general rules exist. For the moment, the distinction is made and will find its semantic expression by calling value series that have been converted with exchange rates "expressed in an international currency" and by calling value series that have been converted with a PPP conversion factor as "expressed in international prices".

1.1. Value aggregates at current exchange rates (or current international prices)

1.1.1. Indices

At the national level, current price data typically consists of a price and a quantity component. At the international level, a second ‘price’ component enters the picture in the form of a conversion rate from the domestic to a common currency or from a domestic to an international set of prices. The implication is that there are two types of value series: one at current international prices (or exchange rates); and one at constant international prices (or exchange rates). Whether one or the other type of value series is constructed depends on the specific variable that is aggregated and on the analytical purpose of the aggregation.

To keep things simple in the construction of indices, one of the most commonly-used forms of index numbers, the Laspeyres-type, is used here. It should be noted that this is but one specific type of index number and more general formulations are possible. However, the Laspeyres-type formulation eases exposition and is frequently encountered in empirical applications with national accounts-based data.

A simple value index of a geographical zone can be defined as the aggregate value of a variable (for example private consumption) in year t relative to year t_0 . To be able to sum up values across different countries, they have to be expressed in a common unit and in the first case we use the current exchange rate for this purpose. The value index (labelled V^{ε_t} for ‘value’ and to indicate that it is based on current exchange rates ε_t) takes the following form:

$$V^{\varepsilon_t} = \frac{\sum_i X_t^i \cdot P_t^i / \varepsilon_t^i}{\sum_i X_{t_0}^i \cdot P_{t_0}^i / \varepsilon_{t_0}^i} \quad (1)$$

Thus, V^{ε_t} compares the value of say, private consumption in year t to the value of private consumption in year t_0 , and in each period values are based on the exchange rates prevailing in these periods. V^{ε_t} is therefore called a **value index at current exchange rates**. It expresses national current prices in current USD.

Instead of converting national data by way of the exchange rate, one could do so with purchasing power parities. This yields a different value index, V^{PPP_t} :

$$V^{PPP_t} = \frac{\sum_i X_t^i \cdot P_t^i / PPP_t^i}{\sum_i X_{t_0}^i \cdot P_{t_0}^i / PPP_{t_0}^i} \quad (2)$$

V^{PPP_t} compares the value of say, private consumption in year t to the value of private consumption in year t_0 , and in each period values are based on the PPPs prevailing in these periods. V^{PPP_t} is therefore called a **value index at current international prices**. It expresses the values for comparison at international prices of the respective periods. International prices are either the prices of a reference country or an average of countries. To illustrate by way of a simplified example¹, assume that the reference country is the United States and that PPPs reflect the relative price level of a country i with respect to the United States:

$PPP_t^i \equiv P_t^{USA} / P_t^i$. Inserting this relationship into (2) yields the expression in (3) that shows a value index for country i , expressed in current prices of the United States.

$$V^{PPP_t} = \frac{\sum_i X_t^i \cdot P_t^{USA}}{\sum_i X_{t_0}^i \cdot P_{t_0}^{USA}} \quad (3)$$

Note that the expressions above can be written in a somewhat different form, namely as weighted averages of each country's current price index, expressed at current exchange rates or current international prices. Thus, the value index at current exchange rates can also be written as:

$$V^{\varepsilon_t} = \sum_i \tau_{t_0}^i \cdot \left[\frac{X_t^i \cdot P_t^i / \varepsilon_t^i}{X_{t_0}^i \cdot P_{t_0}^i / \varepsilon_{t_0}^i} \right] \quad (4)$$

As, for example described in Hong and Beilby-Orrin (1999) or ECB (2000), the index in this expression is a weighted average of the indices of the current nominal variable in each country, expressed at current exchange rates. The exchange rate effect captures the shifts of the bundle of currencies against the international currency, for example the USD. The weights reflect each country's share in the aggregate in

period t_0 and are expressed by $\tau_{t_0}^i \equiv \frac{X_{t_0}^i P_{t_0}^i / \varepsilon_{t_0}^i}{\sum_j X_{t_0}^j P_{t_0}^j / \varepsilon_{t_0}^j}$.

Now use again the PPP example from expression (3) and translate it into the form equivalent to expression (4). One gets a relationship that shows that a value index of a geographic area at current PPPs reflects a weighted average of volume changes in individual countries, multiplied by the domestic price level changes in the reference country – the United States in the example at hand. This is a useful way of interpreting the consequences of using current PPPs for conversion of value series.

$$V^{PPP_t} = \sum_i \tau_{t_0}^i \cdot \left[\frac{X_t^i \cdot P_t^{USA}}{X_{t_0}^i \cdot P_{t_0}^{USA}} \right] \quad (5)$$

1.1.2. Levels

The above presentation started from a definition of indices, or rates of change of the variable under consideration. The implication of this formulation is that there exist *levels* of zone aggregates at current exchange rates (or PPPs): for example, $Z_t \equiv \sum_i X_t^i P_t^i / \varepsilon_t^i$ is the level of the value aggregate at current exchange rates. It is straightforward to see that the ratio of two levels for different periods brings one back to the index number defined above: $Z_t / Z_{t_0} = V^{\varepsilon_t}$.

1.2. Value aggregates at constant exchange rates (or constant international prices)

1.2.1. Fixed base year

The value indices for a geographical area outlined above share the common property that they reflect the movement of exchange rates or international prices for every year under observation. An alternative way of constructing a value index for a geographical zone is to compare values between different periods at constant exchange rates or at constant international prices. In other words, conversion rates are held fixed when comparing two periods in time. This means that expression (1) becomes:

$$V^{\varepsilon_0} \equiv \frac{\sum_i X_t^i P_t^i / \varepsilon_{t_0}^i}{\sum_i X_{t_0}^i P_{t_0}^i / \varepsilon_{t_0}^i} \quad (6)$$

Thus, V^{ε_0} compares the value of say, private consumption in year t to the value of private consumption in year t_0 , and in each period values are based on the exchange rates prevailing in period t_0 . V^{ε_0} is therefore called a **value index at constant exchange rates of period t_0** . It expresses national current prices in USD of period t_0 . The parallel **value index at constant international prices of period t_0** can be constructed in an isomorphic way – by using $ppp_{t_0}^i$ instead of $\varepsilon_{t_0}^i$ in expression (6). It expresses national time series in constant international prices of a reference country such as the United States.

To see an important implication of constructing value indices at constant international prices, it is useful to re-formulate expression (6) as a weighted average of country-specific value indices, just as (4) was a re-statement of (1). The result is shown below and one sees that V^{ε_0} constitutes a weighted average of the year-to-year change of each country's value series, expressed in national currencies. This is the direct consequence of keeping exchange rates or PPPs constant in the value index: exchange rate changes or relative price changes are not reflected in this value measure of a geographical zone.

$$V^{\varepsilon_0} = \sum_i \tau_{t_0}^i \cdot \left[\frac{X_t^i P_t^i}{X_{t_0}^i P_{t_0}^i} \right] \quad (7)$$

1.2.2. Moving base year

Although expressions (6) or (7) are frequently used to eliminate the effects of exchange rate moves from value aggregates, there is nothing that obliges one to pick a fixed period t_0 for this purpose. An alternative way is to use the period previous to the comparison period t as the year (or month) at which exchange rates or PPPs are held constant for a particular comparison. Thus, the value index of the zone aggregate could be defined as in (8):

$$V^{\varepsilon_{t-1}} \equiv \frac{\sum_i X_t^i P_t^i / \varepsilon_{t-1}^i}{\sum_i X_{t-1}^i P_{t-1}^i / \varepsilon_{t-1}^i} \quad (8)$$

Again, it can be transformed into a weighted average of country-specific indices and expressed as:

$$V^{\varepsilon_{t-1}} = \sum_i \tau_{t-1}^i \cdot \left[\frac{X_t^i P_t^i}{X_{t-1}^i P_{t-1}^i} \right] \quad (9)$$

In formula (9), aggregation weights are defined as each country's share in the area total, measured at current exchange rates or current international prices of year $t-1$. Thus, although the country-specific indices are not influenced by exchange rate shifts, weights are updated to reflect international price or change rate changes up to the period $t-1$. Thus, (9) is a chain value index at exchange rates of period $t-1$ or at international prices of period $t-1$ if computations are based on PPPs as conversion factors. This chained value index has, for example, been used by OECD (2001) in its euro area data set.

1.2.3. Levels

As before, there exist level formulations of zone aggregates that are consistent with the index number expressions (6) and (8). In the first case, these are value levels at constant exchange rates (or PPPs).

$Z_t^{\varepsilon_{t_0}} \equiv \sum_i X_t^i P_t^i / \varepsilon_{t_0}^i$ is the level of the value aggregate at constant exchange rates, and

$Z_t^{ppp_{t_0}} \equiv \sum_i X_t^i P_t^i / ppp_{t_0}$ the level of the value aggregate at PPPs of period t_0 . Again, the ratio of two

levels for different periods brings one back to the index number defined above: $Z_t^{\varepsilon_{t_0}} / Z_{t_0}^{\varepsilon_{t_0}} = V^{\varepsilon_{t_0}}$.

In the second case, there exist value levels at exchange rates of the previous period, defined as

$Z_t^{\varepsilon_{t-1}} \equiv \sum_i X_t^i P_t^i / \varepsilon_{t-1}^i$, and the ratio $Z_t^{\varepsilon_{t-1}} / Z_{t-1}^{\varepsilon_{t-1}}$ equals the value index $V^{\varepsilon_{t-1}}$.

2. Volume indices

2.1. What is meant by 'constant prices'?

Before tackling the issue of volume aggregation, it is useful to explicitly state what is understood by 'constant price', 'volume', or 'quantity' data. There are at least two different meanings. First, 'constant prices' stand for aggregates of quantities that are weighted with a price vector of a fixed base year. This is, for example the case for national accounts data in some OECD countries that are based on 1995 price weights and current quantities (Laspeyres-type quantity index). In this case, levels of different components of national accounts can be added and compared within a given period as well as between different periods.

Not all countries' accounts are based on such fixed-weight measures, however. Increasingly, countries' national accounts use chained indices (with a price base that corresponds to the preceding year or with a price base that is an average of the reference and the comparison period). In this case, the information available from national accounts is one about volume indices between adjacent periods. These volume indices can be transformed into levels of 'constant price' data by setting the volume index to equal one in a given reference year and by multiplying through with the current price value of the same year. For example, the United States National Income and Product Accounts produce chained volume and price indices². By normalising these volume indices to the (arbitrarily chosen) reference year 1996, one obtains what is called 'chained 1996 dollars' in the United States' National Accounts. A similar procedure is used in OECD's National Accounts where volume changes between adjacent years are expressed as 'constant price levels', whether these volume changes come from chained, or fixed weight indices used in individual

countries' national accounts. While convenient in some ways, this transformation may lead to non-additivity in the data: the sum of the constant price series of components does not in general match the constant price level of the aggregate. For example the sum of capital formation, household and government consumption and exports minus imports expressed in such 'constant price' data does not generally equal the constant price level of GDP. Full additivity of components over time only prevails when there is a single price base, but this brings one back to the case of a fixed-weight Laspeyres quantity index. At the same time, there are few instances where additivity of constant price level data are actually required – in the vast majority of cases, analysts are interested in aggregate *growth rates* or *indices*, not in aggregate *levels*.

The same issue arises at the international level. Constant price data from individual countries are only additive and comparable between different periods if they are expressed in constant national and constant international prices of a fixed year, i.e., if they are aggregated by way of a fixed base Laspeyres index. Additivity still holds across countries *for any given period* (but not between periods) if a chain Laspeyres index is used. In this case, each year's constant price data is expressed in national and international prices of the year preceding the current period. The following section describes these two aggregation methods in greater detail and applies them to the euro area. It is important to note, however, that they are by no means the only conceivable ways of forming indices of volumes or prices for geographical zones.

2.2. Volume aggregates at constant exchange rates (or constant international prices)

2.2.1. Fixed base year

The first option to form a volume index of a zone aggregate is to compare quantities of two periods valued at prices of a base period. In the case of an international aggregation, this Laspeyres-type formulation implies that both domestic and international prices are held constant. Such a volume index reads as:

$$Q^{\varepsilon_{t_0}} \equiv \frac{\sum_i X_t^i P_{t_0}^i / \varepsilon_{t_0}^i}{\sum_i X_{t_0}^i P_{t_0}^i / \varepsilon_{t_0}^i} \quad (10)$$

Thus, $Q^{\varepsilon_{t_0}}$ compares the volume of say, private consumption in year t to the volume of private consumption in year t_0 , and in each period volumes are based on the domestic prices and exchange rates prevailing in period t_0 . $Q^{\varepsilon_{t_0}}$ is therefore called a **volume index at constant exchange rates of period t_0** . It expresses national volumes in USD of period t_0 . The parallel **volume index at constant international prices of period t_0** can be constructed in an isomorphic way – by using $ppp_{t_0}^i$ instead of $\varepsilon_{t_0}^i$ in expression (10). It expresses national time series in constant international prices of a reference country such as the United States.

We can again conveniently express the index number formulation in (10) as a weighted average of each country's volume index. Weights correspond to the share that each country represented in the total at the time of the base year. The index is a direct comparison between years with a fixed base period. Alternatively, aggregation can be carried out by way of a chain index, as explained in the following section.

$$Q^{\varepsilon_{t_0}} = \sum_i \tau_{t_0}^i \cdot \left[\frac{X_t^i}{X_{t_0}^i} \right] \quad (11)$$

2.2.2. Moving base year

The second option to form a volume index uses a chain Laspeyres formula. There, the base period moves with the current period. This implies that $t_0 = t-1$, so that the aggregate volume index of period t over period $t-1$, is now given by:

$$Q^{\varepsilon_{t-1}} \equiv \frac{\sum_i X_t^i P_{t-1}^i / \varepsilon_{t-1}^i}{\sum_i X_{t-1}^i P_{t-1}^i / \varepsilon_{t-1}^i} \quad (12)$$

Thus, $Q^{\varepsilon_{t-1}}$ compares the volume of say, private consumption in year t to the volume of private consumption in year $t-1$, and in each period volumes are based on the domestic prices and exchange rates prevailing in the previous period $t-1$. $Q^{\varepsilon_{t-1}}$ is therefore called a **volume index at constant exchange rates of period $t-1$** . It expresses national volumes in USD of period $t-1$. The parallel **volume index at constant international prices of period $t-1$** can be constructed in an isomorphic way – by using ppp_{t-1}^i instead of ε_{t-1}^i in expression (12). It expresses national volume series in constant international prices of a reference country such as the United States.

For completeness, the volume index in (12) is once again transformed into a weighted average of country-specific volume indices. Unlike the fixed base year case shown in (11), one gets a chained comparison between adjacent years, and with country weights defined as each country's share in the total value at current exchange rates or international prices of period $t-1$. One notes that the aggregation of volume indices, as shown in (13) uses exactly the same current price weights as the aggregation procedure for the value index $V^{\varepsilon_{t-1}}$ in (9).

$$Q^{\varepsilon_{t-1}} = \sum_i \tau_{t-1}^i \cdot \left[\frac{X_t^i}{X_{t-1}^i} \right] \quad (13)$$

2.2.3. Levels

With a Laspeyres-type approach, the *level* of the volume aggregate at constant exchange rates of the fixed base year t_0 exists and is either obtained directly by adding up country-specific volumes at constant exchange rates ($\sum_i X_t^i P_{t_0}^i / \varepsilon_{t_0}^i$), or by applying the volume index $Q^{\varepsilon_{t_0}}$ to the base-year level

$$\sum_i X_{t_0}^i P_{t_0}^i / \varepsilon_{t_0}^i .$$

When volume indices are defined with a moving base year, levels of the volume aggregate at constant exchange rates or international prices exist for every period t : $\sum_i X_t^i P_{t-1}^i / \varepsilon_{t-1}^i$. However, these levels cannot be directly compared between non-adjacent years because each aggregate is valued at exchange rates or international prices of the preceding year. Level series of the volume aggregate can be constructed by defining an arbitrary reference year where the aggregate level of the value of the variable at current international prices is set to equal the volume level of the aggregate at constant international prices³. Volume levels for other periods are then obtained by applying the volume index (12) to the level of the reference year. This is the set-up used in OECD (2001) for the aggregation of volume indices across countries of the euro area and for the computation of level series.

2.3. Implicit price indices

With sets of value and volume indices defined, it is now possible to derive implicit price indices. Notice that one could also first define aggregate price indices directly and then construct implicit volume indices. Generally, the two methods will not have the same outcome. Computation of implicit price indices always relies on a basic property of index numbers: the product of price and volume indices should equal the value index⁴. Thus, the implicit price index is obtained by dividing the value index by the volume index. At the

domestic level, an implicit price index between periods t and t_0 , $\frac{P_t^i}{P_{t_0}^i}$ is therefore defined as

$\frac{P_t^i}{P_{t_0}^i} = \frac{X_t^i P_t^i}{X_{t_0}^i P_{t_0}^i} \bigg/ \frac{X_t^i}{X_{t_0}^i}$. At the international level, one has to keep in mind that there are several types of

value and volume indices, and consequently there will also be several types of implicit price indices.

2.3.1. Current exchange rates (or current international prices)

The first implicit deflator is derived as the ratio of the value index at current exchange rates (or current international prices) as defined in expression (1) over the volume index at constant exchange rates of period t_0 (or constant PPPs of period t_0), as defined in expression (10). The result is a **price index based on current exchange rates (or current international prices)** – called P^{ε_t} or P^{PPP_t} in expression (14) that has the form of a Paasche index and constitutes a harmonic mean of price ratios between the current and the base year. When the price index is exchange-rate based, it reflects the shifts in exchange rates and devaluation or appreciation of individual currencies can influence the price aggregate even if domestic prices are unchanged. When the price index is PPP-based, it reflects domestic inflation of the reference country or average inflation of the reference countries.

$$P^{\varepsilon_t} \equiv \frac{V^{\varepsilon_t}}{Q^{\varepsilon_{t_0}}}, \quad P^{PPP_t} \equiv \frac{V^{PPP_t}}{Q^{PPP_{t_0}}} \quad (14)$$

Given its dependence on exchange rates or on price movements in a reference country, this aggregate price index is of limited suitability to make statements about domestic price developments in the geographical zone that it represents. For example, this price index would not normally qualify as a useful indicator of price movements in the euro area. This has also been pointed out by Lequiller (1998) and the European Central Bank⁵ who advise against this deflator to assess inflationary movements in the euro area. This does not render the price index meaningless for other purposes. For example, it could be applied to measure price movements of exports or imports on a current dollar basis, given value and volume indices of exports

and imports. The current dollar basis is useful when commodities are traded on a dollar basis in international markets.

2.3.2. *Constant exchange rates (or constant international prices) of a fixed base year*

The second implicit deflator is derived as the ratio of the value index at constant exchange rates (or constant international prices) of a fixed base year (equation (6)) over the volume index at constant exchange rates (or constant international prices) of a fixed base year (equation (10)). The result is a price index for the geographical aggregate, that has again the form of a Paasche index and constitutes a harmonic mean of price ratios between the current and the base year. The price index only reflects changes in domestic inflation, unadjusted for exchange rate effects (or effects of shifts in relative prices) vis-à-vis the reference country. This index is labelled ***price index based on constant exchange rates (or constant international prices) of a fixed base year***.

$$P^{\varepsilon_{t_0}} \equiv \frac{V^{\varepsilon_{t_0}}}{Q^{\varepsilon_{t_0}}}; P^{PPP_{t_0}} \equiv \frac{V^{PPP_{t_0}}}{Q^{PPP_{t_0}}} \quad (15)$$

Given its independence from international prices, this aggregate price index is suitable for statements about *domestic* price developments in a geographical zone. But note that in cases where the geographical zone comprises some, or even only one, high inflation country, the large domestic price changes in this economy are fully reflected in this price index and may dominate the measure to the point of rendering it meaningless.

2.3.3. *Constant exchange rates (or constant international prices) of period t-1*

A third implicit deflator is derived as the ratio of the value index at constant exchange rates (or constant international prices) of period *t-1* (equation (8)) over the volume index at constant exchange rates (or constant international prices) of period *t-1* (equation (13)). The result is a ***price index based on constant exchange rates (or constant international prices) of period t-1*** with the form of a Paasche index and constitutes a harmonic mean of price ratios between two adjacent years. As before, this price index only reflects changes in *domestic* inflation, unadjusted for exchange rate effects (or effects of shifts in relative prices) vis-à-vis the reference country. However, exchange rate movements, though absent from index components, do influence the evolution of the weights over time. This matters little for comparatively homogenous groups of countries such as EU15 but can play a larger role in more heterogenous country groupings. Heterogeneity can for example arise when one country exhibits large domestic inflation, accompanied by rapid devaluation of the currency.

$$P^{\varepsilon_{t-1}} \equiv \frac{V^{\varepsilon_{t-1}}}{Q^{\varepsilon_{t-1}}}; P^{PPP_{t-1}} \equiv \frac{V^{PPP_{t-1}}}{Q^{PPP_{t-1}}} \quad (16)$$

This chained index constitutes another option for quantifying the time profile of domestic inflation in a geographic aggregate.

2.4. *Explicit price indices*

Whereas the previous section described several options to obtain implicit price indices for zone totals, the following paragraphs consider the case of direct or explicit price indices. Implicit price indices are

typically computed in the context of national accounts variables, and they often present themselves as Paasche-type indices (when volume indices are constructed with a Laspeyres-type index number formula which is the case for national accounts in most OECD countries). Explicit price indices are a different way to gauge price developments, and the most prominent explicit price index is the consumer price (CPI) index. Other examples are producer price indices or wholesale price indices. Just as there are several possibilities to construct value or (direct) volume indices for zone aggregates, there are several ways to go about aggregating explicit price indices. Conceptually, most of the above discussion on aggregation carries over directly to explicit price indices. An *explicit price index based on current exchange rates* can thus be defined as:

$$\tilde{P}^{\varepsilon_t} \equiv \frac{\sum_i X_{t_0}^i P_t^i / \varepsilon_t^i}{\sum_i X_{t_0}^i P_{t_0}^i / \varepsilon_{t_0}^i} = \sum_i \tau_{t_0}^i \cdot \left[\frac{P_t^i}{P_{t_0}^i} / \frac{\varepsilon_t^i}{\varepsilon_{t_0}^i} \right] \quad (17)$$

Similar to the implicit price index based on current exchange rates, this index reflects both the domestic price changes and exchange rate movements. As shown on the right hand side of (17), the index can also be presented as a weighted average of each country's price index, converted by way of current exchange rates (or current PPPs). Weights are made up of each country's share in total expenditure in period t_0 .

Note that the explicit price index, when based on current international prices (PPPs), simply reduces to the price index of the reference currency. This can be seen by inserting the simplified relationship $PPP_t^i = P_t^i / P_t^{USA}$ with the United States as the reference country into relationship (17). The result is the

$$\text{domestic price index of the United States: } \tilde{P}^{PPP_t} = \sum_i \tau_{t_0}^i \left[\frac{P_t^i}{P_{t_0}^i} / \frac{PPP_t^i}{PPP_{t_0}^i} \right] = \sum_i \tau_{t_0}^i \left[\frac{P_t^{USA}}{P_{t_0}^{USA}} \right] = \frac{P_t^{USA}}{P_{t_0}^{USA}}.$$

An *explicit price index based on constant exchange rates of period t_0* would be given by:

$$\tilde{P}^{\varepsilon_{t_0}} \equiv \frac{\sum_i X_{t_0}^i P_t^i / \varepsilon_{t_0}^i}{\sum_i X_{t_0}^i P_{t_0}^i / \varepsilon_{t_0}^i} = \sum_i \tau_{t_0}^i \cdot \left[\frac{P_t^i}{P_{t_0}^i} \right] \quad (18)$$

Like the implicit price index based on constant exchange rates of period t_0 , this index reflects only *domestic* price changes. This is apparent from the presentation of the index as a weighted average of each country's domestic price index, with base-period expenditure shares as weights. Unlike the explicit price index based on current international prices (which simply yields the domestic price index of the reference country), the explicit price index based on constant international prices (PPPs) is a meaningful measure. It is isomorph to (18), and represents a weighted average of domestic inflation. However, based-period weights $\tau_{t_0}^i$ are based on PPPs of period t_0 , rather than on exchange rates of t_0 .

Thirdly, an *explicit price index based on constant exchange rates (or constant international prices) of period $t-1$* can be defined as:

$$\tilde{P}^{\varepsilon_{t-1}} \equiv \frac{\sum_i X_{t-1}^i P_t^i / \varepsilon_{t-1}^i}{\sum_i X_{t-1}^i P_{t-1}^i / \varepsilon_{t-1}^i} = \sum_i \tau_{t-1}^i \cdot \left[\frac{P_t^i}{P_{t-1}^i} \right] \quad (19)$$

This index is suitable to trace domestic inflation in a geographical zone and benefits from the added advantage of country-specific weights that are periodically updated.

2.5. Aggregation of volume indices when there is no value or price information

A practical problem arises when it comes to forming zone aggregates for variables that only exist as volume series, and without corresponding information on prices or values. One example is the index of industrial production. Typically, indices of industrial production are fixed-weight quantity indices of the Laspeyres type, with price structures within a country held fixed in a base year. Although base years may vary between countries, indices can be normalised to a common reference year. Aggregation then proceeds by forming weighted averages of indices of production in the form of direct comparisons between the current and the reference year. In line with the construction of volume indices at constant exchange rates ($Q^{\mathcal{E}_{t_0}}$) or constant international prices ($Q^{PPP_{t_0}}$), weights correspond to base year shares of each country in total production or value-added, converted with exchange rates or PPPs of period t_0 . Alternatively, and in analogy to the volume index at constant exchange rates (or constant international prices) of period $t-1$ ($Q^{\mathcal{E}_{t-1}}$ and $Q^{PPP_{t-1}}$), weights could be periodically updated to reflect exchange rate or PPP movements. Weights based on value-added are often preferred to those based on production (or gross output) because of data availability and because of the possibility of double-counting in the case of gross output when there are international intra-industry trade flows. Thus, zone aggregates of indices of production (IP_t/IP_{t_0}) are computed on the basis of value-added shares at exchange rates (or PPPs) of a fixed base-year.

$$IP^{\mathcal{E}_{t_0}} = \sum_i \tau_{t_0}^i \cdot \left[\frac{IP_t^i}{IP_{t_0}^i} \right]; \quad \tau_{t_0}^i = \frac{VA_{t_0}^i / \mathcal{E}_{t_0}^i}{\sum_j VA_{t_0}^j / \mathcal{E}_{t_0}^j} \quad (20)$$

EURO AREA AGGREGATION

It is now possible to take a closer look at euro area aggregation. This section of the document uses the more general principles and terminology discussed above and applies them to constructing zone totals for the EMU countries.

Definitions and notation

$EU11$	The set of 11 countries that participated in the European Monetary Union as of 1 st January 1999
$EU12$	EU11 countries plus Greece who joined the European Monetary Union as of 1 st January 2001
<i>Euro area</i>	The set of countries that participate in the European Monetary Union at the time of the preparation of the OECD data set. Currently (year 2001, this series would be identical to EU12)
$Z_t^i = X_t^i P_t^i$	Value of variable Z in country i and period t at current domestic prices, expressed in (pre-euro) domestic currency
Z_t^{eu12} (Z_t^{eu11})	Value of variable Z for the EU12 (EU11, euro) area in period t at current exchange rates (or current international prices). The variable is expressed in USD.
$Z_t^{\varepsilon_{t_0}, eu12}$ ($Z_t^{\varepsilon_{t_0}, eu11}$)	Value of variable Z in the $EU12$ ($EU11$) in period t at current domestic prices and constant international prices (USD of a fixed reference year)
$X_t^i / X_{t_0}^i$	Volume or quantity index of variable Z in country i between period t and period t_0 .
$P_t^i / P_{t_0}^i$	Domestic price index of variable Z in country i between periods t period t_0
ε_t^i	Exchange rate, expressed as local currency of country i per USD in period t
ε_t^{EUR}	Exchange rate of EUR per USD in period t (1999 and onwards)
e^i	Irrevocable exchange rate between currencies in countries of the euro area and the EUR

Value aggregates

Values at current exchange rates (or current international prices)

For the *EU 11* countries, the value aggregate at current exchange rates is given in expression (21). For the years from 1999 onwards, the bilateral exchange rate between each currency and the US dollar is replaced by the irrevocable conversion rate between each currency and the euro, multiplied by the current euro/dollar exchange rate.

$$Z_t^{eu11} = \begin{cases} \sum_{i \in EU11} Z_t^i / \varepsilon_t^i & \text{for } t < 1999 \\ \sum_{i \in EU11} Z_t^i / (e^i \cdot \varepsilon_t^{EUR}) & \text{for } t \geq 1999 \end{cases} \quad (21)$$

By relating the value aggregate at current exchange rates in period t to its counterpart in period t_0 , one obtains the *EU11 value index at current exchange rates*, as a specific case of the index V^{ε_t} defined in expression (1). Three situations are possible, depending on whether the current period t relates to years prior to 1999 (first expression on the right hand side of (22)) or to years post 1999 (second and third expressions on the right hand side of (22)):

$$V^{\varepsilon_t, eu11} = \frac{Z_t^{eu11}}{Z_{t-1}^{eu11}} = \begin{cases} \sum_{i \in EU11} \tau_{t_0}^i \left[\frac{Z_t^i}{Z_{t_0}^i} \right] / \left[\frac{\varepsilon_t^i}{\varepsilon_{t_0}^i} \right] & \text{for } t < 1999 \\ \sum_{i \in EU11} \tau_{t_0}^i \left[\frac{Z_t^i}{Z_{t_0}^i} \right] / \left[\frac{e^i \varepsilon_t^{EUR}}{\varepsilon_{t_0}^i} \right] & \text{for } t \geq 1999; t_0 < t \\ \sum_{i \in EU11} \tau_{t_0}^i \left[\frac{Z_t^i}{Z_{t_0}^i} \right] / \left[\frac{\varepsilon_t^{EUR}}{\varepsilon_{t_0}^{EUR}} \right] & \text{for } t, t_0 > 1999; t_0 < t \end{cases} \quad (22)$$

When exchange rates are replaced by PPPs, one obtains the EU value index at current international prices, in line with the index V^{PPP_t} defined earlier on. Note however, that PPP conversions are, in principle, unaffected by the introduction of the EUR. No euro-specific formulation such as in (21) is necessary.

The *EU12* aggregation is similar but has to take into account that, during the years 1999 and 2000, Greece was not yet part of the EMU. Considering this fact, aggregation of the value of the euro area variable at current exchange rates is given by:

$$Z_t^{eu12} = \begin{cases} \sum_{i \in EU12} Z_t^i / \varepsilon_t^i & \text{for } t < 1999 \\ \sum_{i \in EU11} Z_t^i / (e^i \cdot \varepsilon_t^{EUR}) + Z_t^{Drachme} / \varepsilon_t^{Drachme} & \text{for } t \geq 1999, 2000 \\ \sum_{i \in EU12} Z_t^i / (e^i \cdot \varepsilon_t^{EUR}) & \text{for } t \geq 2001 \end{cases} \quad (23)$$

Values at constant exchange rates (or constant international prices) of period t_0

This aggregate is given by expression (24). Two cases apply, depending on whether the year at which the exchange rate is held fixed is prior to 1999, or not. In the first case, all observations are computed according to the upper line on the right hand side of (24). If the exchange rate is held fixed in a year from 1999 onwards, the formula in the second line applies. Fixity of the exchange rate now applies to the euro/dollar conversion rate for a particular year t_0 . The **EU 12** case follows in direct analogy and is not specifically spelled out here.

$$Z_t^{\varepsilon_{t_0}, eu11} = \begin{cases} \sum_{i \in EU11} Z_t^i / \varepsilon_{t_0}^i & \text{for all } t \text{ if } t_0 < 1999 \\ \sum_{i \in EU11} Z_t^i / (e^i \cdot \varepsilon_{t_0}^{EUR}) & \text{for all } t \text{ if } t_0 \geq 1999 \end{cases} \quad (24)$$

The **EU11 value index at constant exchange rates of period t_0** is then computed as in expression

$$V^{\varepsilon_{t_0}, eu11} = \frac{Z_t^{\varepsilon_{t_0}, eu11}}{Z_{t_0}^{\varepsilon_{t_0}, eu11}} = \frac{\sum_i Z_t^i / \varepsilon_{t_0}^i}{\sum_i Z_{t_0}^i / \varepsilon_{t_0}^i} = \sum_i \tau_{t_0}^i \cdot \left[\frac{Z_t^i}{Z_{t_0}^i} \right] \quad (25)$$

This term resembles expression (22) but exchange rate effects are excluded from the rates of change of country variables at current prices. Note, however, that weights *do* depend on the exchange rate or conversion factor of the year t_0 .

Values at constant exchange rates (or constant international prices) of period $t-1$

It is again possible to express the value of the euro aggregate at constant exchange rates (or at constant prices) of period $t-1$. This aggregate is given by expression (26). Two cases apply, depending on whether the year at which the exchange rate is held fixed is prior to 1999, or not. In the first case, all observations are computed according to the upper line on the right hand side of (24). If the exchange rate is held fixed in a year from 1999 onwards, the formula in the second line applies. Fixity of the exchange rate now applies to the euro/dollar conversion rate for a particular year t_0 . The **EU 12** case follows in direct analogy and is not specifically spelled out here.

$$Z_t^{\varepsilon_{t-1}, eu11} = \begin{cases} \sum_{i \in EU11} Z_t^i / \varepsilon_{t-1}^i & \text{for all } t \text{ if } t-1 < 1999 \\ \sum_{i \in EU11} Z_t^i / (e^i \cdot \varepsilon_{t-1}^{EUR}) & \text{for all } t \text{ if } t-1 \geq 1999 \end{cases} \quad (26)$$

With a moving base year, the aggregate level of current price data, expressed in exchange rates of period $t-1$, every year t , it has to be compared with the previous year's value aggregate, expressed at constant exchange rates of the same period. This gives rise to the following chain index:

$$V^{\varepsilon_{t-1}} = \frac{\bar{Z}_t^{EU11}}{Z_{t-1}^{EU11}} = \sum_{i \in EU11} \tau_{t-1}^i \cdot \left[\frac{Z_t^i}{Z_{t-1}^i} \right] \quad (27)$$

This chained aggregate value index which holds fixed exchange rate changes from one year to the next has been used by OECD (2001) in its euro area data set. In formula (27), aggregation weights are defined as

$\tau_{t-1}^i \equiv \frac{Z_{t-1}^i / \varepsilon_{t-1}^i}{\sum_j Z_{t-1}^j / \varepsilon_{t-1}^j}$ for the years up to and including 1999. For subsequent periods, the bilateral exchange

rate between domestic currencies and the US dollar (ε_t^i) is replaced by the irrevocable conversion rate between the national currency and the EUR multiplied by the exchange rate between the EUR and the USD: $\varepsilon_t^i = e^i \cdot \varepsilon_t^{Euro}$.

EMPIRICAL EXAMPLE

Consider an empirical example. From the national accounts, the following information be available: (a) private consumption expenditure for three countries (France, Italy and Germany) at current prices in national currencies; (b) private consumption expenditure at constant prices; (c) exchange rates vis-à-vis the USD and irrevocable conversion rates vis-à-vis the EUR. Four years are considered: 1997-2000. This covers two years prior and two year post the introduction of the EUR on Jan 1st 1999. Source data is shown in the table below.

Annex Table 1

Source data on private consumption expenditure

		Year			
Country		1997	1998	1999	2000
Current prices, national currency	France (FRF)	4510.3	4698.0	4840.4	5015.0
	Germany (DEM)	2112.3	2177.9	2241.1	2309.1
	Italy (ITL)	1167.8	1219.5	1267.2	1322.4
Constant prices reference year 1997, national currency	France	4510.3	4672.3	4795.5	4909.2
	Germany	2112.3	2155.1	2210.7	2245.2
	Italy	1167.8	1194.7	1215.4	1237.7
Exchange rate with respect to USD	FRF	5.43	5.90	6.99	6.05
	DEM	1.61	1.76	2.08	1.81
	ITL	1.20	1.74	2.06	1.79
	EUR	-	-	1.07	0.92
Bilateral exchange rate per EUR	FRF	-	-	6.56	6.56
	DEM	-	-	1.96	1.96
	ITL	-	-	1.94	1.94

In a first step, three types value aggregates are calculated: (A) at current USD; (B) at constant USD of 1997; (C) at constant USD of the previous year. In case (A), current price expenditure levels are simply divided by the current exchange rates; in case (B), the 1997 exchange rate is used throughout. In case (C), each year's current price expenditure is converted to a common currency by multiplying through with the previous year's exchange rate. Note that presentation of data in USDs of the previous year implies non-comparability between years: the 2000 area total of 2465.5 USD must not be directly compared with the 1999 aggregate of 2824.6 USD: the units of the former are 1999 USDs, the units of the latter are 1998 USDs. However, the 2000 area aggregate, expressed in 1999 USDs, can be compared to the 1999 area aggregate expressed in 1999 USD. Likewise, the 1999 aggregate in 1998 USDs can be compared to the

1998 aggregate expressed in 1998 USDs and so forth. These comparisons give rise to the year-to-year rates of change shown under area total (C).

Annex Table 2:

Current-price private consumption expenditure levels

		Year			
Country		1997	1998	1999	2000
Values at current USD	France	829.9	796.9	692.3	828.3
	Germany	1309.6	1237.7	1075.0	1279.0
	Italy	975.9	702.4	614.0	739.9
	Area total (A)	3115.4	2736.9	2381.2	2847.2
	1997=100	100.0	87.9	76.4	91.4
Values at constant USD of 1997	France	829.9	864.4	890.6	922.7
	Germany	1309.6	1350.3	1389.5	1431.6
	Italy	975.9	1019.1	1059.0	1105.1
	Area total (B)	3115.4	3233.7	3339.1	3459.3
	1997=100	100.0	103.8	107.2	111.0
Values at constant USD of the previous year	France	-	864.4	821.1	717.2
	Germany	-	1350.3	1273.6	1107.6
	Italy	-	1019.1	729.9	640.7
	Area total (C)		3233.7	2824.6	2465.5
	year t-1 = 100		103.8	103.2	103.5
	1997=100	100.0	103.8	107.1	110.9

Rates of change of area totals can also be computed as weighted averages of component indices. This is shown in Annex Table 3. Obviously, the resulting indices for the three area totals are identical to the ones derived in Annex Table 2. Results for area totals (B) and (C) turn out to be very similar. This arises from the fact of relative stability of bilateral exchange rates of the three countries over the period 1997-99. Large differences arise, however, in comparison with the area aggregate expressed at current exchange rates. This reflects the shift of the three European countries' currencies against the USD.

Annex Table 3:

Current price private consumption expenditure: rates of change

		Year			
Country		1997	1998	1999	2000
Country shares in values at current exchange rates	France	0.27	0.29	0.29	0.29
	Germany	0.42	0.45	0.45	0.45
	Italy	0.31	0.26	0.26	0.26
	Total	1.00	1.00	1.00	1.00
Value index of private consumption at current exchange rates (year t-1 = 100)	France	-	96.0	86.9	119.6
	Germany	-	94.5	86.9	119.0
	Italy	-	72.0	87.4	120.5
Value index of private consumption at constant exchange rates of 1997 (1997=100)	France	100.0	104.2	107.3	111.2
	Germany	100.0	103.1	106.1	109.3
	Italy	100.0	104.4	108.5	113.2
Value index of private consumption at constant exchange rates of year t-1 (Index: year t-1=100)	France	-	104.2	103.0	103.6
	Germany	-	103.1	102.9	103.0
	Italy	-	104.4	103.9	104.4
Area total (A): value index of private consumption at current exchange rates (USD)	Year t-1 = 100	-	87.9	87.0	119.6
	% change from preceding year		-12.1	-13.0	19.6
	1997=100	100.0	87.9	76.4	91.4
Area total (B): value index of private consumption at constant exchange rates of 1997	1997=100	100.0	103.8	107.2	111.0
	% change from preceding year		3.8	3.3	3.6
	Year t-1 = 100	-	103.8	103.2	103.5
Area total (C): value index of private consumption expenditure at constant exchange rates of year t-1	% change from preceding year		3.8	3.2	3.5
	1997=100	100.0	103.8	107.1	110.9
	Year t-1 = 100	-	103.8	103.2	103.5

Volume aggregates are shown in the next table. Only two options are presented, volume aggregates based on fixed 1997 USD exchange rates and volume aggregates based on USD exchange rates of the previous year. The third possibility, constant price national data converted by way of current exchange rates is difficult to interpret in a meaningful way and has been left aside. The two resulting area totals in Annex Table 4 are again very similar. As before, this is due to the fact that bilateral exchange rates between the three countries moved relatively little over the period under consideration.

Annex Table 4:**Constant price private consumption expenditure: rates of change**

		Year			
Country		1997	1998	1999	2000
Country shares in values at current exchange rates	France	0.27	0.29	0.29	0.29
	Germany	0.42	0.45	0.45	0.45
	Italy	0.31	0.26	0.26	0.26
	Total	1.00	1.00	1.00	1.00
Volume index of private consumption at constant exchange rates of 1997 (1997=100)	France	100.0	103.6	106.3	108.8
	Germany	100.0	102.0	104.7	106.3
	Italy	100.0	102.3	104.1	106.0
Volume index of private consumption at constant exchange rates of year t-1 (Index year t-1=100)	France	-	103.6	102.6	102.4
	Germany	-	102.0	102.6	101.6
	Italy	-	102.3	101.7	101.8
Area total (D): volume index of private consumption at constant exchange rates of year 1997	1997=100	100.0	102.5	104.9	106.9
	% change from preceding year		2.5	2.3	1.9
Area total (E): volume index of private consumption at constant exchange rates of year t-1	Year t-1 = 100	-	102.5	102.4	101.9
	% change from preceding year	-	2.5	2.4	1.9
	1997=100	100.0	102.5	105.0	106.9

Finally, implicit deflators can be constructed by dividing value aggregates by volume aggregates. The resulting indices are shown in Annex Table 5. As would be expected from the comparisons of value aggregates, there are significant differences between the deflator series based on current and constant exchange rates. The implicit price index for private consumption expenditure for the three country zone (France, Germany, Italy) dropped from 100 to 85.5 when based on current USD. With constant exchange rates, the same index rises to about 104 in the year 2000.

Annex Table 5:**Implicit deflator of private consumption expenditure**

		Year			
	Country	1997	1998	1999	2000
Domestic implicit deflator based on national currency (1997=100)	France	100.0	100.5	100.9	102.2
	Germany	100.0	101.1	101.4	102.8
	Italy	100.0	102.1	104.3	106.8
Area total (F=A/D): implicit deflator based on current exchange rates	1997=100	100.0	85.7	72.9	85.5
	% change from preceding year		-14.3	-15.0	17.4
Area total (G=B/D): implicit deflator based on constant exchange rates of 1997	1997=100	100.0	101.2	102.2	103.9
	% change from preceding year	-	1.2	0.9	1.7
Area total (H=C/E): implicit deflator based on constant USD exchange rates of year t-1	1997=100	100.0	101.2	102.1	103.7
	% change from preceding year	-	1.2	0.8	1.6

REFERENCES

- European Central Bank (2000); *Compilation Guide on Government Finance Statistics*; Chapter 4. Compilation of Aggregates, Draft.
- Frisch, R. (1930); “Necessary and sufficient conditions regarding the form of an index number which shall meet certain of Fisher’s tests”; *American Statistical Association Journal* 25.
- OECD (2001); *A Complete and Consistent Macro-Economic Data Set of the Euro Area, Methodological Issues and Results*; Paris
- Hong, Eun-Pyo and Helen Beilby-Orrin (1999); *Compilation Methodologies for Euro zone Aggregates*; <http://www.oecd.org/std/euroaggr.html>.
- Lequiller, Francois (1998); *Conversion rates for historical series*; Note to the 16th Meeting of the Committee on Monetary, Fiscal and Balance of Payments Statistics, Luxembourg.

NOTES

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1. The simple substitution shown here is only valid when PPPs are themselves based on a Laspeyres-type index or when each country produces or consumes only one single commodity. Recall that each country's value term in current prices is in fact a sum over M individual commodities: $X_t^i P_t^i = \sum_j^M X_t^{ij} P_t^{ij}$. But as soon as more than one commodity is involved, there are several possibilities to construct weights for price level comparisons. For example, the prices of country 1 can be weighted with the consumption or production structure of country 2 for purposes of comparison. Or the production structure of country 1 can be used, or an average of both. The Laspeyres-type PPP is one where price levels are compared in terms of the quantity structure of the reference country. In the example chosen here with the United States as reference country, this Laspeyres-type PPPs for country i would be defined as: $PPP_t^i \equiv \frac{\sum_j^M X_t^{kj} P_t^{ij}}{\sum_j^M X_t^{kj} P_t^{kj}}$. In this case, the substitution of the expression for the PPP into equation (2) produces the result as shown in (3). If PPPs were defined as a Paasche index, i.e., by using the quantities of country i as weights in the price comparison, no simple expression as in (3) would result.
2. Specifically, the US-NIPA are based on a chain Fisher index. A Fisher index is a geometric average of a chain Laspeyres type index (where price weights are from the period $t-1$) and a chain Paasche index where price weights are from period t . European countries that base their national accounts on chain indices (e.g., France, Netherlands) use a chain Laspeyres formula for this purpose.
3. Note the issue of additivity that arises when level aggregates are computed for sub-components of a time series, for example for the expenditure components of GDP. The sum of the constant price levels will not, in general, equal the constant price level obtained for total GDP.
4. This *Product Test* of an index number was first formulated by Frisch (1930).
5. "It should be emphasised that the euro area aggregates expressed in current ECUs should be considered with caution. Because the composition of the ECU does not coincide with the currencies of the Member states adopting the euro, pre-1999 amounts converted into ECU at current exchange rates are affected by movements in the currencies of Member states which have not adopted the euro. The data presented [...] are, in particular, inappropriate as a means of deriving implicit deflators" (ECB Monthly Bulletin Table 5.1 National Accounts).