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AGGREGATION BY INDUSTRY
IN GENERAL EQUILIBRIUM MODELS
WITH INTERNATIONAL TRADE

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SUMMARY

Models of trading economies have become very large in dimensions and complex in structure. This paper seeks conditions under which it is possible to aggregate the production and consumption of groups of commodities in "industries": commodity groups sharing some common characteristics and behaviour. The most extreme form of aggregation is the simultaneous aggregation of the same commodities using the same aggregator functions on both the production and consumption sides of the model. This is called "complete aggregation". If this can be done, the competitive equilibrium can be determined in two stages and the commodity dimensions of a model can be reduced drastically, perhaps to very few. In other cases it is possible to aggregate commodities only on the production or the consumption side of the model. Such aggregation will simplify the production or consumption side and allow the derivation of new results.

Conditions which are sufficient for aggregation in production or consumption, or for complete aggregation, are derived. They require the existence of linearly homogeneous indices of production and/or consumption in the industries.

These methods are applied to three groups of models. The first is the Armington model which groups commodities on the demand side. The second is a group of models which assume that in some industries all commodities use a common industry-specific fixed factor. The third is a group in which the commodities in an industry are defined on a continuum. All of these models exemplify some form of aggregation of traded commodities. The Dixit-Grossman model with a continuum of intermediates permits complete aggregation.

These results have applications for the measurement of the effective rates of assistance for commodities in an industry. They also permit the construction of large general equilibrium models which have a simpler structure and they show how tests of comparative advantage can be conducted in two stages, first by analysing the pattern of inter-industry trade and then by analysing the pattern of intra-industry trade.

RESUME

Les modèles sur les échanges économiques sont devenus très importants tant par la taille que par la complexité de leurs structures. Ce document cherche à déterminer les conditions qui permettent d'agréger la production et la consommation de groupes de produits par "industrie", c'est à dire les groupes de produits ayant des caractéristiques et des comportements communs. La forme la plus extrême d'agrégation est l'agrégation simultanée des mêmes produits , qui utilise les mêmes fonctions agrégatives, à la fois pour la modélisation de la production et de la consommation. Ceci est appelé "agrégation complète". Si on y parvient, l'équilibre peut être atteint en deux étapes et le nombre de produits du modèle considérablement diminué, et même réduit à quelques unités. Dans d'autres cas, on ne peut agréger les produits que du coté de la production ou de la consommation. Une telle agrégation simplifierait une partie du modèle et permettrait d'obtenir de nouveaux résultats.

Les conditions nécessaires à l'agrégation d'un seul coté --- production ou consommation --- ainsi que pour une agrégation parfaite sont développées. Elles impliquent l'existence d'indices homogènes linéaires issus de la production et / ou de la consommation dans les industries.

Ces méthodes sont appliquées à trois types de modèles. Le premier est le modèle Armington qui rassemble les produits relatifs à la demande. Le second est un groupe de modèle qui suppose que dans certaines industries tous les produits utilisent un facteur commun spécifique à l'industrie. Le troisième est un groupe dans lequel les produits d'une industrie sont définis sur un continuum. Tous ces modèles illustrent une forme d'agrégation sur le commerce des produits. Le modèle Dixit-Grossman qui comporte des intermédiaires en continuum permet une agrégation complète.

Ces résultats ont des applications dans l'établissement des taux d'assistance pour les produits dans une industrie. Ils permettent également d'élaborer des grands modèles d'équilibre général avec une structure simpifiée et ils montrent comment les test d'avantage comparatif peuvent se faire en deux étapes, d'abord en analysant le schéma des échanges inter-industriels et ensuite en analysant celui des échanges intra-industriels.

PREFACE

The Development Centre's 1990-92 programme on Developing Country Agriculture and International Economic Trends is analysing the implications for developing country agriculture and food security of alternative economic development and trade liberalisation scenarios. A central element of the research is the analysis of agricultural interactions with the rest of the economy using the Rural Urban North South (RUNS) applied general equilibrium model.

This analytic model provides insights into key economic relations. Building on the advance in modelling and computer technology, increasingly complex interrelationships may be specified. Nevertheless, the most complex specifications till capture only a stylised reflection of reality; the formulation of assumptions and the translation of theory into manageable quantitative systems remains the principal challenge of model development.

The question of aggregation lies at the heart of all economic analysis, and is fundamental to applied general equilibrium modelling. This paper provides insights into the aggregation of production and consumption of groups of commodities into "industries", with the term industry used in the broadest sense, so that it includes agriculture and other commodity groupings.

The paper offers insights into the specification and interpretation of computable general equilibrium models. In doing so, it also illuminates the methodological problems associated with the measurement of effective rates of protection and the assessment of comparative advantage. The provision of methodological tools, based on solid conceptual foundations, is an essential first step in the resolution of apparently intractable policy issues. This paper, in addressing a number of key methodological issues, makes an important step which I believe will facilitate policy-orientated research at the OECD Development Centre and elsewhere.

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President, OECD Development Centre
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INTRODUCTION

Traditional international trade theory has been developed in terms of commodities (and non-produced factors), rather than industries. Initially the Classical, Heckscher-Ohlin and other models were stated in terms of the exchange of two commodities but later the commodity dimensions of the models were extended to deal with the general case of n commodities. However, it was found that the main properties of trade models are not robust with respect to dimensions. For example, in the Heckscher-Ohlin model the most basic proposition is the Heckscher-Ohlin Theorem relating to the pattern of trade. When the dimensions of the model are extended beyond two commodities and two factors this theorem ceases to hold generally when either the physical or the factor price definition of factor abundance are used. Attempts have been made to generalise the theorem in terms of bilateral comparisons, factor content propositions and relationships that hold on average. Excellent surveys are provided by Ethier (1984), Chipman (1987) and Jones (1987). This paper seeks to generalise the properties of models of trading economies by utilising another device, the concept of an industry. It involves an explicit process of aggregation of commodity groups within well-defined industries.

In the Heckscher-Ohlin model it is normally assumed that each commodity has a single-output (non-joint) technology which is represented by a production function for the commodity. The activity producing each commodity is often referred to as an "industry". The same assumptions are made for other models such as the Jones specific factor model . Thus in these models one can express comparative advantage in terms of specialisation by industry but such industries are the trivial one-commodity industries. Beginning with the pioneering paper of Krugman (1979), a number of models which feature industries with multi-commodity production and consumption and intra-industry trade between countries have now been constructed. In each of these the term "industry" is defined in a different way but these industries have a meaning in terms of a group of commodities which share some common characteristics and behaviour. The term "industry" is used in the broadest sense so that it includes agriculture and any other commodity group. This has increased the realism of trade models but it has produced high dimensional models with complex structures.

The basic question this paper pursues is whether and under what conditions it is possible in models of internationally trading economies to aggregate the

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production and consumption of the groups of commodities in industries. The most extreme form of aggregation is the aggregation of the same commodities simultaneously and using the same aggregator functions on both the production and consumption sides of the model. This may be called "complete aggregation" as it applies to all parts of the model. If this can be done one can reduce the commodity dimension of a model drastically, perhaps to two or a few. In such cases, one can reverse the traditional procedure and regard such industries as "commodities". One may be able to resurrect the Heckscher-Ohlin Theorem or other theorems of lower dimensionality by expressing them in terms of these aggregate commodities. In other cases it may be possible to aggregate commodities only on the production side or the consumption side. If meaningful aggregation is possible on the production side, this will suffice to resurrect those properties which involve only production. Fortunately, these include many of the main propositions of trade theory, such as the Stolper-Samuelson/Rybczynski Theorem, univalence and the factor price equalisation theorem.

This paper examines a number of models which may conveniently be put into three groups. First, there is the Armington (1969) model. This was the first fully-specified general equilibrium model of intra-industry specialisation and trade, though it is a special case in which each country still produces only one of the products of each industry. Second, there is a group of models which assumes that in some industries all commodities use a common industry-specific fixed factor. This is a device used frequently in applied general equilibrium models, especially for the agricultural sector. The third group of models defines commodities in an industry on a continuum. All of these models will exemplify a form of aggregation of traded commodities.

Section I outlines the relevant features of these groups of models. It concentrates on the meaning of an "industry" in each model and on the pattern of intra-industry trade within such industries. In Section II the two types of aggregation are presented formally and conditions sufficient for consistent aggregation are derived. Four examples of these types of aggregation are given in Section III, one from each group together with the Lancaster-Helpman model in which the assumptions of increasing returns violate the linear homogeneity condition used in the other models. These aggregated models explain the pattern of intra-industry trade. It turns out, as Helpman (1981, Propositions 2 and 3) observed, that the Heckscher-Ohlin and other theorems relating to production still hold in the Lancaster-Helpman model because of the large amount of symmetry. We can regard all of these results as an application of consistent aggregation.

Section IV discusses some implications of aggregation by industry. The first implication is that it is now possible to put together a model of high commodity

dimensions which can combine industries with very different structures, each being chosen as the best representation of the key features of the real world counterparts. Moreover, the analysis of the trade properties can be conducted at the top level of reduced dimensionality in terms of the industries. The second implication relates to empirical tests of comparative advantage. The early empirical evidence of intra-industry trade was dismissed by some observers as "categorical aggregation" due to the misaggregation within a statistical "industry" of commodities that were unrelated. The intra-industry trade models show that intra-industry trade is a real phenomenon and can be explained by models of industry behaviour. These models yield the form of the inter-industry and intra-industry trade variables which are to be explained and the explanators for tests of comparative advantage.

Armington Models

Armington (1969) produced a multi-country model whose main feature has been imitated in many subsequent theoretical and computable models of trading economies. The "Armington Assumption" is that commodities can be nested on the demand side into groups called "industries" and that in each group the commodities consist of one product produced by each of the countries in the model. Thus, in each industry, countries produce close but not identical substitutes. These product groups are weakly and homogeneously separable in the utility functions of the consumers. All consumers in one country are assumed to have the same utility function and the nesting is common to all countries but the parameters of the utility function differ among the countries. In Armington (1969) the supply side of the model is unspecified. Subsequent general equilibrium models with the Armington demand assumptions have usually followed the Heckscher-Ohlin model with two mobile primary factors of production (see, for example, Srinivasan and Whalley (1986) and references therein), though it is possible to introduce specific factors (as in the OECD's Walras Model (Burniaux *et al.* (1989)).

This specification produces a pattern of intra-industry specialisation and trade such that each country produces one product in each industry, the national product, and then trades this with the products of other countries according to the preferences of the residents. This yields a very general pattern of intra-industry trade though it has the disadvantage that there is nothing in the model to explain why the products of the countries are differentiated by country and, viewing it from the supply side, why each country has a unique capacity or technology to produce a particular commodity.

Krugman Model

The Krugman (1979) model was the first model to produce intra-industry specialisation in a rigorous general equilibrium model of the world economy. Krugman (1979), like Armington, grouped the products of the industries in his model on the demand side by assuming that there were weakly separable groups

of commodities but he explained the pattern of specialisation in terms of factor endowments and the existence of economies of scale.

Joint Production with a Common Input

A number of models assume that some industries have a fixed industry-specific factor which can be used to produce a number of products. This assumption is common in the specification of the agricultural industry or sector, as in the Walras model (see Burniaux *et al.* (1989)). The same assumption is used in the model of Falvey (1980) with respect to the manufacturing industry. The assumption of a common factor implies that there is a kind of non-intrinsic jointness in the production of the group of commodities within the industry and this can be represented by an aggregate implicit production function for the group.

Continuum Models with Intra-industry Trade

The first model of intra-industry trade which featured a continuum of commodities was that of Lancaster (1980, 1984). He wished to model diversity of tastes among consumers with respect to commodities within a group. The group is identified with the "manufacturing sector". "A 'group' ... is a product class in which all products, actual and potential, possess the same characteristics, different products within the group being defined as products having these characteristics in different proportions." (Lancaster (1980) p. 153). There are assumed to be only two relevant characteristics for the commodities in the group (see, especially, Lancaster, (1984), p. 138). The characteristics specification of commodities can then be defined in terms of one variable, the ratios of these two characteristics embodied in them. All specifications lie on a one-dimensional spectrum which is assumed to be continuous, that is, a continuum[1]. Lancaster represented these differentiated products as segment of the real line. Any consumer can choose any available product within the group. Each consumer has a single most-preferred product. All consumers have identical preferences except for their most-preferred product. The population of consumers is assumed to be distributed continuously over the commodity spectrum with respect to the consumers' most-preferred products. To define the group precisely, Lancaster (1979, p. 25) imposed the condition that the utility function was separable in this group. To make the model tractable, Lancaster assumes that consumers have identical compensation functions and they are distributed uniformly over the continuum, and the cost

functions for all varieties are the same. This is a highly specialised definition of the industry as it derives from a specialised model of consumer product differentiation. Lancaster's model also introduces economies of scale within the continuum industry. This model was extended by Helpman (1981) and Helpman and Krugman (1985).

Falvey (1981) constructed a continuum of commodities in a quite different way[2]. An industry, called "manufacturing", can produce an infinite number of commodities, using a fixed stock of industry-specific capital and homogeneous labour. Falvey defined the industry in terms of supply characteristics. "For the purpose of this paper, an industry is best defined by the range of products a certain type of capital equipment can produce." (Falvey (1981) p. 496). The products are differentiated by quality. There is assumed to be a continuum of commodities defined over an interval of the real line. The capital intensity of the production process increases with the quality of output for all factor price ratios. Moreover, "Units are chosen so that production of a unit of quality requires the services of units of this industry's capital stock, and one unit of its (hired) labor force" (Falvey (1980) p. 498). The ranking on the continuum is common to both countries as they have a common technology.

Dixit and Grossman (1982) also base their continuum on supply relationships and define their "manufacturing sector" in terms of commodities that use sector-specific capital. In contrast to Falvey, they consider that the final product of the manufacturing sector goes through a succession of stages, each adding value to an intermediate product to yield goods-in-process ready for the next stage. They allow a continuum of stages on the closed interval [0,1] of the real line with the raw material indexed by 0 and the single final product indexed by 1. All stages in the half-open interval [0,1) produce pure intermediates. Let z index the stage (= commodity). (To emphasise the comparability of results from these models, the symbol z is used throughout to denote a commodity on a continuum.) The intermediate good at stage z+dz is produced using one unit of stage i output. The stages are assumed to have differences in factor intensities which are invariant with respect to factor prices and they can, therefore, be ordered on the continuum by their capital-labour intensities. Assuming the technology is common, this ranking holds in both countries[3].

Thus we have, in this group of intra-industry models, models in which the continuum is defined in terms of the variety of a pure consumer good, the quality of a pure consumer good and the stages of a pure intermediate good. The continuity of the commodity index makes the analysis of the number of commodities produced easier than a discrete model with a finite number of commodities.

Table 1 summarises the dimensions of the three groups of models of internationally trading economies. The number of commodities, primary factors and countries gives the dimensions of the trade model in the standard way. In the Armington model the number of industries is the same as the number of commodities produced in one country but in all of the other models the number of industries is much less because there is a domestic industry which can produce many commodities. In the Krugman (1979) model and in each of the continuum models there is an industry or sector outside the continuum industry. Lancaster (1980) gives a Classical version of the model with only one primary factor and another Neoclassical version with two factors, capital and labour. It is the latter which has been entered in this table. The Falvey and the Dixit and Grossman are variants of the Jones-type model with one homogeneous factor, labour, and another factor in each industry which is specific to the industry, giving a total of three factors. Thus there is a variety of dimensional combinations in these models.

These industry groups are well defined in each model if all consumers have the same preferences at least to the extent of the groupings of commodities and all producers have the same technology, as the case may be, and if the product groups do not intersect. The former requires that the utility function which represent preferences be weakly separable in the same partition of commodities for all consumers. The latter requires, in the case of the Dixit and Grossman model, that the input-output matrix be partitionable into blocks each of which has positive entries only along a diagonal and, in the case of the Falvey model, that the potential product ranges of industries do not overlap.

Each of these concepts of the industry can be expressed in terms of relationships of substitutability-complementarity in demand or supply. In the demand-based definitions, a separable group is a group of close substitutes, the exact nature of the restrictions on substitutability varying according to whether the sub-function is also assumed to be symmetric or homogeneous or subject to some other restriction. The Falvey definition makes the commodities in the group substitutes in production while the Dixit and Grossman definition makes the products in the industry perfect complements in production.

Trade Patterns in Models with Intra-industry Trade

These definitions of industries give rise to intra-industry international trade which depends critically on the nature of the industry. The Dixit and Grossman and the Falvey models retain the borderline feature of the Dornsbusch, Fischer and Samuelson (1980) continuum version of the Heckscher-Ohlin model from which

they were developed. (See the Appendix for a discussion of this feature in the Dornsbusch, Fischer and Samuelson model). In these models with intra-industry trade, the borderline commodity falls within the industry[4]. This borderline commodity, $\tilde{z}$, partitions the continuum. The subset $[0,\tilde{z})$ is produced and exported by the country which is more capital-abundant in the sense of having the higher wage rate/capital rental ratio and the subset $(\tilde{z},1]$ is produced and exported by the other country. The borderline commodity may be produced in one or both countries, depending on the demand. In the Dixit-Grossman model, all of the commodities on the continuum will be produced and each country produces infinitely many commodities on a subset of the continuum. This result duplicates that of the Dornsbusch, Fischer and Samuelson model but it is due here to the essentiality of all stages of production rather than the essentiality of all commodities in consumption. In this model the exchange of manufactured products is intra-industry trade and there is inter-industry trade with one country specialising in the production and export of the other commodity, agriculture.

The Falvey model produces an analogous pattern of intra-industry trade when it is extended to a full general equilibrium model, provided the same restrictions are imposed on the technologies. Again there is a borderline commodity, partial specialisation and intra-industry and inter-industry trade. However, the explanation is quite different. In this model the commodities on the continuum below the borderline commodity which are produced and exported by the relatively capital-abundant home country are the higher qualities of the good[5]. In this model the demand for commodities is specified only in terms of being a function of the relative prices. In general, therefore, some qualities will not be demanded and will not be produced in either country.

The pattern of *intra*-industry trade in these two models is now explained neatly by the borderline commodity proposition along the lines of the traditional Heckscher-Ohlin theorem using the price definition of factor abundance. But what of the pattern of inter-industry trade? Neither Falvey nor Dixit and Grossman consider the inter-industry trade in their models.

International trade in the Lancaster model takes place between identical economies. With such a complex model structure almost any of the possible patterns of intra-industry and inter-industry trade is permissible. Trade may consist entirely of intra-industry trade or of inter-industry trade or a mixture. The "normal" case is that in which all trade is intra-industry trade in manufactures and no exchange of agricultural for manufacturing commodities. In the continuum group each of the two identical countries will produce exactly half the number of these goods produced in the world economy and export one half of its output. However, the composition of a country's production and exports and imports is indeterminate

because there are no differences in factor endowments or technologies to give rise to cost differences. Gains from trade may occur between identical economies with identical pre-trade prices because of economies of scale. When countries differ in size the pre-trade price ratios will differ but these do not necessarily predict the direction of trade correctly, as they do in models with constant returns to scale.

Helpman (1981) considers in detail the pattern of trade when there are two homogeneous factors of production, as in the two-factor Heckscher-Ohlin model. He confirms the Lancaster results but obtains additional propositions concerning trade. A variety of patterns of intra-industry and inter-industry trade are possible, depending upon whether they have the same or different endowments of the two factors and the factor intensity of production in the manufacturing industry relative to that in the non-manufacturing industry.

Table 1

	Commodities	Number of Industries	Primary Factors	Countries
Armington (1969)	np	n	2	p
Krugman (1979)	n	2	2	2
Lancaster (1980, 1984) Helpman (1981)	∞	2	2	2
Falvey (1981)	∞	2	3	2
Dixit and Grossman (1982)	∞	2	3	2

It is apparent that the pattern of trade in models with intra-industry trade and more than one factor is related to factor endowment but it also depends on other factors such as economies of scale and country size. We seek to use the concept of the industry to explain further the pattern of intra- and inter-industry trade and other propositions.

Formally, one needs to define some function which aggregates the commodities in a model and to show that this process of aggregation is consistent. Consistency is the property that the aggregation produces the same value of all unaggregated variables as the original model and the sums of the unaggregated variables are equal to the appropriately defined aggregated variables. This aggregation procedure is used in other areas of economic theory; for example, the aggregation of commodities in a separable sub-function of the utility function of a household, and the construction of value added functions in the theory of effective protection. We shall consider first aggregation in demand as this is a direct application of the aggregation procedures in the theory of the consumer.

Aggregation in Consumption

Let the utility function of a consumer agent be homogeneously separable. A function is homogeneously separable if it is weakly separable, viz.

$$U = V(v^1(\bar{x}_1),...,v^j(\bar{x}_j),...,v^m(\bar{x}_m)))$$ (1)

where

$$\bar{x}_j = (x_{j1},...,x_{jk},...,x_{jn_j})$$ $$j = 1,...,m < n$$

and

$$\sum_{j=1}^{m} n_j = n, \quad x_{jk} \geq 0$$

and the sub-functions, $v^j(\bar{x}_j)$, are linearly homogeneous[6]. This is a general form of separability in which there are m groups of the n elementary commodities, with n_j commodities in the j'th group. This form is appropriate in the present context as we shall later interpret these multiple groups as the "industries" of the model. The functions $v^j(\bar{x}_j)$ may be regarded as quantity indices for the groups. Because of the property of linear homogeneity, each of these functions has a dual price index

$$p^j = \varphi^j(\bar{p}_j)$$ $$\bar{p}_j = (p_{j1},...,p_{jk},...,p_{jn_j})$$ (2)

which minimises the cost of a unit of the quantity, v^j, and is a function of prices only. The consumer's problem is to maximise utility, taking as given the vector of commodity prices. It is well known that, under these conditions, the consumer can treat these groups as aggregates and maximise utility in two stages. In the first upper-level stage, the consumer maximises his/her utility by allocating the budget among the commodity groups, given the price indices of the quantities of these groups. In the second lower-level stage, the consumer allocates the group expenditures among the commodities in each group, given the prices of the individual commodities. Moreover, the consumer choices in the two-stage process are consistent with those in the one stage process if and only if the quantity indices are linearly homogeneous. Consistency is the property that the quantities chosen by the consumer of each elementary commodity when he/she maximises utility in two stages be equal to those chosen when utility is maximised in one stage, and the expenditures on each group, $v^j p^j(p^1,...,p^m,I)$ be equal to the sum of the expenditures on all of the elementary commodities in the group. The first property is required for meaningful aggregation. The second property is clearly desirable. Since it holds for the elementary commodities, one may treat the aggregates in the same manner as elementary commodities only if they satisfy this condition.

The results are contained in the following proposition.

Proposition 1

Commodities may be aggregated into groups in the consumer problem of maximising utility and this problem may be solved in two stages if the consumer's utility functions is homogeneously separable.

The proof is omitted because it is wellknown in consumer theory (See Blackorby, Primont, and Russell (1977)). Essentially, one aggregates the variables by choosing the quantity and price index of the group as v^j and p^j respectively. The upper level maximisation is defined in terms of these quantities and prices. It can then be shown that the first-order conditions of the one-stage problem and the two-stage problem yield the same values of the elementary variables. Furthermore, the expenditures on the groups of commodities are equal to the sums of the expenditures on the elementary commodities, viz.

$$v^j p^j = \sum_{k=1}^{n_j} p_{jk} \, x_{jk}(p_{j1},...,p_{jk},...,p_{jn_j},e_j) \tag{3}$$

Homogeneous separability is not a necessary condition as aggregation is possible under other conditions; for example, the Hicks condition of price proportionality. The necessary and sufficient condition for consistent aggregation is that the utility

function by homogeneously separable for all feasible prices and quantities. This condition includes the case of Hicks price proportionality and that of Leontief quantity proportionality as well as functional homogeneous separability as described above. (See Lloyd (1977) for a treatment which includes all three cases).

Furthermore, it turns out that utility maximisation is a consistent two-stage process if and only if the dual expenditure minimisation problem is a consistent two-stage process. In this case the expenditure function in terms of the elementary commodities, $e^h(p,u^h)$, may be rewritten as

$$e^h = f^h(p^1,...,p^m,u^h) \tag{4}$$

where the prices, p^j, are the price indices for the groups which are themselves the minimum expenditure functions for the groups and u is the arbitrary level of utility of the consumer. Thus, the expenditure function is a function of the group expenditure functions (see Lloyd (1977)).

In a general equilibrium model with many consumers we must also assume that all consumers in one country have identical preferences. Otherwise, each consumer would have a different set of quantity and price aggregator functions and there would not be a unique set of national prices for the groups in the competitive equilibrium. We take the utility function then as the function of all of the consumers of the country.

Under these assumptions of identical homogeneously separable utility functions, there exists quantity and price indices for each group in each country and utility maximisation in each country may be regarded as a consistent two-stage process.

Aggregation in Production

Aggregation on the supply side is possible under similar conditions. Suppose initially that each of the n commodities can be produced in a country with a nonjoint technology that can be represented by a linearly homogeneous production function

$$g_i = f^i(v_i, \hat{g}_i) \qquad\qquad i = 1,...,n \tag{5}$$

where $v_i = (v_{i1},...,v_{is})$ and $\hat{g}_i = (g_{1i},...,g_{ni})$ are the vectors of primary and intermediate inputs respectively which are used in the production of commodity i[7].

The primary inputs may be mobile or specific. With intermediate input usage, g_i, is the gross output of commodity i. The net output of this commodity is

$$y_i = g_i - \sum_{i=1}^{n} a_{ij} g_j$$

$$= h^i(v_i, \hat{g}_i) \tag{6}$$

where a_{ij} is the intermediate input requirement of input i into output j.

This model is sufficiently general to encompass the particular intra-industry models considered in this paper with the extension of the number of commodities to infinity in the case of the Dixit and Grossman model and the exception of the Lancaster-Helpman model with increasing returns to scale.

The production problem for the national economy is

$$\max{}_{y,w} \{qy: y_i = h^i(v_i, \hat{g}_i) , \sum_{i=1}^{n} a_{ji}(w,p) g_i \leq v_j, v_i \geq 0, g_i \geq 0\} \tag{7}$$

q is the vector of prices to producers and y is the vector of net outputs of the economy. This problem is solved if the following Kuhn-Tucker conditions are satisfied

$$q_i - c^i(w,p) \leq 0$$
$$[q_i - c^i(w,p)] \, g_i = 0 \qquad\qquad i = 1,...,n$$
$$g_i \geq 0$$

and $\tag{8}$

$$\sum_{i=1}^{n} a_{ji}(w,p) \, g_i - v_j \leq 0$$

$$[\sum_{i=1}^{n} a_{ji}(w,p) \, g_i - v_j] w_j = 0 \qquad\qquad j = 1,...,s$$

$$w_j \geq 0$$

where $w = (w_1,...,w_s)$ and $q = (q_1,...,w_n)$ are the vectors of the prices of the primary inputs and the produced outputs respectively. As with the consumer problem, the vector of prices has been taken as given.

The first set of equations and inequalities are the familiar zero profit conditions. $c^i(w,q)$ is the unit cost function for commodity i which is the dual to the production function, $f^i(v_i, \hat{g}_i)$. They are also the solution to the problem of maximising profits from the production of each commodity:

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$$\max_{g_i}\{g_i(q_i - c^i(w,q))\} \quad \text{such that } g_i \geq 0 \tag{9}$$

Thus, under these conditions, the social problem is solved by the profit-maximising behaviour of individual producers, as is well known. The second set of equations and inequalities are the factor market equilibrium conditions. In most models it is assumed that the competitive equilibrium is such that all gross outputs and factor prices are strictly positive, thereby reducing the system to two sets of equations.

The joint solution to these sets of equations yields

$$w_j = w_j(q,v) \qquad\qquad\qquad j = 1,...,s \tag{10}$$
$$g_i = g_i(q,v) \qquad\qquad\qquad i = 1,...,n$$

Knowing the gross outputs of each commodity, the vector of net outputs is now given from their definitions,

$$y_i = g_i(q,v) - \sum_{i=1}^{n} a_{ij}(q,v)g_j(q,v) = y_i(q,v) \tag{11}$$

Substituting the solution values of $y(q,v)$ into the definition of national product gives the national product function

$$g(q,v) = qy(q,v) \tag{12}$$

which is the maximal value of national output subject to the constraints on the production problem of Equation (7).

Consistent aggregation on the supply side is possible if, for a group of commodities, group k, there exists a linearly homogeneous production,

$$g_k = H^k(v_k) \qquad\qquad v_k = (v_{k1},...,v_{ks}) \tag{13}$$
where
$$g_k = J^k(y_{k1},...,y_{kt}) \tag{14}$$

defines the index of the output of the group as an aggregate of the outputs of the commodities in the group and is itself linearly homogeneous. Thus J^k is the aggregator function which defines units of the output of the group and H^k is the aggregate production function which maps from input space to output space. A group of commodities which has such functions will be called "industry" k. We may suppose there are $M < n$ such groups

In the presence of the aggregation of commodities in such groups, the maximisation problem of Equation (7) can be presented as a two-stage problem similar to that of the consumer maximisation problem. At the first upper-level stage the economy allocates resources to the industry groups, knowing the prices of these groups. At the second lower-level stage, the producers in the group allocate the resources of the group to the individual commodities, knowing the prices of the individual commodities in the group, in order to maximise the group profits. This result is contained in the next Proposition.

Proposition 2

Commodities may be aggregated into groups in the production problem of maximising the aggregate value of net output and this problem may be solved in two stages if there exists a set of industries each of which has a linearly homogeneous output aggregator function and a linearly homogeneous aggregate production function that represents the technology of the group.

The proof follows the same lines as the consumer maximisation problem. One chooses J^k as the index of the quantity produced of the group. This quantity has a dual cost function

$$C^k = C^k(w,q,h) = \min_{v_k}\{wv_k : H^k(v_k) \geq h\} \tag{15}$$
$$= c^k(w,q)$$

where $c^k(w,q)$ is the unit cost function. c^k is itself linearly homogeneous in (w,q). The zero profit condition implies that there is a price for the industry's output, g_k, assuming the output is produced. Moreover, this price is equal to the unit cost index for the group. The upper level problem is defined in terms of these aggregate quantity and price variables. The linear homogeneity of the indices of quantity and price ensures that the consistency requirement is satisfied.

It follows from this aggregation that the national product function may be written in terms of the aggregated quantities and prices

$$g(q,v) = G(\tilde{q},v) \qquad\qquad \tilde{q} = (q^1,...,q^M) \tag{16}$$
$$= \max\{\tilde{q}\tilde{y} : y^j = H^j(V_j, \hat{g}^j), \sum_{i=1}^{n} A_{ij}(w,p)g^j \leq v_i, V_j \geq 0, g^j \geq 0\}$$
$$\tilde{y} = (y^1,...,y^j,...,y^M)$$

where V_j and A_{ij} are the vectors of inputs used in the production of the aggregate outputs and the aggregate input-output coefficients respectively. The values of

outputs of the industries, $\tilde{q}^j \; \tilde{y}^j$, are themselves the maximum values of the outputs which can be produced by the industries given the allocation of inputs to the industries. Thus, the national output function may be regarded as a function of the industry output value functions.

Complete Aggregation

The third type of aggregation is complete aggregation. This is an aggregation which applies to the same group of commodities everywhere in the model, that is, to both the production and consumption relationships and to all countries.

In the context of models of internationally trading economies, the formal problem can be approached in a straightforward manner by using the national trade expenditure function of Lloyd and Schweinberger (1988). For an economy which trades freely with other economies, this function is defined as

$$B^e(p,v,u) = S_h e^h(p,u^h) - g(p,v) \qquad\qquad u = (u^1,...,u^H) \qquad (17)$$

$e^h(p,u^h)$ is the expenditure function of household h. $\Sigma_h e^h(p,u^h)$ is the national expenditure function. This is the minimum aggregate expenditure, given the prices p, which will enable each of the households in the economy to attain the level of utility u^h. $g(p,v)$ is the national product function for the economy, as above. Because of free trade, the prices received by producers have been set equal to the prices paid by consumers. B^e is, therefore, the minimum expenditure needed to attain the vector of utilities u, given prices and allowing the production side of the economy to respond efficiently to these prices. The model of production can encompass intermediates and specific factors and it is unrestricted on the demand side. The problem of minimising this aggregate expenditure is dual to the vector maximum problem which determines the competitive equilibrium. This is the problem of maximising the weighted utilities of the household agents, subject to the constraints on the economy.

One may note that the trade expenditure minimisation is the difference between two terms, one representing the minimisation of the households' expenditures and the other the maximisation of the economy's national product. Consistent aggregation of commodities which appear in these two functions into a product group or industry follows if one can show that the expenditure function can be minimised equivalently by a grouping of commodities and similarly that the national product can be maximised by the same grouping of commodities. These

have been demonstrated separately above. The consumer problem and the producer problem can now be combined into one problem because in these models the primary factors which constrain production do not enter the utility functions of the households. Consequently the economy's minimisation problem divides into the problem of minimising the aggregate expenditure and the problem of maximising the value of output. The central requirement is that the utility and production functions for the industries be homogeneously separable. This is satisfied by the assumption of separability and constant returns to scale on the supply side and separability and homotheticity of the sub-functions on the demand side.

Thus, if the same commodity groups appear on the demand side and the supply side of the model and these groups can be aggregated into linearly homogeneous functions of the elementary commodities, complete aggregation is possible in the model. In this event both the demand and supply sides of the model may be represented in terms of the aggregated "industry" variables and these aggregated variables are consistent with the model as specified in terms of the elementary unaggregated variables. The industry is a device which allows us to view the world economy at two levels, the inter-industry level and the intra-industry level. At the top level of the model the dimensions have been reduced from the number of elementary commodities, n, to the number of aggregated commodities, $m=M<n$.

SECTION III

This Section constructs several examples of aggregation using the aggregators developed in Section II.

The Armington Model

On the demand side Armington assumed that in each country there is one consumer whose utility function is homogeneously separable, precisely as in Equation (1) above. He made the further assumption that the sub-utility functions are CES. However, unlike most of those who have adopted his separability assumption, he eschewed the assumption that the upper level utility function, U, is homothetic. Non-homotheticity allows non-unitary income elasticities of demand for the groups. It is the linear homogeneity of the group functions, not that of the upper level function, which is required for consistent aggregation[8].

For each group there is a group function, $v^j(\bar{x}j)$. Each of these quantity indices has a dual price index

$$p^j = \varphi^j(p_{j1},...,p_{j1k},...,p_{jnj})$$

which minimises the cost of producing a unit of the quantity, v^j. This price index is itself linearly homogeneous. Moreover, the aggregation is consistent. Consequently, utility maximisation is a two-stage process and one may examine the demand at the top-level which greatly simplifies the demand functions.

On the supply side, the groupings of commodities in the industries are the same as on the demand side but each country produces only one of the commodities in each industry group, the national commodity. The technology in each country is given by the production function for the commodity

$$y_j = f^i(v_j,...,v_{sj}) \tag{18}$$

where y^j is the net output as there are no intermediate inputs in the model. These functions have the property that they are linearly homogeneous and satisfy regularity properties. However, the technologies differ among countries for the national goods in the one group.

We may now use the aggregator defined by preferences on the demand side for the group to define an index of production for the group by the country concerned. The quantity of the group product produced by the country is $y^J =$ $v^J(y_j>0) = g_j\, y_j$ where J is the subscript or superscript which indexes the industry. $v^J(y_j>0)$ is the value of the function with the strictly positive value for the argument which represents the output of the commodity produced by this country and zero values for all other commodities which are produced by the other countries. The unit cost for this aggregated commodity is the dual of y^J which is equal to $\varphi^J(p_j>0)$. The last price is the value of the price index φ^J with the strictly positive price of the commodity produced in the country and a zero value for the prices of the commodities produced in other countries. By duality, this price is $(\gamma_j)^{-1}\varphi^J$.

The aggregate value of the net imports of the industry j in this country is given by

$$M_J = \sum_k M_{jk} = m_{jk}p_{jk} = \sum_k (x_{jk}-y_{jk})p_{jk} = \sum_k x_{jk}p_{jk} - y_{jk}p_{jk} \quad \text{[since } y_{jk}>0 \text{ for one j]}$$

$$= v^J p^J - (\gamma_j)^{-1}y^J\, \gamma_j\, p^J = m_J p_J \tag{19}$$

That is, the definition of the aggregate quantity and price of the industry output gives a value of the net imports of the industry as the product of the quantity of net imports of the industry times the price which is equal to the sum of the imports and exports of the individual commodities in the industry group. The same applies to the other countries in the model. Such aggregation is possible because the same aggregator functions are used in both demand and supply.

In the Armington model the aggregation on the supply side does not add to the interpretation of the model because there is only one commodity produced in each industry of each country. The dimensions of the model are still large with m produced commodities in each country.

The Dixit and Grossman model

In this model it is possible to aggregate the continuum commodities to form a single composite commodity in both production and consumption. Assume that the home country is abundant in the capital used in the continuum industry. Before aggregation at the industry level one can regard the commodities produced on the lower end of the continuum by the home country and those produced on the upper end of the continuum by the foreign country as integrated commodities. These commodities and the outside commodity are denoted commodities 1, 2 and 3

respectively. The production conditions for the competitive equilibrium are given by

<div align="center">

Home Country   **Foreign Country**

</div>

$$c_1(w,r,\bar{z}) = \int_0^{\bar{z}} c(w,r,z)dz = p_1$$

$$c_2(w^*,r^*,\bar{z}) = \int_{\bar{z}}^{1} c(w^*,r^*,z)dz = p_2$$

$$c_3(w,v) = p_3 \qquad\qquad\qquad c_3(w^*,r^*) = p_3$$

$$c_{1w}(w,r)y_1 + c_{3w}(w,r)y_3 = L \qquad c_{2w}(w^*,r^*)\,y_2 + c_{3w^*}(w^*,r^*)y_3 = L^* \qquad (20)$$

$$c_{1r}(w,r)y_1 = K \qquad\qquad\qquad c_{2r^*}(w^*,r^*)y_2 = K^*$$

$$c_{3v}(w,v)y_3 = V \qquad\qquad\qquad c_{3v^*}(w^*,v^*)y_3 = V^*$$

The asterisks indicate variables which are those of the foreign country. The first three equations are the standard cost-minimising conditions and the next three are the full employment conditions.

The price of the final good produced on the continuum is given by

$$\int_0^{\bar{z}} c(w,r,z)dz + \int_{\bar{z}}^{1} c(w^*,r^*,z)dz = p_1 + p_2 \qquad (21)$$

When the factor prices and the commodity prices are known from the equilibrium we can define $p_1/p_I = \alpha$ and $p_2/p_I = \beta$ which are the proportions of the cost of producing the final product of the continuum industry in the foreign and the home countries respectively. Using α and β as price weights we can now aggregate commodities 1 and 2 to form the manufacturing industry.

Denote the continuum industry as industry I and relabel the outside industry as industry II. Now $p_I = p_1 + p_2$. The aggregate quantity of the outputs of the industry I in the home country and the foreign country are now $y_I = ay_1$ and $y_I^* = by_2$. Each country is considered to produce a part of the final output, y_I, in proportion to the share of the value added in industry I. The unit cost of production is $c_I = \int_0^{\bar{z}} c(w,r,z)dz$

$+ \int\limits_{\bar{z}}^{1} c(w^*, r^*, z) dz$. This defines the constant returns to scale technology of the industry. The production conditions for competitive equilibrium in both industries are now given by

Home Country	Foreign Country

$$c_I(w,r) = p_I \qquad\qquad c_I(w^*,r^*) = p_I$$

$$c_{II}(w,v) = p_{II} \qquad\qquad c_{II}(w^*,r^*) = p_{II}$$

$$c_{Iw}(w,r)y_I + c_{IIw}(w,v)y_{II} = L \qquad c_{Iw}(w^*,r^*)\, y_I^* + c_{IIw}^*(w^*,r^*)y_{II}^* = L^* \quad (22)$$

$$c_{Ir}(w,r)y_I = K \qquad\qquad c_{Ir}^*(w^*,r^*)y_I^* = K^*$$

$$c_{IIv}(w,v)y_{II} = V \qquad\qquad c_{IIv}^*(w^*,v^*)y_{II}^* = V^*$$

The aggregation on the supply side enables us to treat the production conditions in all respects as if only two commodities are produced.

On the demand side the aggregation is simpler because only one final product is produced by the vertically integrated industry. Hence, $x_I = x_2$ and $x_I^* = x_2^*$. The quantities consumed enter the utility function of all agents as a single argument, viz. $U^h = U^h(x_I, x_{II})$ where h is any agent. In all respects the final output of industry I can be treated as a single consumable commodity. It is not necessary in this model to assume all agents have identical utility functions within or across countries.

The aggregate value of the net imports of the continuum industry are now

$$M_I = M_1 + M_2 = -y_1\, p_1 + x_2\, (p_1 + p_2) = -\alpha y_1 p_I + x_2 p_I = [(x_2) - (\alpha y_1)]p_I$$

$$= (x_I - y_I)p_I = m_I p_I$$

$$(23)$$

and

$$M_I^* = M_1^* + M_2^* = y_2 p_1 + (x_2^* - y_2)(p_1 + p_2) = \alpha y_2\, p_I + (x_2^* - y_2)p_I$$

$$= [(x_2^*)) - (\beta y_2)]p_I \;\; = (x_I^* - y_I^*)p_I = m_I^* p_I \qquad\qquad (24)$$

$$= -M_I \neq 0$$

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where the variables x and y denote consumption/use and production of the commodities respectively. Equations (23) and (24) state that the value of net imports and exports of industry I in the two countries defined as the sums of the imports and exports of the components and final products of the industry are equal to the value of the net imports and exports of the aggregated commodities. These results use the equalities $y_1 = y_2^*$, which follows from the fact that the production of one unit of commodity 1 requires one unit of commodity 2 because of the fixed assumption in the continuum industry, and $x_2 + x_2^* = y_2$.

This aggregation is possible because of the assumption of fixed proportions of all stages in the production of the continuum commodity. It is in fact an example of Leontief aggregation[9].

Having aggregated the variables in the commodity group and thereby reduced the commodity dimensions of the model, one may now use the lower dimensional version of the model with the aggregated commodities to derive some propositions. Which country is the net exporter and which the net importer of the products of the continuum industry will be determined by the solution to the model. The country which is a (net) exporter of the products of the continuum industry will of course be the importer of the products of the outside industry. Thus each country will specialise incompletely in one of the two industries. The aggregated version of the model is merely a standard Jones specific factor model of the minimum dimensions, that is, there are 2 "commodities" and 3 factors. The pattern of trade in the two products will follow the pattern in the 3-factor 2-commodity Jones specific factor model. Other properties follow. For example, consider the pattern of the sign change of real incomes when the commodity prices change. The Stolper-Samuelson Theorem cannot hold in this model because of the existence of specific factors. We know the sign pattern from the Jones model . Thus, if the price of one commodity rises, we know that the real income of the specific factor used intensively in the production of this commodity must rise and the real income of the specific factor used in the production of the other commodity must fall, and the sign of the change of real income of the mobile factor is ambiguous as it depends upon the elasticity of demand for labour and the pattern of consumption. (See Ruffin and Jones (1974)). By the Reciprocity Relation, the Rybczynski Theorem does not hold. This proposition is not evident from the higher dimensional unaggregated version of the model with infinitely many commodities. We also know univalence and factor price equalisation do not hold.

The Falvey Model

In the Falvey model there are also two industries, the continuum industry and the outside industry, and the continuum industry can be aggregated in a related way. On the supply side, there is an infinite number of producible commodities, with the index reflecting both the quality and the capital intensity of the production process for the commodity. Each quality z has an individual production function exhibiting constant returns to scale and the strong Samuelson factor intensity assumption

$$y_z = f^z(K_z, L_z) \qquad\qquad z \in [0,1] \qquad\qquad (25)$$

The capital factor is specific to the industry but labour is non-specific.

The use of a common factor for the commodities in the industry allows the derivation of a multiple-input multiple-output production function, $F(z,K,L) = 0$, by maximising the output of one commodity for a given output of the other commodities and subject to the constraints on production. This function is of the usual implicit form except that z is continuous. The assumption of constant returns to scale means that this function is almost-homogeneous, viz. $F(\lambda z, \lambda K, \lambda L) = 0$ for $\lambda > 0$. Suppose now that the function is also input-output-separable, viz. there exist functions g(z) and h(K,L) such that

$$F(z,K,L) = g(z) = h(K,L) = 0 \qquad\qquad (26)$$

g(z) is the index of outputs and h(k,L) is the index of inputs. This function belongs to the common class of almost-homogeneous input-output-separable multiple-input multiple-output production functions (see Hasenkamp (1976)). Because of almost-homogeneity, g(z) can be taken as the linearly homogeneous index of the quantity of output of the industry. This has a dual cost function of the form

$$C(w,r,g(z)) = g(z)\, c(w,r) \qquad\qquad (27)$$

c(w,r) is the linearly homogeneous index of the unit cost of production of the industry's output. The production conditions for the competitive equilibrium at the upper level are precisely the same as those of the Dixit-Grossman model in Equation (22) above, with the appropriate labelling of variables.

Again the pattern of the sign change of real incomes when commodity prices change is precisely that of the Jones 3x2 specific factor model and univalence and the factor price equalisation theorem do not hold.

The Lancaster-Helpman Model

The Lancaster and Helpman models of intra-industry trade do not enable aggregation in the same way because the commodity production functions are not linearly homogeneous. Nevertheless, Lancaster (1980, p. 171) postulated that the *inter*-industry pattern of trade would follow Heckscher-Ohlin lines. There are two homogeneous factors, labour and capital, in the Neoclassical version of the Lancaster model. "If country 1 is relatively capital abundant in its endowments, the trade equilibrium will be such that country 1 produces a higher ratio of manufacturing output to agriculture than does country 2. If the countries are similar enough in other respects to give approximately the same ratio of manufactures to agriculture in consumption, country 1 will be a net exporter of manufactured goods and a net importer of agricultural products." This is the characterisation of the pattern of production and the Heckscher-Ohlin Theorem in the stronger form of physical abundance in a standard 2x2 Heckscher-Ohlin model.

Helpman (1981) provided the proof of the Lancaster proposition. He modified the Lancaster model by locating the most-preferred product of each consumer on a circle instead of a line, following Vickrey (1964). This avoids having to make special assumptions about the supply at the ends of the continuum. He assumed that consumers in each country have identical incomes and that the preferences are the same in the two countries. These assumptions assure an equal demand for varieties of manufactured products around the circle. On the supply side the production of all varieties in both countries have the same technology. Under these assumptions the equilibrium in the continuum industry is a symmetric Nash equilibrium. The finite number of commodities produced are spaced at equal distance on the continuum, produced in the same quantities by a single producer and sold for the same price.

The continuum industry, manufacturing, and the outside industry, agriculture, can be denoted by I and II again. Let n and n^* denote the number of commodities produced in the home country and the foreign country respectively. The production conditions for a competitive equilibrium are given by

Home Country	Foreign Country
$c_I(w,r,y_I) = p_I y_I$	$c_I(w,r,y_I^*) = p_I y_I^*$
$c_{II}(w,r) = p_{II}$	$c_{II}(w,r) = p_{II}$

$$R(p_I, p_{II}, n) = \theta(w, r, y_I) \qquad\qquad R^*(p_I, p_{II}, n^*) = \theta(w, r, y_I^*)$$

$$C_{Iw}(w, r, y_I)n + c_{IIw}(w, r)y_{II} = L \qquad C_{Iw}(w, r, y_I^*)n^* + c_{IIw}(w, r)y_{II}^* = L^* \qquad (28)$$

$$C_{Ir}(w, r, y_I)n + c_{IIr}(w, r)y_{II} = K \qquad C_{Ir}(w, r, y_I^*)y_I^* + c_{IIr}(w, r)y_{II}^* = K^*$$

These equations can be compared with those of the Dixit-Grossman/Falvey competitive equilibrium before aggregation in Equations (20). As in Equation (20), the equations separate into two subsets of equations, the first three describing the zero profit conditions and the last two describing the full employment in the factor markets. In the zero profit conditions there is on extra equation to determine the output level of a firm in the manufacturing sector. This equation states that the Lerner degree of monopoly in the continuum industry, R, is equal to the degree of elasticity of the cost function, θ, which is a measure of the degree of economies of scale. This is feature of the equilibrium. In the full employment conditions, the factor prices are equal across countries because of factor price equalisation in the model. It then follows from the properties of factor price equalisation and the output levels of all firms being the same that the country with the higher capital-labour ratio produces more manufacturing varieties and less food per capita than the other. Finally, given identical preferences across countries, and assuming manufacturing products are relatively capital-intensive, the country which has the higher capital-labour ratio is the *net* exporter of manufactures and the *net* importer of food, though both countries export and import manufacturing varieties. Helpman (1980, p. 324) concluded that "we use Heckscher-Ohlin to explain intersectoral trade while intra-industry trade is explained by the existence of economies of scale and differentiated products."

These results can also be expressed in terms of an aggregation. An industry in this model is like a perfectly competitive industry. There are many producers each of whom is indistinguishable from the others. As they have the same commodity production function for a commodity, this function can be used as the industry production function. Similarly, consumers have the same utility functions except for the choice of the most preferred product. This symmetry allows aggregation in both production and consumption and it is enough to show that the properties of univalence, factor price equalisation and the Rybczynski (and, by the reciprocity relation, the Stolper-Samuelson) Theorems also hold. Unfortunately, these results will not continue to hold when the symmetry is relaxed but the economies of scale are retained.

SECTION IV

These aggregation results for models with intra-industry trade have a number of applications.

First, the indices of prices and quantities may themselves be useful. For example, international economists frequently measure the effective rates of assistance going to different industries by a procedure which is equivalent to taking some arbitrary average of the effective rates of assistance given to the individual commodities in the industry group. Typically these measures are weighted by the value added shares in the industry whereas these weights should properly be derived from a model of the industry. In the models above the aggregate industry price index provides immediately the correct measure of the percentage change in the value added due to assistance measures. The correct measure is

$$E = (p_I^d - p_I^f)/p_I^f \tag{29}$$

where p_I^d and p_I^f are the distorted and free trade prices respectively. The Dixit-Grossman model is especially instructive in this context as it is designed to capture complex intra-industry input-output relations. In this model p_I measures the value added in the country per unit of output. Consider the home country. Take any given structure of assistance where t_z is the tariff on commodity z and this rate may vary among commodities. The measure of effective assistance is

$$E = \{ \int_0^{\bar{z}_1} c(w^d, r^d, z)dz - \int_0^{\bar{z}} c(w^f, r^f, z)dz \} / \int_0^{\bar{z}} c(w^f, r^f, z)dz$$

$$= \{ \int_0^{\bar{z}} [c(w^d, r^d, z)dz - c(w^f, r^f, z)]dz + \int_{\bar{z}}^{\bar{z}_1} c(w^d, r^d, z)dz \} / \int_0^{\bar{z}} c(w^f, r^f, z)dz \tag{30}$$

This expression allows for the changes in the range of commodities produced after assistance and for changes in the terms of trade. There are two components, the first measuring the increase in the value added for those comodities which would be produced under free trade and the second measuring the increase in value added due to the increase in the number of stages produced in the protected situation. The first component is the simple sum of the increases in the value added (at domestic prices) which is equivalent to equal weighting of the individual commodities protected. This equal weighting holds because of the

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$$= \{ \int_0^{\bar{z}} [c(w^d, r^d, z)dz - c(w^f, r^f, z)]dz + \int_{\bar{z}}^{\bar{z}_1} c(w^d, r^d, z)dz \}/ \int_0^{\bar{z}} c(w^f, r^f, z)dz \tag{30}$$

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assumption that a unit of the previous stage is required at each stage of production. If the structure of assistance is uniform among all commodities in the industry, the effective rate of assistance does not reduce to t_z. This result is partly due to the necessity to add in the value added for those commodities which are not produced in the free trade situation and partly to the complication that some of the protection for the commodities below the borderline is redundant in that the margin by which the domestic price exceeds the foreign price will be less than the tariff. For other models the appropriate measure can be derived in the same way.

The main applications of the aggregations are to the methods of constructing or using applied general equilibrium models. The first application is for the method of constructing multi-commodity general equilibrium models of the world economy. We may consider all intra-industry trade models as extensions of the Heckscher-Ohlin type model. All have been constructed by taking the Heckscher-Ohlin or Jones specific factor model of the world economy with two commodities and then disaggregating one of the two industries into a horizontal or vertical group or continuum. Lancaster (1980) and Helpman (1981) also introduced economies of scale in the disaggregated industry. Plainly one could use the same device in a Heckscher-Ohlin or Jones model of any dimensions. One simply takes a set of industries and disaggregates each horizontally or vertically. Moreover, the method of disaggregation can and should vary among the industries. Some industries are obviously of the sequential Dixit and Grossman type and some involve instead jointness in production and some trade in differentiated consumer products.

Unfortunately, most of the many-industry applied general equilibrium models that have been constructed recently are either ones in which all of the industries are standard constant returns to scale single product industries or industries with, say, economies of scale and strategic behaviour of the same type for all industries. As two well-known examples, we consider the work of Harris and Cox in Canada and Smith and Venables in the EC. Each team has developed an applied model in a series of papers.

Harris and Cox model a small open economy with a version of Chamberlinian competition based on product differentiation in the noncompetitive industries (see, especially, Harris (1984) and Harris and Cox (1984)). There are two groups of industries, competitive and noncompetitive. Each representative firm in a noncompetitive industry has a multi-product production function with economies of scale and scope due to the presence of fixed plant and product-specific costs. Firms fix prices either on the basis of an optimal markup or, in the manner of the Eastman and Stykolt study of the Canadian economy, on the basis of the tariff-inclusive price of import substitutes. There is free entry. There are separable symmetric CES sub-utility functions for each industry group.

Smith and Venables developed a similar model of production and trade in differentiated manufacturing goods for the world economy in order to model the effects of completing the internal market of the EC (see, especially, Smith and Venables (1988)). All ten manufacturing industries in their model have the same form of imperfect competition which is an extension of the Krugman (1979) type; all firms are symmetric and multi-product producers but there is no jointness of production, there are economies of scale due to fixed costs and continuously decreasing marginal costs for each product, firms act as price-discriminating Cournot or Bertrand competitors in nationally segmented markets, there is free entry and the sub-utility functions for each industry group are symmetric CES. The industries produce product groups ranging from machine tools to electrical household appliances, motor vehicles and footwear. In addition, the economy contains a numeraire commodity which is produced under constant returns to scale and perfect competition conditions.

For both models the usual assumption of universal perfect competition has been replaced by the assumption of the same very specific type of competition in all manufacturing industries. In reality, by contrast, some industries are obviously of one type and some of another. Applied general equilibrium models should combine different specifications and different types of behaviour for different industries, as appropriate for the industries.

The second application of the aggregated models is for the construction of theoretically sound empirical tests of comparative advantage when there is intra-industry trade. When intra-industry trade first received attention a number of authors claimed that it is a statistical phenomenon due to the misaggregation within "industries" of products which have different factor proportions that cause them to be trade. Finger (1978) used the finitely-many-commodities two-factor version of the Heckscher-Ohlin model. In this model, as in the continuum version studied by Dorsnbusch, Fischer and Samuelson (1980), there is a borderline commodity which separates the exported commodities from the imported commodities in one country. Each country exports the commodities which are intensive in the factor with which it is well endowed. Thus, according to Finger, countries could only export and import products of an industry if the industry included products of differing factor intensities. This was called "categorical aggregation". The Dixit-Grossman and Falvey models do group commodities together which have different factor intensities but this is perfectly permissible because the industry has other features of common industry-specific inputs or vertical input-output relations which make it a meaningful unit.

The models discussed above show that factor proportions can continue to explain both inter-industry trade and intra-industry. For the explanation of inter-

industry trade, the appropriate dependent variable in all the models is the net or inter-industry trade flows, netting out all intra-industry trade. Many early and recent studies of comparative advantage have intuitively used net trade flows but until now the justification for this choice has been absent. For the intra-industry trade the appropriate dependent variable is the Grubel-Lloyd index of intra-industry trade because the trade of countries is assumed to be balanced.

These models also show how the factor proportions should be combined with the factors that explain intra-industry trade if one is to test properly the hypotheses that these models determine actual trade flows. The explanatory variables for intra-industry trade will generally vary among industries. In the Falvey and the Dixit and Grossman models factor proportions determine the patterns of inter-industry and inter-industry trade. This is also true of other models involving jointness due to a common industry input. However, in the models with increasing returns to scale and imperfect competition, the pattern of intra-industry trade does not depend on factor proportions. Factor proportions cannot explain intra-industry trade in this model precisely because all of the products of the industry are produced in a competitive equilibrium with identical factor proportions. Instead, the aggregate country size determines intra-industry trade in the model. Even in this model, however, the absolute and relative amount of intra-industry trade still depend on factor proportions. The more similar the endowment ratios of the countries and the smaller the size of the capital-abundant country, the larger the share of intra-industry trade in total trade (Helpman (1980, Proposition 4) and Helpman and Krugman (1985, chapter 8)). Thus factor proportions and other variables must be used simultaneously in all models to test the determinants of inter- and intra-industry comparative advantage. In a general equilibrium model neither inter-industry nor intra-industry trade is independent of each other.

APPENDIX

The Borderline Commodity in the DFS Model

Dornsbusch, Fischer and Samuelson (DFS) (1980) considered the pattern of trade in their continuum version of the Heckscher-Ohlin model. The pattern of trade is remarkably simple under free trade (and zero international transport costs). The central feature of the trade pattern is the existence of a borderline commodity. The model is considered here because this feature carries over to the Dixit-Grossman and Falvey models which are extensions of the DFS Model.

There are infinitely many commodities on a continuum defined by the unit interval on the real line, $z \in [0,1]$. It is assumed that for each commodity the production functions have co nstant returns to scale, are differentiable and the derivatives satisfy the Inada conditions. There is a dual unit cost function $c(w,r,z)dz$ where w and r are the price of the factors labour and capital respectively, with differentiability and other corresponding regularity properties. The strong Samuelson factor intensity assumption is made for each commodity with respect to all other commodities. With the strong factor intensity assumption and the further assumption that there are identical technologies in the two countries, the commodities on the continuum can be ordered by decreasing capital intensity. In any one country the relative price (= relative unit costs) of any two commodities is a function of the wage rate/capital rental ratio only, with the relative price of the labour-intensive commodity increasing continuously with the factor price ratio. If factor prices are equalised in the competitive equilibrium the location of production is indeterminate. The competitive equilibrium may, therefore, be taken to be one in which the factor endowments are sufficiently dissimilar so that factor prices are not equalised by trade.

A commodity z will be produced in the home country if the condition

$$c(w,r,z) \leq c(w^*,r^*,z) \tag{31}$$

is satisfied. With distinct factor prices and the assumed restrictions on the technology, the price of commodity z in the home country relative to that in the foreign country is a continuously increasing/decreasing function of z, depending on whether $(w/r) >(<) (w^*/r^*)$. For a given competitive equilibrium, there will be a single commodity, $\bar{z}$, for which the unit costs are equal across the two countries:

$$c(w,r,\bar{z}) = c(w^*,r^*,\bar{z}) \tag{32}$$

This is the borderline commodity. If, for example, the home country has an endowment ratio of the capital used in the continuum industry relative to labour which is greater than that of the foreign country, it will also be capital abundant in the sense that $(w/r) > (w^*/r^*)$ because the preferences have been assumed to be identical and homothetic. The home country produces commodities in the range $[0,\bar{z}]$ with higher capital intensities and the foreign country produces commodities in the range $[\bar{z},1]$. $\bar{z}$ may be produced by one or both countries. Since the only interesting case of trade is that in which factor prices are not equalised, the borderline commodity will be produced with a more capital-intensive technique in the home capital-abundant country compared to that used in the other country.

Free commodity trade will be associated with a partition of the commodity continuum. See Figure 1. This partition determines the range of commodities produced and exported by each of the two countries. There will be partial specialisation in that the two countries produce only a strict subset of the producible commodities which are disjoint except for the borderline commodity which may be produced by both countries, though each country does produce and export infinitely many commodities.

One may consider the effects of introducing a uniform *ad valorem* tariff (t) on all imports in one country, say, the home country, into this model in the same way as Dornsbusch, Fischer and Samuelson (1977) did for the Classical model. The effect is qualitatively the same. There are now two borderline commodities, one which determines the margin of production in the home country and one which determines the margin in the foreign country. For the home country the borderline commodity is that for which the costs of production in the home country are just equal to those of the imports of the commodity

$$c(w',r',\bar{z}_1) = c(w^{*'},r^{*'},\bar{z}_1)(1+t) \tag{33}$$

For the foreign country it is the commodity for which

$$c(w',r',\bar{z}_2) = c(w^{*'},r^{*'},\bar{z}_2) \tag{34}$$

The continuum is now partitioned into three intervals, the two intervals of commodities produced and traded in the two countries which now do not overlap and an intermediate interval of nontraded commodities. See Figure 2. The tariff-imposing country now produces a larger range of commodities and its range of goods *produced* overlaps that of the foreign country as all of the nontraded commodities must be produced in both countries. Assuming the home country is

the capital-abundant country, it produces commodities in the range $[0,\bar{z}]$ with higher capital intensities before the tariff and it now produces in the range $[0,\bar{z}_1]$. The tariff causes it to increase its range in the downstream direction. The increase in the production of labour-intensive commodities at the extensive margin will cause the price of labour which is the scarce factor in the tariff-imposing country to rise and that of capital, the abundant factor, to fall. In the foreign country, under normal assumptions, the factor prices will move in the opposite direction and it now produces in the range $[\bar{z}_2,1]$. Thus, normally, the range of commodities produced will be greater than under free trade for the other country too[10]. The commodities in the range $(\bar{z}_1 - \bar{z}_2)$ will be produced by both countries and not traded.

The effects of introducing a uniform tariff into the Dixit and Grossman or the Falvey model are qualitatively the same on the continuum as in the Dornsbusch, Fischer, and Samuelson version of the Heckscher-Ohlin model[11]. There are now two borderline commodities, one for each country, and a set of nontraded commodities on the interval between these two borderline commodities, as Falvey (1981) noted. A uniform tariff reduces the volume of trade in continuum products at two margins. It reduces the range of commodities traded, as noted. It also reduces the volume of trade in the products that remain traded. Dixit and Grossman (1982) noted that a tariff is unambiguously protective in terms of increasing the range of products produced in the tariff-imposing country but it may be antiprotective in terms of the aggregate labour employed and value added in the protected industry. This also applies to the Falvey model.

NOTES

1. In this model the assumption of infinitely many goods within the group is made because there are infinitely many possible goods specifications. "The economy which has traditionally formed the subject of analysis by economists is one in which there are a finite number of goods, the exact number and properties of which are assumed to be part of the given data. One of the more obvious features of modern industrial economies, however, is that products can be designed to any set of specifications within some range... The present work is devoted to the analysis of such economies, in which the number and specifications of the goods to be produced form part of the solution instead of part of the data." (Lancaster (1979) p. vii).

2. Falvey uses partial equilibrium analysis and does not specify the nature of the other sector or sectors. However, his model can be embedded in a general equilibrium model with one other sector or industry to make it comparable with the other continuum models.

3. The assumption that the ordering of commodities by factor intensity coincides with the ordering by production sequence is strong but it can be replaced by the assumption that stages can be ordered by factor intensity and one unit of production at each stage requires one unit of the stage preceding it in the engineering order. It is the ordering of factor intensity which is economically important in the model.

4. The possibility that the competitive equilibrium is such that borderline commodity is at the end of the continuum, that is, all of the continuum commodities are produced in one country and there is only inter-industry trade between the countries, is ruled out by the Inada condition.

5. This reverses the ordering convention used by Falvey in order to be consistent with the model of Dixit and Grossman but this ordering is arbitrary.

6. Regularity conditions must also be imposed to ensure a maximum and other restrictions may be added, if desired.

7. The assumption of constant returns to scale can be relaxed to non-increasing returns to scale by the standard device of reintroducing fixed factors but it cannot readily be extended to increasing returns because the latter leads to the breakdown of competitive behaviour and requires a complete specification of the model.

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8. The assumption that the utility function is linearly homogeneous or, more generally, that it is homothetic, is sufficient but not necessary for the sub-functions to be linearly homogeneous (see Lloyd (1977)).

9. It is in fact one of the few examples of economically meaningful Leontief aggregation.

10. The comparative static analysis of the world economy is complex and, as is well known, related to the local stability of the model. For the Classical continuum model, Wilson (1980) shows that local stability is satisfied by the assumption that all goods are gross substitutes in consumption. However, the simple structure of the Classical model implies all goods are substitutes in production. In a more general model the substitution effects between pairs of commodities may be positive or negative in both production and consumption. The intra-industry trade models considered here are of this type. Such models are locally stable if the excess demand functions for commodities have the property that all commodities are substitutes for each other (Hatta (1977)). These substitution effects combine substitution in production and consumption.

11. Falvey (1981, p. 510) shows that the range of commodities produced will increase in both countries and Dixit and Grossman (1982, p. 591) note the increase in the range in the tariff-imposing country. However, their analyses are incomplete as Falvey takes account of the changes in the price of the specific factor, capital, but not the price of the mobile factor, labour, and Dixit and Grossman take account of the changes of both factor prices but only in the home country.

REFERENCES

Appleyard, D., P. Conway and A. Field (1989), "The Effects of Customs Union on the Pattern of the Terms of Trade in a Ricardian Model with a Continuum of Goods", *Journal of International Economics*, August, 147-64.

Armington, P. (1969), "A Theory of Demand for Products Differentiated by Place of Production", *IMF Staff Papers*, 16, 159-78.

Blackorby, C., D. Primont and R.R. Russell, (1977), *Duality, Separability and Functional Structure: Theory and Applications*, American Elsevier.

Burniaux, J.M., F. Delorme, I. Lienert and J. Martin (1989), "WALRAS - A Multi-Sector, Multi-Country Applied General Equilibrium Model for Quantifying the Economy-wide Effects of Agricultural Policies", *OECD Economic Studies*.

Bhagwati, J. N. (1972), "The Heckscher-Ohlin Theorem in the Multi-commodity Case", *Journal of Political Economy*, 80, September/October, 1052-1055.

Chipman, J. (1987) , "International Trade" in Eatwell, J., M. Milgate and J. Newman (eds.), *The New Palgrave A Dictionary of Economics*, 2, Macmillan, London, 922-955.

Collins, S. M. (1985), "Technological Progress in a Three country Ricardian Model with a Continuum of Goods", *Journal of International Economics*, 19, 171-179.

Dixit, A. and G. Grossman (1982), "Trade and Protection with Multistage Production", *Review of Economic Studies*, 43, 583-594.

Dornsbusch, R., S. Fischer and P.A. Samuelson (1977), "Comparative Advantage, Trade and Payments in a Ricardian Model with a Continuum of Goods", *American Economic Review*, 67, December, 823-839.

----- (1980), "Heckscher-Ohlin Trade Theory with a Continuum of Goods", *The Quarterly Journal of Economics*, 95, September, 203-224.

Ethier, W. (1984), "Higher Dimensional Issues in Trade Theory", in Jones, R.W. and P. Kenen (eds.), *A Handbook in International Economics*, 1, 131-184.

Falvey, R.E. (1981), "Commercial Policy and Intra-industry Trade", *Journal of International Economics*, 11 (4), 495-512.

Finger, J. M. (1975), "Trade Overlap and Intra-industry Trade", *Economic Inquiry*, 16, July, 581-589.

Haberler, G. (1936), *The Theory of International Trade with Applications to Commercial Policy*, William Hodge and Company, London.

Harris, R. (1984), "Increasing Returns, Monopolistic Competition and International Trade", *Journal of International Economics*, 9, 469-479.

Harris, R. and D. Cox (1984), *Trade, Industrial Policy and Canadian Manufacturing*, Ontario Economic Council, Toronto.

Hasenkamp, G. (1975), *Specification and Estimation of Multiple-Output Production Functions*, Springer-Verlag, Wien.

Hatta, T. (1977), "A Recommendation for a Better Tariff Structure", *Econometrica*, 45, 1859-1869.

Helpman, E. (1981), "International Trade in the Presence of Product Differentiation, Economies of Scale and Monopolistic Competition: A Chamberlin-Heckscher-Ohlin Approach", *Journal of International Economics*, 11, August, 305-340.

Helpman, E. and P. Krugman (1985), *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition and the International Economy*, MIT Press, Cambridge, Mass.

Jones, R. W. (1956), "Factor Proportions and the Heckscher-Ohlin Theorem", *Review of Economic Studies*, 24, 1-10.

----- (1961), "Comparative Advantage and the Theory of Tariffs: A Multi-country, Multi-commodity Model", *Review of Economic Studies*, 29,161-175.

---- (1987), "Heckscher-Ohlin Trade Theory" in Eatwell, J., M. Milgate and J. Newman (eds.), *The New Palgrave A Dictionary of Economics*, 2, Macmillan, London, 620-627.

Krugman, P. R. (1979), "Increasing Returns, Monopolistic Competition and International Trade", *Journal of International Economics*, 9, 469-479.

Lancaster, K. (1980), "Intra-Industry Trade under Perfect Monopolistic Competition", *Journal of International Economics*, 10, May, 151-175.

----- (1984), "Protection and Product Differentiation" in Kierzkowski, H., (ed.), *Monopolistic Competition and International Trade*, Oxford University Press, Oxford.

Lloyd, P. J. (1977), "Two-stage Expenditure Minimization and Some Welfare Applications", *Quarterly Journal of Economics*, 91, September, 531-554.

Lloyd, P. J. and A. G. Schweinberger (1988), "Trade Expenditure Functions and Gains from Trade", *Journal of International Studies*, May, 24, 275-98.

Ruffin, R. and R. W. Jones (1974), "The Neoclassical Ambiguity", *Journal of Economic Theory*.

Smith, A. and A. J. Venables (1988), "Completing the Internal Market in the European Community", *European Economic Review*, 32, 1501-1525.

Srinivasan, T. N. and J. Whalley (1986), *General Equilibrium Trade Policy Modelling*, MIT Press, Cambridge, Mass.

Vickrey, W. S. (1964), *Microstatics*, Harcourt, Brace and World, New York

Wilson, C. (1980), "On the General Structure of Ricardian Models with a Continuum of Goods: Applications to Growth, Tariff Theory and Technical Change", *Econometrica*, 48, 1675-1702.