A Generalised Dynamic Factor Model for the Belgian Economy

Identification of the Business Cycle and GDP Growth Forecasts

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Abstract

This paper aims to identify the Belgian business cycle and to forecast GDP growth using a large data base on short-term conjunctural indicators. The data base consists of 509 indicators containing information on business and consumer surveys of Belgium and its neighbouring countries, macroeconomic variables and some worldwide watched indicators such as the US Institute of Supply Management (ISM) and the OECD confidence indicators. The statistical framework used is the One-sided Generalised Dynamic Factor Model developed by Forni, Hallin, Lippi and Reichlin (2005). The model reduces the variables to their core business cycle information, defined as that part of the variables' variation which is common to the data set. Well-known indicators such as the EC economic sentiment indicator for Belgium and the National Bank of Belgium (NBB) overall synthetic curve contain a high amount of business cycle information.

Furthermore, the richness of the model allows to determine the cyclical properties of the series and to forecast GDP growth all within the same unified setting. We classify the variables into leading, lagging and coincident with respect to a reference business cycle defined as the common variation contained in quarter-on-quarter GDP growth. 22% of the variables are found to be leading. Amongst the most leading variables we find asset prices and international confidence indicators such as the ISM and some OECD indicators. In general, national business confidence surveys are found to be coincident, while consumer confidence seems to lag. Although the model captures the dynamic common variation contained in the data set, forecasts based on that information are insufficient to deliver a good proxy for GDP growth as a result of a non-negligible idiosyncratic part in GDP's variance.

Lastly, we explore the dependence of the model's results on the data set and show through a data reduction process that the idiosyncratic part of GDP's quarter-on-quarter growth can be dramatically reduced. However, this does not improve the forecasts.

Key Words: Dynamic factor model, Business cycle, Leading indicators, Forecasting, Data reduction

JEL Classification: C33, C43, E32, E37

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Résumé

Le présent document vise à identifier le cycle économique belge et à prévoir la croissance du PIB à l'aide d'une vaste base de données sur les indicateurs conjoncturels à court terme. La base de données comporte 509 indicateurs contenant des informations sur les enquêtes menées auprès des entreprises et des consommateurs en Belgique et dans les pays voisins, des variables macroéconomiques et certains indicateurs qui sont surveillés dans le monde entier, comme l'indicateur ISM des Etats-Unis et les indicateurs de confiance des l'OCDE. Le cadre statistique utilisé est le Modèle à facteurs dynamiques généralisé et unilatéral mis au point par Forni, Hallin, Lippi et Reichlin (2005). Le modèle réduit les variables à leur information conjoncturelle de base, définie comme étant la part de la variation des variables qui est commune à la série de données. Des indicateurs bien connus, comme l'indicateur du climat économique de la CE pour la Belgique et la courbe synthétique globale de la Banque nationale de Belgique (BNB), contiennent une grande masse d'informations sur le cycle de l'activité économique.

Par ailleurs, la richesse du modèle permet de déterminer les propriétés cycliques des séries et de prévoir la croissance du PIB, le tout dans le même cadre unifié. Nous classons les variables en variables avancées, retardées et simultanées pour un cycle de référence défini comme étant la variation commune contenue dans la croissance trimestrielle du PIB. La proportion de variables considérées comme avancées est de 22 pour cent. Parmi les plus avancées se trouvent les prix des actifs et les indicateurs de confiance internationaux tels que l'ISM et certains indicateurs de l'OCDE. En général, les variables des enquêtes nationales sur la confiance des entreprises sont considérées comme simultanées, tandis que celles de la confiance des consommateurs paraissent retardées. Bien que le modèle rende compte de la variation dynamique commune contenue dans la série de données, les prévisions fondées sur ces informations sont insuffisantes pour donner une mesure approchée variable de la croissance du PIB du fait de l'existence d'une part idiosyncrasique non négligeable dans la variance du PIB.

Enfin, nous étudions la dépendance des résultats du modèle à l'égard de la série de données et nous montrons, à l'aide d'un processus de réduction des données, que la part idiosyncrasique de la croissance trimestrielle du PIB peut être réduite de façon spectaculaire. Cela n'améliore cependant pas les prévisions.

1 Introduction

According to the well-known definition of Burns and Mitchell (1946), that describes the business cycle as a type of fluctuation found in many economic activities, business cycle research should be characterised by a huge amount of data. The business cycle is in the first place an empirical phenomenon, whereby preferably information belonging to different economic spheres such as national accounts data, financial and monetary variables, retail sales, etc. should be analysed. The common fluctuation in these data series could be described as the business cycle. In the past, econometric theory, as opposed to the availability of macroeconomic information, has not been very helpful in identifying the business cycle since most econometric models are small-scaled and contain only a handful of variables to describe the economy (e.g. VARs). When a lot of series are used, models become hard to identify since the number of parameters that needs to be estimated increases accordingly. To solve this problem, business cycle researchers mostly rely on a reference variable, which gives them an accurate and broad measure of economic activity. The most widely used reference variable is GDP. However, since GDP itself needs to be estimated, it may contain measurement errors and the amount of data it is based upon, while extensive, may not be broad enough to capture all macroeconomic realities.

Burns and Mitchell proposed to analyse the business cycle through an index model, which, by taking contemporaneous averages of the series, summarised their data into a single index. This is what is called the NBER-method and it has been widely used to identify the business cycle (e.g. Zarnowitz, 1992). However, the method is informal and relies to a large extent on arbitrariness both in method and variable selection. The method does e.g. not allow to distinguish data series according to their "usefulness", since all series are equally weighted in the aggregate.

A formal representation of index models can be found in factor models, which, by assuming the data is driven by a few factors, dramatically reduce the dimension and make identification feasible. The factors take on the role of the index in the model of Burns and Mitchell. Under the factor model approach each time series is represented as the sum of two orthogonal components: the common component, which is strongly correlated with the rest of the panel and is a linear combination of the factors, and the idiosyncratic component. In the classic or exact factor model, idiosyncratic components are mutually uncorrelated, which limited its economic applications and maintained the use of informal methods.

Recently serious progress has been made in the theory of factor models through the Generalised Dynamic Factor Model (GDFM) of Forni, Hallin, Lippi and Reichlin, henceforth FHLR (2000b, 2001, 2004, 2005). The model differs from the classic factor model in that it allows the idiosyncratic errors to be weakly serial and cross-sectional correlated to some extent. It thereby combines the so-called "approximate static factor model" of Chamberlain and Rothschild (1983), widely applied in financial econometrics (e.g. Arbitrage Pricing Theory, APT) and the Dynamic Factor Model of Geweke (1977), Sargent and Sims (1977) for which respectively cross-sectional and serial correlation was allowed. The model is dynamic since the common shocks can hit the series at different times as opposed to the static model. The common shocks and components, which are a linear combination of them, are inherently unobservable and are estimated by means of dynamic principal components. While the familiar static principal components are based on an eigenvalue decomposition of the contemporaneous covariance matrix, dynamic principal components are based on the spectral density matrix (i.e. dynamic covariations) of the data and consequently are averages of the data weighted and shifted through time.

In this paper, we explore the richness of the GDFM by applying it to a large data set containing information on the Belgian economy and its indicators. Since the GDFM is based on the spectral density of the data (i.e. dynamic covariations), the model is two-sided in the sense that the common component is a projection onto the leads and lags of the common factors. Therefore, problems arise at the end of the sample since future observations are needed to estimate and forecast the common component. Therefore we have used the one-sided version of the GDFM as proposed by FHLR (2005). In this one-sided model, the common component is a linear combination of contemporaneous and lagged observations only. We identify the Belgian business cycle out of a data base of 509 indicators and study how individual and economic groupings of series behave in relation to the business cycle. The model is very flexible and allows to classify the series into leading, lagging and coincident series with respect to the part of GDP's variance which is common to the data set (i.e. the business cycle) and measure their time delay. Results are reported for individual series and economic groupings, which is highly desirable since it can be used as a guide for assessing the importance of individual macroeconomic indicators as "warning signals" for the Belgian economy.

Moreover, we forecast quarter-on-quarter real GDP growth using the common information in the data set. Through the model's features, this takes place within the same unified setting, which makes the model extremely useful for business cycle analysis. However, we show that the results rely for a great part on the composition of the data set. In general, this is a feature of many econometric models, but for factor analysis a more profound understanding of the relation between the models' outcome and the composition of the data set would be desirable.

The current application is an extension of previous research conducted at the NBB by De Mulder and Dresse (2002). Compared to their model, a larger data base was constructed and the estimation and forecasting is now performed within the same setting, whereas previously it were disjoint operations. Moreover, attention has been paid to the business cycle properties of individual variables and groups of variables.

Because the GDFM is quite novel, applications are few. The best known application is the construction of a coincident indicator for the euro area business cycle (EuroCOIN, Altissimo *et al.* 2001). Using a selection of 951 indicators,[1] they found 4 common factors to explain the aggregate dynamics at business cycle frequency and identified the business cycle as the common component of GDP[2] (i.e. the projection of GDP onto the common factorspace). Furthermore, results on time delays are reported for economic groupings. Examples of the construction of leading, lagging and coincident indicators and forecasting include FHLR (2000a) and FHLR (2003). A larger literature exists on the dynamic factor model estimated through static principal components as proposed by Stock and Watson (1998b, 2002). Since static principal components are solely based on contemporaneous covariations, the method does not allow to directly measure the time lag between the variables and to classify them as coincident, leading or lagging with respect to the business cycle.[3] These papers therefore mainly focus on forecasting and its performance, examples include Artis *et al.* (2002), Boivin and Ng (2003), Hansson *et al.* (2003), Dreger and Schumacher (2002), Stock and Watson (2002) and Watson (2003). Unlike what is done in most of the existing empirical literature, we want to cover the main aspects of business cycle analysis within one single model. This will also allow us to empirically explore the relationship between "estimation performance" and "forecast performance" of the GDFM.

The paper proceeds as follows. Section 2 describes the one-sided GDFM and how it is estimated. In Section 3 the data set is described. Section 4 presents the reference business cycle, identified through the common component of quarter-on-quarter GDP growth and orders the variables according to their degree of commonality. This degree measures the amount of common variation in the total variance of the variables and consequently reveals which indicators include most "business cycle information". Section 5 examines whether the series are pro- or countercyclical and orders them into leading, lagging and coincident variables by measuring their time lag with respect to the reference cycle. Section 6 forecasts quarter-on-quarter real GDP growth and explores the relationship between estimation and forecast performance of the GDFM. Finally, Section 7 concludes and raises some questions for further research.

1 This data set was further reduced in Altissimo *et al.* to 246 variables according to specific criteria concerning delays in publication and estimation/forecasting purposes. We rejected on forehand the variables with long publication delays so that the longest delay with respect to the reference period amongst our 509 indicators is no longer than 3 months. Concerning the second criteria, we perform a similar reduction as in Altissimo *et al.* albeit at some looser criteria delivering 382 series (see Section 6.2.2). Apart from that, we perform a more discriminating reduction exercise with the view on forecasting, resulting in a whole range of subsamples of varying size and composition (see 6.2.3).

2 The resulting business cycle indicator is published each month on http://www.cepr.org

3 Apart from the greater analytic purposes (study of the business cycle properties next to forecasting), FHLR(2003b) show that the dynamic method performs better than the static method of Stock and Watson when there is a substantial heterogeneity in the fraction of variance explained by the common component between the variables (which seems to be the case here, see Section 4).

2 The One-sided Generalised Dynamic Factor Model

2.1 Description

The model used in this paper is the GDFM of FHLR (2000b, 2001, 2004, 2005). The representation theory can be found in Forni and Lippi (2001). The model has been developed in order to deal with large data panels, both in time and cross-sectional dimension. Similar to other factor models, a vector of N time series is represented as the sum of two mutually orthogonal components: a common component driven by a small number $q<N$ of common shocks or factors and an idiosyncratic component related to N variable specific shocks.

The model is called general, since (i) it does not restrict the order of the dynamic loadings of the common factors and (ii) the idiosyncratic component is allowed to be mildly cross-correlated at all leads and lags. Giving up the factor analysis orthogonality conditions between the idiosyncratic components requires assumptions on the eigenvalues of the spectral density matrix of the data to separate the idiosyncratic sources of variation from the common ones and to identify the model. When the series follow a GDFM with q common factors it is required that the first q eigenvalues of the spectral density matrix diverge, while the other eigenvalues remain bounded. After all, the rate of divergence of the eigenvalues indicates "how common" the shocks are. The more they diverge, the more likely the shocks are present in infinitely many cross-sectional units since they keep on contributing in a non-decreasing manner to the variance of a progressively larger panel. This divergence assumption also ensures a minimum amount of cross-correlation between the common components. On the other hand, the boundedness assumption ensures that the idiosyncratic causes of variation, although possibly shared by many units, have their effects concentrated on a finite number of series, and tend to zero as N tends to infinity. FHLR show that the model is asymptotically identified when $(N,T)\to\infty$.

2.2 Estimation[4]

We assume that the N time series included in our panel are, after suitable transformations, a realisation of a real-valued stationary N-dimensional vector process with zero mean $\{x_{it}=(x_{1t},...,x_{nt})';n\in \mathrm{N},t\in \mathrm{Z}\}$. Under the GDFM, satisfying the necessary conditions and assumptions, it is shown that each time series can be decomposed into two components:

$$x_{it}=\chi_{it}+\xi_{it}=\sum_{j=1}^{q}b_{ij}(L)u_{jt}+\xi_{it} \tag{1}$$

4 The model has been estimated using MATLAB. MATLAB procedures were taken from http://www.dynfactors.org and further extended. To perform the data reduction exercise as described in part 6.2.3 an additional algorithm was written in MATLAB.

where χ_{it} is the common component and ξ_{it} the idiosyncratic component. $b_{ij}(L) = B_n(L) = B_0^n + B_1^n L + ... + B_s^n L^s$ represents the (dynamic) loadings of order s, which are allowed to differ in coefficient and lags across the series. The q common shocks $\left(u_{jt}; j = 1,...,q; t \in \mathrm{Z}\right)$ are assumed to be mutually orthogonal white noise processes (at all leads and lags) with unit variance.[5] The idiosyncratic component is driven by variable-specific shocks, for which the GDFM allows a certain amount of correlation. The dynamic factor structure implies that the idiosyncratic component of any series is orthogonal to all common shocks at any lead or lag.

The common shocks u_{jt} are latent and need to be estimated. This is done through the estimation of dynamic principal components. These are obtained by the dynamic eigenvalues and eigenvectors decomposition of the spectral density matrix of x_{nt}, which is a generalisation of the orthogonalisation process of the variance-covariance matrix of x_{nt} in case of static principal components. Contrary to static principal components, the data are shifted through time before averaging along the cross-section, taking into account the whole set of dynamic covariances, whereas static principal components are only based on the contemporaneous covariances.[6] With $N \to \infty$, the dynamic principal components become increasingly collinear with the common shocks. The idea behind this method is that by averaging along the cross-section and by shifting the series through time the idiosyncratic components which are poorly correlated cancel out, whereas the common sources of variation do not. Hence, the factor space spanned by the common shocks and the factor space spanned by the dynamic principal components (which approximate the common shocks) coincide when $N \to \infty$.

The spectral density matrix $\sum_n(\theta) = \left(\sigma_{ij}(\theta)\right)$ of x_{nt} is estimated using the frequency representation of the time series.[7] For each frequency $-\pi < \theta < \pi$, we obtain dynamic principal components through the dynamic eigenvector and eigenvalue decomposition of the spectral density matrix, as outlined in Appendix A.1. The common components are the orthogonal projections of the data on the present, past and future of the first q dynamic principal components. Since the dynamic principal components themselves are a linear combination of the data, the same holds for the common components. The idiosyncratic components are found after subtraction of the common components from the data or equivalently as the projections of the data on the remaining $N-q$ dynamic principal components. The eigenvalue-eigenvector decomposition also allows to split up the spectral density matrix into a spectral density matrix of the common component $\sum_n^{\chi}(\theta)$ (first q dynamic

5 This vector process has a non-singular spectral density matrix, equal to the first q dynamic eigenvalues of the data.

6 More information on dynamic principal components can be found in Brillinger (1975), who shows that the first q dynamic principal components are the best approximation of x_{nt} by means of q linear combinations of the data.

7 The frequency representation of a time series allows to represent a stationary time series by means of its autocovariance function – which summarises its dynamic correlation properties – in the frequency domain. Through the Fourier transform it is decomposed into sized and delayed (co)sine waves of different frequencies (see Harris, 1967).

eigenvalues and eigenvectors) and of the idiosyncratic component $\sum_n^{\xi}(\theta)$ (remaining dynamic eigenvalues and eigenvectors).

The projection coefficients of the common components, $b_{ij}(L)$, are the result of an inverse Fourier transform[8] of the first q dynamic eigenvectors. Since they are dynamic, they are two-sided, both lagged and future values of the common shocks can be loaded. This causes a problem at the end of the sample to estimate and forecast the common component since no future observations are available. To solve this problem FHLR (2005) suggested a refinement of their procedure that retains the advantages of the dynamic approach, while the common component is based on a one-sided filter of the observations. Following this procedure, the factor space is approximated by r static aggregates instead of q dynamic principal components. These r contemporaneous averages are however based on the information of the dynamic approach. The procedure consists of two steps. In the first step, it relies on the dynamic approach, which delivers estimates of the covariance matrices of the common and idiosyncratic component (at all leads and lags) through an inverse Fourier transform of the spectral density matrices. In the second step, this information is used to construct the factor space by r contemporaneous averages, wherein the variables are weighted according to their common/idiosyncratic variance ratio obtained from the contemporaneous covariance matrices estimated in the first step. These r aggregates are the solutions from a generalised principal component problem and have the efficient property of reducing the idiosyncratic disturbance[9] in the common factor space to a minimum, by selecting the variables with the highest common/idiosyncratic variance ratio. The number of aggregates is equal to $r = (q(s+1))$, which is the static rank of the spectral density matrix of the common factors, s indicates the order of the lag operator in (1). It is worth noting that this one-sided refinement is only used to estimate and forecast the common component. The business cycle characteristics such as cyclical behaviour and timing are deduced in the spectral domain, without the need to actually estimate the common component and thus the use of the one-sided approach.

2.3 Parameter values

Prior to the actual estimation, the lead/lag of the cross-covariance matrices and the number of frequencies at which the spectral density is evaluated has to be determined. Details on this issue are provided in Appendix A.1. The number of leads/lags of the covariances is set at 3, and the number of frequencies at 7. Apart from these parameters, the number of common shocks, q, has to be determined. If the data share q sources of variation, then according to the assumptions of the GDFM, the first q dynamic eigenvalues of

8 This transform translates the results found in the spectral domain (dynamic eigenvectors) into a filter in the time domain $(b_{ij}(L))$.

9 When an idiosyncratic component is large it could possibly survive aggregation and be part of the first principal components. However letting $N \to \infty$ and attributing lower weights to the highly idiosyncratic variables reduces this risk and makes the principal components increasingly collinear with the common factors.

the spectral density matrix diverge, while the other remain bounded. This rate of divergence is examined by letting $N \to \infty$ and is thus of asymptotic nature. In finite samples it is not clear how a slowly diverging sequence can be distinguished from an eventually bounded one. Therefore we need to rely on a heuristic inspection of the eigenvalues against the number of series as suggested by Forni and Lippi (2001). Having N series and T observations, spectral density matrices $\sum_g^T(\theta)$, $g \le N,$ for $g \to N$ can be estimated and the corresponding dynamic eigenvalues can be computed for different frequencies. To determine the number of common factors, Forni and Lippi (2001) suggest to take the following rules into account:

1. The average over frequencies θ of the first q eigenvalues diverges when $g \to N$, whereas the average of the $(q+1)$-th eigenvalue remains relatively stable.
2. At $g = N$, there should be a substantial gap between the variance explained by the q-th principal component and the variance explained by the $(q+1)$-th one.

This last rule suggests to add dynamic principal components as long as the increase in explained variance is larger than some pre-specified value. Setting this at 10% (see Altissimo *et al.*, 2001), the number of common factors driving our data set of 509 indicators is equal to 2.

Figure 1a shows the first 20 dynamic eigenvalues averaged over the different frequencies, plotted against the number of series g. From this it can be seen that the first 2 eigenvalues diverge most probably. In Figure 1b, the contribution of the first 20 dynamic principal components to the total variance of the data set is shown. The second dynamic principal component accounts for 10%. Together with the first one, about 38% of the total variance is explained.[10]

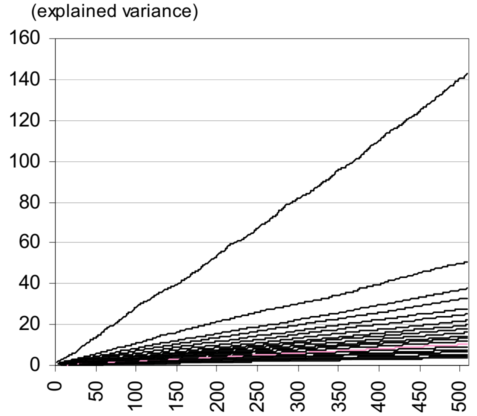
While 2 factors driving 509 indicators may seem low, it is not uncommon for macroeconomic data to be well-approximated by low-dimensional factor structures. Forni and Reichlin (1998) and FHLR (2001) for example also found 2 factors to be helpful in understanding the aggregate dynamics of 450 and 123 series respectively. Moreover, the factors are dynamic and thereby have a more extreme "reducing capacity" compared to static factors. For the actual estimation of the common component we will treat the past values of the common factors as separate static factors, which delivers 8 static factors $(= r)$ out of 2 dynamic ones $(= q)$, which is already considerably higher. In order to avoid a too large number of static factors, we set a relatively strict criterion for the explanatory power of the dynamic factors at 10%. The low number of factors is not only convenient but also ensures the idea of a commonly present business cycle (i.e. the comovement of the series).

10 Altissimo *et al.* (2001) found the first two factors to explain a comparable 34% of the variance for the euro area economy. Contrary to our case, however, they study monthly data – for which commonality is likely to be lower – and also found the next two factors to exceed the 10% condition. As a result, they end up with four common factors, which on aggregate explain 55% of the variance of their data set. Ignoring the discriminative power of our factors, the first four factors would explain 51% of the total variance in our data set. The relatively low share of explained variance in our sample is possibly a consequence of (i) the diversity of the data set and (ii) the more volatile character of the Belgian economy compared to the euro area, for which comovement is likely to be smaller.

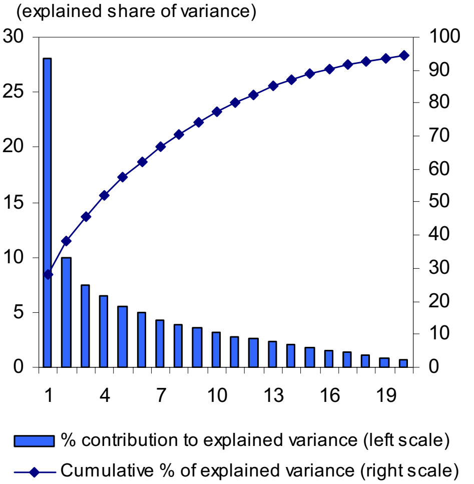
Furthermore, as shown later, the results are quite robust to the number of factors.[11] Although, logically, commonality increases with the number of factors, the results regarding the cyclical features of the series remain largely the same.

Figure 1 **Heuristic inspection of the eigenvalues**

1a
Dynamic eigenvalues[a)] averaged over different frequencies with respect to the number of series



1b
Dynamic eigenvalues[a)] averaged over different frequencies for the total number of series



Vertical axis: $Variance/2\pi$

Horizontal axis: g = number of series of sub-spectral density matrix $\sum_g^T(\theta)$

a) Only the first 20 dynamic eigenvalues are shown.

[11] Results of the model estimated with 3 till 5 dynamic factors can be obtained from the author upon request.

3 Data Set

3.1 Description

Constructing a rich data base is a crucial step in identifying the business cycle through the GDFM. The factors are always defined with respect to a data set, therefore the correctness of the identified business cycle depends on the data set. Furthermore, the data set should be constructed in view of the exercise on hand. We want (i) to identify the Belgian business cycle as the common part of the variance of the reference variable quarter-on-quarter GDP growth (ii) to evaluate the indicators with respect to this reference cycle and examine their cyclical properties (iii) to make reliable forecasts for quarter-on-quarter GDP growth using the common variation.

To that end, we tried to include a large number of variables which are likely to comove with each other and more specifically with Belgian GDP. The large number of variables should allow to "kill" the idiosyncratic variance over the cross-section, ensuring the correctness of the identified business cycle. At first sight, selecting only national economic indicators would be appropriate in order to identify the Belgian cycle. On the other hand, it is known that the Belgian business cycle, due to the openness of its economy, strongly comoves with that of the euro area and more specifically with those of the neighbouring countries, suggesting international indicators could convey important information for the Belgian business cycle as well. Therefore we also included economic indicators for the euro area as a whole, Germany, France and the Netherlands. Possible differences in synchronisation are not a problem given the dynamics of the model. Moreover, including these variables enables us to fully benefit from the dynamic structure of the model. Furthermore, since some international indicators are widely used to assess the state of the business cycle beyond the scope of their national economy (e.g. US Institute of Supply Management (ISM) indicators), we also included some economic indicators from the US, UK and Japan. The results of the model will allow us to check whether these indicators are well suited for assessing the Belgian business cycle.

In contrast to these benefits, including international variables might disturb the extracted business cycle. However, since the method extracts the common shocks present in all variables, these disturbances tend to be rather limited if the number of international variables is not too high. To limit the number of international variables, non-survey variables only include national variables, with the exception of GDP[12] and financial variables. Survey variables include both national and international variables. Additional requirements concerning homogeneity, both across time and cross-section dimension were taken into account.[13] All in all, this provided 509 time series available on a quarterly or monthly basis between 1990Q1 - 2003Q3. The starting

12 Apart from the countries mentioned, we also included GDP series for other euro area countries, as the main indicator of their business cycle, in order to shed some light on their behaviour with respect to the Belgian cycle.

13 Apart from the harmonised surveys of the EC and OECD, survey results from the national sources of the countries were used (obtained through Thomson Financial Datastream). Regarding consistency, we only included those indicators which correspond to the published indicators of the NBB business survey (see http://www.nbb.be/doc/dq/E/dq3/PEC.pdf). If the overall indicator of the national source consists of various overall sub indicators (e.g. IFO overall indicator, which is split into an IFO overall current climate and an IFO overall forecasts indicator), all of these components are applied.

date was the result of a trade-off between obtaining a richer data set and maintaining a relatively large time dimension.

For the purpose of reporting, we regrouped our series in homogeneous groups and subgroups according to their economic classification, source and geographical relevancy. The composition of the data set is shown in Table 1. A first major distinction was made between survey and non-survey data:

Non-survey data represent 36% and were further regrouped into:

- Activity variables (55 variables): National accounts data, industrial production, retail sales, international trade and car sales.
- Labour market variables (32 variables): Employment and unemployment.
- Price variables (47 variables): Consumer prices, producer prices, commodity prices and wages.
- Financial variables (51 variables): Interest rates and asset prices (stock prices, real estate prices, exchange rates and precious metals).

Survey data represent 64% of the data set and were split into two main groups:

- Consumer confidence indicators (77 variables)
- Business confidence indicators (247 variables)

The survey data were additionally split up according to their source: Bank of Japan (BOJ), Statistics Netherlands (CBS), European Commission (EC), Ifo Institute for Economic Research (IFO), National Institute for Statistics and Economic Studies (INSEE), Institute of Supply Management (ISM), Organisation for Economic Co-operation and Development (OECD) and National Bank of Belgium (NBB). For business confidence, further distinctions were made according to the geographical relevant domain of the indicator (Belgium, non-Belgium) and, wherever possible, according to the indicator category:

- Overall indicators
- Manufacturing industry
- Trade sector
- Building
- Capacity utilisation

Together, these distinctions and the variety of data should allow to identify the business cycle and to get a deeper insight into the appropriateness of various indicators as assessing instruments for the Belgian business cycle. Other classifications are possible, but the implemented structure fitted the purposes of the paper the best. Moreover, for some series, individual results are reported in Table 2.

3.2 Data treatment

Before the data can be used, the series need to be transformed. Both the question of interest and the features of the model determine the undertaken procedures. Since the paper aims to identify the business cycle and forecast real GDP quarter-on-quarter growth, the paper focuses on the growth cycle concept of the business cycle (unlike for instance the NBER method, which measures cycles in the level of the series, see Burns and Mitchell, 1946), defined as the quarter-on-quarter variation of the underlying variables. Being measured at a quarterly frequency, the series available on a monthly basis were transformed to a quarterly basis by taking averages, leaving 55 observations between 1990Q1 and 2003Q3. To obtain a meaningful concept of the quarter-on-quarter variations, seasonality was removed where necessary. This was done using the deseasonality procedure Tramo/Seats (see Gomez and Maravall, 1996). Furthermore, all activity variables are expressed in real terms. These are obtained by deflating nominal variables by the CPI index, with the exception of the national accounts concepts which are deflated by their own deflator. For all other variables (e.g. interest rates and exchange rates) both nominal as real concepts were included in the data set.

The estimation of the spectral density and the GDFM requires stationary time series. We opted to apply the same stationary procedure to all series. We first-differenced the series' levels by taking percentage changes compared to the previous quarter and by a simple difference when the level possibly exhibits negative values.[14] We also applied this procedure to the variables which were stationary from the outset. The reason for this is twofold i) having all variables defined in quarter-on-quarter variations enables to capture the growth cycle concept of the business cycle and ii) taking on variables in their level, even when stationary, would seriously disturb the mutual relations in the frequency domain causing phase shifts and thus invalid deduced time lags.[15] Interest rate spreads, which were taken on in levels, are the only exception to this rule. These levels are however stationary and are the result of a cross-sectional difference instead of a difference in time. Given their widely illustrated covariation with the growth cycle concept of the business cycle (e.g. Estrella and Mishkin, 1997), this is common practice.

In a last step, the series were normalised in order to have a zero sample mean and unit variance by subtracting their mean and dividing by their standard deviation. This standardisation is necessary to avoid overweighting of the series with large variance when estimating the spectral density matrix. Afterwards, the common component is denormalised, so as to correspond to the actual series.

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14 For price variables (consumer prices, stock prices, ...) percentage changes with respect to the previous quarter of the index were taken. The suitable transformations were sufficient to render all 509 variables into stationary variables.

15 In general, taking growth rates generates a time series that leads the underlying series (for instance, both the peaks and troughs come earlier) and consequently induces a phase shift with respect to the series in level (Cohen, 2001). Deducing and comparing time lags from both concepts would be improper to do.

4 Identification of the Business Cycle and Degree of Commonality

As stated in the introduction, the business cycle is an empirical phenomenon characterised by comovement of economic time series. According to Stock and Watson (1989), the business cycle can be interpreted as one common factor affecting all time series at the same moment. In our case, such a factor is non-existent, since we allow different common factors to affect the time series at different moments in time.

How can the business cycle then be identified? Through the GDFM, each time series is split into a common component which is a linear combination of the common factors and thereby represents the business cycle information present in the variable, next to an idiosyncratic component which captures variable specific variation unrelated to the business cycle. Having more than one common factor, we can alternatively identify the business cycle as the common component of a particular variable. A priori, being a broad measure of economic activity, the most natural choice would be to define the business cycle as the common component of GDP. Given the wide use of GDP for assessing the business cycle and being the key-issue of our forecasting exercise, we will follow this approach throughout the paper. We are however not interested in GDP itself but in its common component, which is an improvement compared to the use of GDP in its "uncleaned form" as an indicator of the business cycle. Figure 2 illustrates the growth cycle of GDP and its common component. From this figure it can be seen that the common component of GDP generally comoves with the variation in GDP, although its variation is milder and flattens out the sharp growth peaks and troughs of GDP.

Later on, we will evaluate each variable with respect to this reference cycle by analysing the mutual relation between the common component of the series and the common component of GDP, representing the "essential business cycle relation". Of course, other variables could have been used as reference variable.[16] Ideally, the common component should represent a large part of the variance of the reference variable since in practice the common component is not observable and practitioners have to rely on the variable itself in order to assess the state of the business cycle.

The importance of the common component and thus of the amount of business cycle information present in each variable x_i can be measured by the degree of commonality C_i :

$$C_i = \frac{\operatorname{var}(\chi_i)}{\operatorname{var}(x_i)}$$

16 Since all mutual relations between the common components of all variables are known, variables can be easily assessed with respect to another variable than GDP or with respect to each other.

Figure 2 **Quarter-on-quarter GDP growth and its common component**

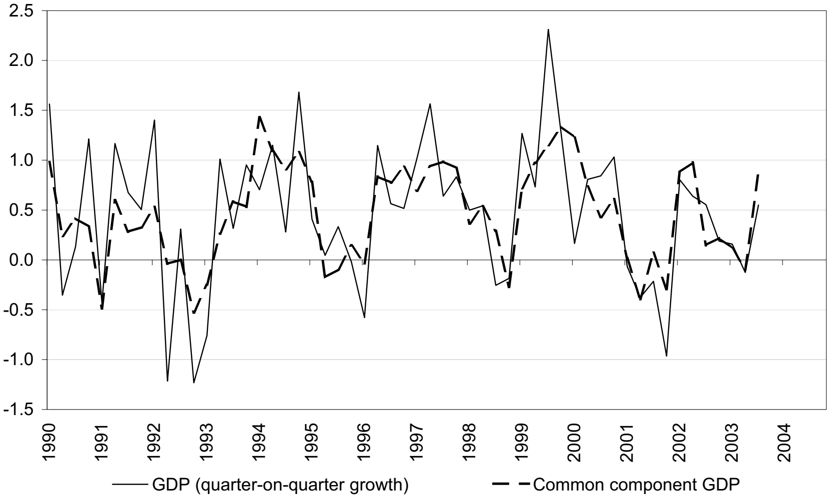

Table 1 shows the degree of commonality for our different regroupings. Individual results, ranked according to the importance of the common component, are reported in Table 2.

Averaging over all series, the common component represents 30% of the series' total variance. This may seem low, but one has to remember that we focus on the growth cycle of quarter-on-quarter variations. The degree of commonality would possibly have been higher when year-on-year variations were used, since macroeconomic variables rather tend to comove in the long run than in the short run.[17] On average, the commonality of survey data is higher than that of non-survey data, meaning that a cleaner business cycle signal is obtained from survey data compared to non-survey data. Within the non-survey data, variables related to the labour market show the highest commonality. Price variables show the lowest degree of commonality.

Looking at individual variables, GDP has a relatively high common component. 47% of the variation of GDP is related to the business cycle, justifying the choice of GDP as reference variable. However, there still remains a non-negligible idiosyncratic component and 109 variables have a larger commonality ratio than GDP, suggesting that these indicators better represent the business cycle. In Section 6.2.3 we will shed further light on the appropriateness of GDP as reference variable. From Table 2, it can be seen that the

17 Estimations of the model with year-on-year data show that the average commonality of the series increases to 47%, using the same model parameters. Although quarter-on-quarter results are less appealing in terms of commonality, they are preferred above year-on-year variations given that they provide more insight in the momentum of the economy and they allow cleaner forecasts (no base effect), see also IMF, 2001.

commonality ratio of the variables ranges between 81.6% and 1.1%. The highest commonality ratio is found for the economic sentiment indicator of Belgium from the EC. Among all indicators, this indicator should consequently give the most adequate view of the state of the business cycle. This is a nice result given the fact that it is a broadly composed indicator, which is widely used by practitioners for assessing the business cycle. In fact, the construction of this indicator (i.e. weighted average across the cross-section dimension) is closely linked to the technique of the GDFM (i.e. weighted average across the cross-section dimension of time-shifted series). Within the top-5 of the indicators containing the most business cycle information, also the NBB overall synthetic curve is found, for which the same remarks hold as for the EC economic sentiment indicator. More surprising is the fact that the EC industrial confidence indicator for the euro area and the OECD leading indicator for the Netherlands are found to have a high fraction of business cycle information. This possibly hints at a strong relationship between the Belgian business cycle and the business cycle of the euro area and the Netherlands. Looking at the indicators with the lowest share of common variation, we find among others Japanese GDP and Belgian government consumption. The high idiosyncratic component of Belgian government consumption is interesting given the fact that it is a part of GDP.[18] Nonetheless, its common component is small, suggesting variation of government consumption is unrelated to the business cycle and more likely evolves according to (idiosyncratic) government actions.

Some robustness analysis shows that the relative suitability of an indicator to represent the business cycle is not very sensitive to the number of factors. Raising the number of dynamic factors to as much as 5 does not affect the results to a great extent. Although, quite naturally, commonality increases and ranges between 87.3% and 7.0%, the ranking of the variables according to their commonality only slightly changes. The top-10 of indicators is largely similar and variables such as Japanese GDP and Belgian government consumption are still ranked among those with the lowest commonality.[19]

18 In general, there is a large dispersion in the commonality ratio of the national accounts variables. While the value-added concepts, apart from the non-market services, have a relatively high degree of commonality, the expenditure components are characterised by a high idiosyncratic component. This is striking, since once they are aggregated (i.e. GDP), they adequately represent the business cycle, while this is not the case when they are taken separately.

19 Within the top-10 of the series with the highest commonality, some indicators change from position. The EC economic sentiment indicator for Belgium has the second highest commonality ratio, while the first place is occupied by the OECD composite leading indicator for Belgium. However, apart from these minor ranking changes, the overall conclusions regarding the relative appropriateness of a variable and/or type of series to represent the business cycle are still valid. Detailed results of the estimation of the GDFM with more than 2 dynamic factors are available from the author upon request.

Table 1 **Data set and cyclical properties**

Series	Nr. of series a)	C b)	T c)	Series	Nr. of series a)	C b)	T c)
TOTAL	509 (100)	30	-0.1				
Non-Survey	185 (55)	23	-0.3				
Activity	55 (1)	20	-0.5	*Labour Market*	32 (24)	35	-1.7
National Accounts	26 (1)	23	-0.6	Unemployment	21 (21)	41	-1.9
Belgium	15 (1)	22	-0.4	Employment	11 (3)	24	-1.1
International	11 (0)	24	-0.8	*Financial*	51 (14)	23	**1.2**
Industrial Production	15 (0)	20	-1.1	Interest Rates	22 (1)	27	-0.8
Retail Sales	8 (0)	11	-0.4	10 Year	7 (0)	30	0.1
International Trade	2 (0)	37	-0.1	3 Month	8 (0)	24	-2.1
Car Sales	4 (0)	12	**2.8**	Yield gap	7 (1)	27	-0.1
Prices	47 (16)	18	-0.7	Asset Prices	29 (13)	21	**2.6**
Commodity Prices	13 (3)	18	-1.0	Stocks	9 (0)	23	**2.3**
Producer Prices	24 (4)	21	-0.5	Real Estate	7 (0)	15	**3.3**
Consumer Prices	8 (7)	9	-0.3	Gold	1 (1)	9	-3.2
Wages	2 (2)	5	-1.9	Exchange Rate	12 (12)	23	**3.0**
Survey	324 (45)	34	0.0				
Business Confidence	247 (26)	37	0.3				
NBB	55 (10)	31	0.3	IFO	31 (1)	39	0.5
Overall	1 (0)	78	0.6	Overall	3 (0)	67	0.2
Manufacturing	11 (1)	57	0.7	Manufacturing	8 (1)	54	0.3
Trade	9 (1)	20	-0.2	Trade	6 (0)	40	0.5
Building	10 (0)	22	**1.1**	Building	10 (0)	13	**1.4**
Capacity Utilisation	24 (8)	25	0.0	Capacity Utilisation	4 (0)	53	-0.7
European Commission	109 (10)	38	0.0	ISM	17 (3)	23	**1.9**
Belgium	25 (3)	40	0.4	INSEE	10 (1)	57	-0.2
Overall	1 (0)	82	0.2	CBS	2 (1)	52	0.0
Manufacturing	8 (1)	63	0.3	BOJ (Tankan)	5 (0)	21	0.5
Trade	6 (1)	17	-0.2	OECD	18 (0)	59	**1.1**
Building	5 (0)	27	0.9	Belgium	11 (0)	63	0.9
Capacity Utilisation	5 (1)	37	0.9	Overall	1 (0)	77	0.9
Non-Belgium	84 (7)	37	-0.1	Partial Indicators	10 (0)	62	0.9
Overall	4 (0)	68	-0.3	International (Overall)	7 (0)	51	**1.4**
Manufacturing	30 (4)	57	-0.5	*Consumer Confidence*	77 (19)	24	-1.1
Trade	24 (3)	13	-0.1	NBB	12 (2)	25	-0.9
Building	20 (0)	30	0.1	European Commission	65 (17)	23	-1.2
Capacity Utilisation	6 (0)	36	0.4				

a) Total number of series in each group, the number of countercyclical variables is indicated between brackets.
b) Commonality ratio of the series, measured as the ratio between the variance of the series' common component and the series' total variance. The reported values are averages.
c) Time lag of the series' common component with respect to the common component of GDP, measured in quarters. Variables are considered to lead GDP when $T > 1$, to lag when $T < -1$ and to coincide otherwise. The reported values are averages. Bold values indicate variables which lead GDP.

5 Cyclical Behaviour of the Variables

Having identified the business cycle information in each variable as the common component, we can now evaluate each variable's cyclical behaviour with respect to the reference cycle. To do so, out of the spectral density matrix of the common component $\sum_n^\chi(\theta)$, the cross-spectral density of each common component is calculated with respect to the common component of GDP: $\sigma_{i,GDP}(\theta)$. From this density,[20] we can estimate the phase angle shift $\phi_{i,GDP}(\theta)$ or time lag $\psi_{i,GDP} = \phi_{i,GDP}(\theta)/(\theta)$ of the common component with respect to the common component of GDP, allowing us to classify the series as pro- or countercyclical and as coincident, leading or lagging.

5.1 Pro- and countercyclical variables

Following FHLR (2000a) we classify the series as pro- and countercyclical by computing the phase angle shifts at the zero frequency with the reference cycle: $\phi_{i,GDP}(0)$. At this frequency, the long-run correlation between the two common components is measured. Depending on whether this long-run correlation is positive or negative, the phase angle shift will be either 0 or π. A positive long-run correlation $(\phi_{i,GDP}(0)=0)$ is interpreted as a procyclical variable, a negative $(\phi_{i,GDP}(0)=\pi)$ as a countercyclical one (see also Granger and Hatanaka, 1964).

Overall we find 409 variables to be procyclical and a minority of 100 to be countercyclical (see Table 1). The procyclical variables include, among others, all expenditure and value-added components of GDP, which is consistent with common knowledge. The only exception to this are the inventory changes which seem to behave countercyclical.[21] Also the assessment of their level in business surveys is classified as countercyclical.[22] Furthermore, all series related to unemployment are countercyclical.

20 The cross spectral density $\sigma_{ij}(\theta)$ out of $\sum_n^\chi(\theta)$ represents the mutual relation between two common components which can be written in the frequency domain as the sum of waves of different frequency, amplitude and phase. The phase angle shift $\phi_{ij}(\theta)$ measures how much a wave i (and thus common component) is shifted with respect to a reference wave j, measured at a particular frequency θ. The phase angle shift can be translated to a time lag ψ_{ij} in the time domain by dividing it by its frequency θ.

21 This might point at involuntary inventory adjustments (depletion during booms and build-up during recessions). Possibly, part of this involuntary inventory adjustment is due to the buffer function inventories fulfil in order to cope with large export and import flows in a small open economy as the Belgian one.

22 This underpins the methodology for the construction of the NBB overall synthetic curve, according to which the sign of the individual indicators related to the inventories is reverted (National Bank of Belgium, 1990).

Amongst the financial variables, the exchange rate and gold price behave in an opposite way to the reference cycle. More surprising is the countercyclical behaviour of consumer prices.[23]

The classification of variables as pro- or countercyclical is robust to the number of factors. When the number of factors is raised to 3, no single variable is classified differently. Only when the number of factors is further raised to 4 or 5, a marginal fraction of the variables (respectively 1% and 1.8%) change from sign.[24]

5.2 Coincident, leading and lagging variables

5.2.1 Classification

A classification of the variables into coincident, leading or lagging can be obtained by evaluating the phase angle shift at a typical business cycle frequency θ^*. There is no such thing as a standard frequency or length of a growth cycle. However, looking at the estimated common component of GDP and defining the length of a cycle as the sum of the length of a boom (defined as an above-average common component) and the length of the subsequent recession[25] (defined as a below-average common component), 4 cycles occurred since 1990 with an average length of 13 quarters or about 3 years. Taking 3 years as the length of a typical business cycle, we calculated the phase shift at a frequency $\theta^* = \pi/6$. Dividing the obtained phase angle shifts by this frequency delivers the time lags $\psi_{i,GDP}$. Variables were classified as leading when the time lag exceeded 1 (quarter), lagging when it was lower than -1 and coincident otherwise. The time lags are reported in Tables 1 and 2.

From the 509 variables, we find 22% to be leading, 27% to be lagging and 51% to be coincident. In general non-survey data contain a higher proportion of leading and lagging indicators compared to survey data. Categories with a high proportion of leading indicators include financial variables, for which asset prices contain an overwhelming amount of leading indicators, and international confidence indicators, such as the indicators of the ISM-institute and the international OECD confidence indicators. On the other hand, a high proportion of lagging indicators is found within the variables related to the labour market, consumer prices

23 The countercyclical behaviour of consumer price inflation is robust, both to the consumer price aggregate and to the number of factors, and henceforth is supportive for an economy dominated by supply shocks instead of demand shocks, linked to the discussion between the RBC school and the more common Keynesian view. While looking at the reasons for the illustrated countercyclical behaviour of inflation is beyond the scope of the present work, we can argue that as suggested in the literature (Backus and Kehoe, 1992 and Chadha and Prasad, 1994), results depend on the period studied, the method employed and the measure of inflation used. The illustrated countercyclical behaviour might thus be specific to the sample and/or due to the difficulties the method has to capture a great deal of inflation's variation, even when the number of factors is raised.

24 In particular the countercyclical behaviour of employment in the trade and public sector seems to depend on the number of factors. This might point to the absence of a clear connection between the business cycle and employment in these sectors, which certainly is defendable for the latter.

25 Taking into account a minimal duration of 2 quarters for the booms and recessions.

and consumer confidence. Whereas consumer confidence generally tends to lag the business cycle, business survey indicators generally coincide.[26]

Generally, the most important expenditure components are classified as coincident, which is compatible with the idea that the business cycle is a phenomenon that roughly occurs at the same time for most expenditure components.[27] Private investment and imports seem to lag the business cycle. For investment, these results are in line with reported business cycle facts for the US and the euro area (see Stock and Watson, 1998a and Bergman *et al.*, 1998).

Similar to the pro- and countercyclical behaviour, the classification of variables into coincident, leading or lagging is robust to the number of dynamic factors. The amount of leading variables raises somewhat from 22% to 24% when 5 instead of 2 dynamic factors are used, while the percentage of lagging variables diminishes slightly from 27% to 24%. Also, all variable groups keep their classification. The only exception to this are wages, which are classified as leading instead of lagging. Furthermore, also the extent of the lead/lag, reported in 5.2.2, seems not very sensitive to the number of factors.

5.2.2 Time lag

Looking at the estimated time lags in Tables 1 and 2, we find the highest leads in the category of asset prices.[28] Both the exchange rate and stock prices as well as real estate prices are found to lead the business cycle by 2 to 3 quarters. Within the non-survey data, the only other category of variables that leads the business cycle are car sales, which are found to lead by almost 3 quarters. Looking at the survey data, the ISM indicators are found to lead the Belgian business cycle on average by almost 2 quarters. Furthermore, the IFO confidence indicators concerning the German building sector have a lead that hovers around 1.5 quarters. The NBB survey indicators related to the Belgian building sector also lead, albeit by 1 quarter. Finally, the international OECD confidence indicators are found to lead the Belgian business cycle, on average, by about 4 months.

26 The coincident behaviour of the national business survey indicators might somewhat be puzzling compared to their documented leading behaviour (Vanhaelen *et al.*, 2000). Note however that these conclusions are still valid as shown in footnote 29. National business survey indicators are only found to be coincident with respect to the national business cycle, measured by the common component of Belgian GDP. As stated in footnote 29, national business survey indicators still lead euro area GDP and most of its survey indicators.

27 The coordination between the business cycle and government outlays is less evident. Here they are found to lead the business cycle. As documented in Stock and Watson (1998a) and Bergman *et al.* (1998), the cyclical properties of government outlays are diverse and do not show a clear pattern across time or countries.

28 In a limited number of cases (18 variables), the calculation of the time lags delivered leads and lags exceeding 4 quarters. Since it is difficult to say whether these variables lag the previous cycle or lead the next one and since they are disturbing for the reported results, we excluded them from the calculation of the average time lag of the different regroupings in Table 1 and from the rankings in Table 2.

Table 2 **Some detailed results**

Variable[a)]	Group	Source[b)]	C[c)]	T[d)]
Top 10 highest commonality ratio				
Economic sentiment indicator: BE	Business confidence	EC	81.6	0.2
Industrial confidence indicator: EUR12	Business confidence	EC	79.8	-0.2
Composite leading indicator (Trend restored): NL	Business confidence	OECD	79.6	0.5
Industrial confidence indicator: BE	Business confidence	EC	78.1	0.4
Overall synthetic curve	Business confidence	NBB	77.8	0.6
Manufacturing industry: Synthetic curve	Business confidence	NBB	77.6	0.7
Manufacturing industry: Synthetic curve	Business confidence	INSEE	77.5	-0.1
Composite leading indicator (Amplitude adjusted): BE	Business confidence	OECD	77.4	0.9
Composite leading indicator (Trend restored): BE	Business confidence	OECD	77.3	0.9
Economic sentiment indicator: EUR 12	Business confidence	OECD	76.7	-0.4
Top 5 lowest commonality ratio				
Building: Trend equipment	Business confidence	IFO	1.1	2.4
GDP Japan	National accounts	EC	1.7	-3.2
Foodstuffs*	Consumer prices	NIS	1.8	2.6
Food excluding restaurants*	Consumer prices	OECD	2.0	2.8
Government consumption	National accounts	NAI	2.5	1.8
Top 10 most leading series				
Real estate return index: BE	Real estate	DS	11.1	3.8
Building: Assessment of order book	Business confidence	IFO	11.4	3.8
Retail: Present business situation: NL	Business confidence	EC	7.1	3.7
Real estate return index: US	Real estate	DS	15.5	3.6
Real estate return index: JA	Real estate	DS	11.8	3.5
Real estate return index: FR	Real estate	DS	5.4	3.5
Brussels stock exchange return index	Stock prices	DS	9.1	3.4
Euro (Belgian Franc) / US $: Monthly average*	Exchange rate	DS	20.5	3.4
Consumer confidence: Savings at present: NL*	Consumer confidence	EC	4.1	3.4
Real effective exchange rate (Unit labour cost based): BE*	Exchange rate	DS	9.4	3.3
Top 5 most lagging series				
Major purchases at present: NL	Consumer confidence	EC	9.0	-3.9
Statement on financial situation of households: NL	Consumer confidence	EC	4.2	-3.9
Statement on financial situation of households: EUR 12*	Consumer confidence	EC	9.9	-3.9
Harmonised unemployment rate: Female*	Unemployment	EC	28.8	-3.8
Total national employment	Employment	NAI	42.1	-3.6

Table 2 **Some detailed results (continued)**

Variable[a)]	Source[b)]	C[c)]	T[d)]	Variable[a)]	Source[b)]	C[c)]	T[d)]
GDP	NAI	46.7	0.0	Manufacturing industry	NAI	56.3	-0.4
Private consumption	NAI	18.6	-0.4	Building	NAI	18.7	-0.7
Government consumption	NAI	2.5	1.8	Services	NAI	24.4	-0.9
Business investment	NAI	6.8	-2.9	Market services	NAI	22.4	-1.1
Housing investment	NAI	11.1	-1.4	Non-market services	NAI	9.1	-0.1
Government investment	NAI	9.5	2.2	GDP Euro area	EC	41.7	-0.9
Exports	NAI	27.8	-0.7	GDP France	EC	45.8	-1.0
Imports	NAI	17.8	-1.2	GDP Germany	EC	15.0	-1.6
Changes in inventories*	NAI	3.5	0.0	GDP Netherlands	EC	33.4	-0.7
Industry	NAI	55.0	-0.2	GDP US	EC	24.3	0.8

a) Countercyclical variables are indicated by an asterisk.

b) The different sources are *DS*=Thomson Financial Datastream, *EC*=European Commission, *IFO*=Ifo Institute for Economic Research, *INSEE*=National Institute for Statistics and Economic Studies, *NAI*=National Accounts Institute, *NBB*=National Bank of Belgium, *NIS*=National Institute of Statistics and *OECD*=Organisation for Economic Co-operation and Development.

c) Commonality ratio of the series, measured as the ratio between the variance of the series' common component and the series' total variance.

d) Time lag of the series' common component with respect to the common component of GDP, measured in quarters. Variables are considered to lead GDP when $T > 1$, to lag when $T < -1$ and to coincide otherwise.

Highly lagging variables include the labour market variables, which are estimated to lag the business cycle on average by about 2 quarters. Furthermore, industrial production seems to lag the business cycle by 1 quarter, which is somewhat surprising given it is widely used as alternative for GDP when measuring the business cycle. Another surprising result is the lagging behaviour of short-term interest rates, which lag the reference cycle by about 2 quarters. Lastly, consumer confidence is found to lag the business cycle by about one quarter.[29]

29 It is worth noting that each variable can be easily evaluated with respect to another. If, for instance we were interested in the behaviour of the variables with respect to the euro area business cycle, we could have evaluated them with respect to the common component of euro area GDP. Since the time lags are mutually consistent, the time lags with respect to this new reference cycle can be simply calculated out of Table 2. From this table we see that the euro area business cycle lags the Belgian cycle by almost one quarter. Taking into account this time delay, the EC industrial confidence indicator for Belgium would lead the euro area business cycle by 1.3 quarters (=0.9+0.4), showing its leading behaviour for the euro area business cycle, similar to the results found on a more classical basis by analysing timing differences in turning points in Vanhaelen *et al.* (2000). These results show that the leading behaviour of national survey indicators for the euro area is largely attributable to the lead Belgian GDP has with respect to that of the euro area. A similar conclusion holds for overseas indicators such as the US ISM indicators, which partly lead the Belgian economy since US GDP leads the Belgian business cycle by almost one quarter. Contrary to most euro area confidence indicators, the ISM indicators however also seem to lead their national GDP by one quarter.

6 Forecasting

6.1 Forecasting GDP by means of its common component

In this part we try to forecast quarter-on-quarter real GDP growth using the GDFM. Within the GDFM, forecasts of the x_i's can be obtained by separately forecasting the common component and the idiosyncratic component given the components' orthogonality to each other at all leads and lags. Since the idiosyncratic components are mutually orthogonal or weakly correlated, their forecast can be obtained using traditional univariate methods or low dimension models, such as ARMA or VAR models. The common component, being based on the common factorspace, can be forecasted by projecting the variables on carefully constructed aggregates approximating this (theoretical) common factorspace. In Appendix A.2 it is shown that the common component can be best estimated and forecasted by constructing aggregates in which variables with a higher commonality ratio receive larger weights. The commonality ratio is important in two perspectives for the forecast of a particular variable (i) the higher the common component of the variables, the more accurate the common factor space will be approximated and thus the more accurate the forecast of the common component will be (ii) the higher the commonality ratio of the variable of interest, the less one has to rely on popular, but in general unsatisfying models to forecast the idiosyncratic component. If for example the commonality ratio of x_i were a hypothetical 100%, the forecast of x_i would coincide with the forecast of the common component and one could totally rely on the GDFM to forecast the variable of interest.

Since this paper focuses on the features of the GDFM and not on those of ARMA or VAR models, we limit ourselves to the forecast of the common component of GDP and examine how well this serves as a proxy for realised GDP growth in an out-of-sample exercise. It is thought that the quality of this proxy depends both on the commonality of the included variables and on the commonality of GDP itself. By changing the size and composition of our data set we will look in a first section at the influence of the commonality of the included variables, in a second section we will pay attention to the importance of GDP's commonality.[30] We first report the forecast results for the full data set.

30 The idea that a subsample, and thus smaller N, might provide better forecast results is not new and is amongst others stated in Boivin and Ng (2003). However, while they focus on the role of N, we investigate the influence of the properties of the included variables on the forecast performance.

6.2 Out-of-sample exercise

6.2.1 Full data set

In this exercise, out-of-sample forecasts[31] of the common component of GDP are calculated over the period 1997Q3 till 2003Q3 for different forecast horizons $h = 1,...,3$ (1 till 3 quarters ahead). The initial estimation window contains 30 observations[32] from 1990Q1 till 1997Q2 and forecasts are made for 1997Q3 till 1998Q1. A second estimation and forecasting round estimates from 1990Q1 till 1997Q3 and produces forecasts for 1997Q4 till 1998Q2. At the end, forecasts are truncated to 2003Q3 so that when the model is estimated in the last step till 2003Q2, only the one-step ahead forecast is retained. This delivers a total of 25, 24 and 23 observations for respectively the one-step, two-steps and three-steps ahead forecasts. For each forecasting horizon we measure how well these forecasts proxy real GDP growth by means of forecast performance statistics such as the mean absolute error (MAE) and root mean squared error (RMSE). Furthermore, the results are compared with the one obtained by an ARMA(4,4) model for GDP by means of the Diebold-Mariano test for equal forecast accuracy. Table 3 presents the results.

For the full model (GDFM, $N = 509$), the MAE and RMSE are quite high given an average GDP growth rate of 0.5% with a standard deviation of 0.7. Both the MAE and RMSE slightly increase over the forecast horizon. Although the RMSE almost equals the standard deviation of GDP and thus points to poor forecasting results, they outperform those obtained by an ARMA model. The gain as indicated by the Diebold-Mariano test is significant for the longest horizon at the 10% level. Furthermore, one has to remember that this is only a partial forecast (i.e. forecast of the common component) and forecast results for GDP growth would improve if an accurate forecast of the idiosyncratic component was added.[33] Moreover, theory suggests that the proxy of the common component would be better if a number of variables with a high idiosyncratic component were excluded. Therefore in the next session we will evaluate the accuracy of the forecasted common component of GDP as a proxy for real GDP growth using a reduced data set excluding variables with a low commonality ratio.

31 It concerns pseudo out-of-sample forecasts since only one vintage of historical data is used to estimate and forecast the model. A more useful measure of the forecast performance would be offered by real-time out-of-sample forecasts which use the actual data vintages available at the time the forecast is constructed. As it would obviously take some data storage capacity to keep track of several different vintages of the 509 series, pseudo out-of-sample forecasts were nevertheless preferred.

32 A minimum amount of 30 observations was set in order to allow for a reasonable forecasting horizon and to meet the estimation criteria of the GDFM (relatively large T).

33 In practice, it seems however difficult to forecast the idiosyncratic component given its "white noise character". Estimates show that adding a forecast of the idiosyncratic (generated by an ARMA model) to the forecast of the common component actually leads to even worse forecasts of GDP, although the difference is not significant.

Table 3 **Forecast performance for quarter-on-quarter growth of real GDP**

	1-step ahead	2-steps ahead	3-steps ahead
GDFM, $N = 509$			
MAE	0.46	0.51	0.51
RMSE	0.61	0.66	0.67
Diebold-Mariano[a)]	-1.41	-1.28	-1.76*
GDFM, $N = 382$			
MAE	0.43	0.48	0.53
RMSE	0.60	0.63	0.68
Diebold-Mariano[a)]	-1.55	-1.56	-1.64
ARMA			
MAE	0.57	0.61	0.68
RMSE	0.71	0.77	0.93

a) This statistic tests for equal forecast accuracy between two competing models and is based on the difference of the squared forecast errors of the two models. Here it is used as performance statistic of the GDFM with respect to an ARMA (4,4).

*** (**, *) denotes significant negative values at the 1 (5,10) percent level for which the null of equal forecast accuracy is rejected in favour of a better forecast performance of the GDFM.

6.2.2 Influence of the commonality of the included variables

The average commonality ratio of the variables in our data set amounts to 30% (see Section 4). There is however a wide dispersion with commonality ratios ranging from 81.6% to 1.1%. Given the high proportion of the idiosyncratic component in some cases, these variables do not deliver valuable information and likely only "disturb" the estimation and the forecast of the common component. To extract a cleaner forecast, we perform a data reduction selecting the variables with a relatively high commonality ratio, defined as the 75% percentile of the ordered commonality ratios. This rule suggests to exclude all variables with a commonality ratio lower than 14.4%. The reduced data set contains 382 variables. For this reduced data set (GDFM, $N = 382$), we extract the common component of GDP and calculate whether its forecasts are a better proxy for real GDP growth as compared to the forecasts obtained from the full model. From Table 3 it can be seen that the resulting forecasting errors do not differ a lot from the full model. The model slightly outperforms the full model at the short horizons but performs worse for the three-steps ahead forecasts.

Overall, the differences are not significant. Consequently, "getting rid of the dirt" does not seem to alter the forecasting results to a great extent.[34]

6.2.3 Importance of the commonality of GDP

Throwing away the bad variables (i.e. low commonality), does not alter the forecast performance significantly. This might be not so surprising since in the one-sided model variables with a high idiosyncratic component receive low weights, so that throwing them away might not be that different. On the other hand, it might indicate that the variables thrown away were not that poor in forecasting GDP. A possible explication why this might be the case lies in the fact that while those variables' idiosyncratic component might be high with respect to the whole sample, it might be low with respect to GDP. Let us clarify. While the whole set of variables is explained by two factors, GDP can possibly be determined by for instance one of these factors and some other factors common to GDP and a small subset of variables. However, when the full model is estimated, these factors are likely to remain hidden in favour of the dominant factors, common to a larger subset of variables (preferably all), causing the idiosyncratic component of the subset of variables to be high. While these hidden factors (and thus variables which load upon them) are unimportant to identify the business cycle, they are important when one wants to predict a certain variable. Chances that such factors exist are higher when commonality is low. Since 53% of the variation of GDP remains unexplained, there seems to be room for improvement by selecting variables which are determined by the same factors that underlay GDP, driving up its commonality ratio and thus the forecast accuracy of GDP through the common variation.[35] Higher forecast accuracy could thus be reached by constructing a subsample in which the factors driving GDP are dominant.

However, since the factors are unidentified in the GDFM, there is no formal method to extract these factors and to retain the variables which load upon them. A sign that a subsample contains the "right" factors is however provided by the commonality ratio of GDP. The higher the commonality, the more likely it is that the factors which drive GDP are driving the sample. We therefore propose a data reduction procedure based on a maximisation of GDP's commonality. This procedure is empirical and proceeds as follows. We construct all

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34 Not only the forecasting results alter a little, also the estimated common component is not clearly different from the full model. Nevertheless, the commonality ratio of GDP raised somewhat from 47% to 49%.

35 Note that alternatively, we could increase the number of factors in order to improve GDP's commonality. However, the more factors are added, the more likely they are common to an increasingly smaller subset of the data, which not necessarily includes our variable of interest (GDP). To avoid a pick-up of such "junk factors" that would deteriorate the forecast of GDP we try to select the "right factors" by a data reduction process aimed at maximising GDP's commonality. Moreover, simulations of the GDFM with more than 2 factors show that the improvement for GDP's commonality stays limited compared to the data reduction exercise (commonality of 59.9% in case of 5 factors) and that they do not lead to significant better forecasts.

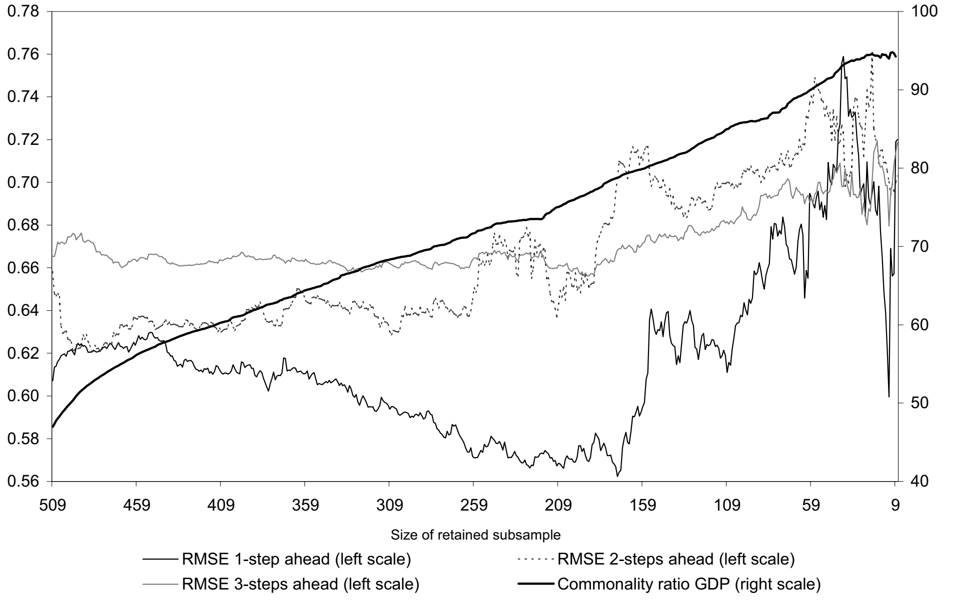
possible $N-1$ subsamples of $N-2$ variables out of all variables excluding GDP. To these subsamples we add GDP and estimate for each subsample the GDFM. The subsample delivering the highest commonality ratio for GDP is retained. This subsample will contain all variables except one. This variable drags down the commonality of GDP the most and is therefore excluded. In a second step we repeat the procedure on the retained sample, which this time allows $N-2$ combinations of $N-3$ variables to which we add GDP. Again one variable will be left out. We repeat this procedure until the number of variables of the subsample (including GDP) equals the number of factors. In total this requires the estimation of a GDFM for:

$$\frac{(N-1)N}{2}-\frac{(r-1)r}{2}=129\,258 \text{ data sets}$$

The retained subsamples contain for a given $n\,(r<n<N)$ the variables which maximise the common component of GDP out of the previous subsample. The amount of progress obtained by this data reduction in terms of the commonality of GDP is shown by the bold line in Figure 3. From this picture, we see that by leaving out the variables which drag down the commonality of GDP the most, the commonality ratio of GDP increases dramatically when $n \to r$. The commonality rises from 46.7% to 94.2%. The dependence of the commonality ratio on the composition of the data set is in itself an interesting feature. In many applications, a high degree of commonality of GDP is taken as evidence that GDP is a good business cycle indicator (FHLR, 2000b). The current practice does however show that a variety of values for this commonality ratio can be obtained by varying the size and composition of the data set. A high commonality for GDP could therefore be the result of "luck" selecting the right data. One should therefore be cautious about drawing conclusions on the appropriateness of a particular indicator as a good business cycle indicator. It should be borne in mind that these conclusions only hold with respect to a particular data set. Much care should therefore go to the construction of the data set in view of the practice at hand.

While the selected subsamples are able to increase the commonality of GDP in a rather impressive way, it remains to be seen if these subsamples also lead to better GDP forecasts based on the common variation compared to the full model. In Figure 3 the RMSE for the different forecast horizons and the different subsamples maximising GDP's common component are plotted. As can be seen, the RMSE is quite volatile and increases extensively for the combinations with the highest commonality ratio for GDP. Looking at the one-step ahead forecast, there is a sound relation between the RMSE and the commonality ratio of GDP (higher commonality/lower RMSE) up to a certain sample size. However, from then onwards the RMSE rises fast. While the subsamples lead to better estimation results (a larger part of GDP can be explained), they do not seem to lead to better forecasts. Given the still large size of most of these subsamples, which consequently satisfies the conditions of the GDFM, it is not clear what causes these forecasts to be bad. Having tried to increase the accuracy of the forecasting results by letting vary the commonality of the included variables and the commonality of GDP, none of these strategies proved to be fruitful to obtain better forecasting results. Better "estimation" results do not deliver better "forecast" results. It is unclear what causes this mismatch and it should therefore be a topic for future research, as is the dependence of the estimation results on the selected data set.

Figure 3 **Commonality of GDP and forecast performance**



7 Conclusion

In this paper, we investigated the business cycle information content of 509 variables and identified the reference business cycle as the common variation contained in quarter-on-quarter Belgian GDP growth. Furthermore, we explored the cyclical behaviour of the variables with respect to this reference cycle and forecasted quarter-on-quarter GDP growth. All of this took place within one unified setting by applying the Generalised Dynamic Factor Model (GDFM) of FHLR (2000b, 2001, 2004, 2005) to a large data set containing information on the Belgian economy and its main indicators. Through its richness, the model provides useful information for both the business cycle analyst and the market parties interested in the Belgian business cycle and its indicators. The model reduces the variables to their core business cycle information, defined as that part of the variables' variation which is common to the data set. The results show that some well-known indicators such as the EC economic sentiment indicator for Belgium and the NBB overall synthetic curve contain a high amount of business cycle information. Given the importance of GDP for forecasting purposes, we defined the reference business cycle as the common variation contained in the quarter-on-quarter GDP growth and classified the whole set of indicators with respect to this reference cycle. 22% of the variables were classified as leading, 27% as lagging. Amongst the most leading variables we find asset prices and international confidence indicators such as the ISM and some OECD indicators, which lead the Belgian business cycle by 2 to 3 quarters. In general, national business confidence surveys are found to be coincident, while consumer confidence seems to lag. For each of the 509 indicators, individual time lags can be obtained which could be used as a guide for assessing the importance of an indicator as "warning signal" for the Belgian economy.

Although the model captures the dynamic common information contained in the data set, forecasts based on that information are insufficient to deliver a good proxy for GDP growth as a result of a non-negligible idiosyncratic part in GDP's variance. It should however be noted that the model focuses on quarter-on-quarter variations, for which, comovement in general is low. Consequently they are highly idiosyncratic and hard to predict. However, through the use of a data reduction process we show that the GDFM is able to reduce the unexplained variation in GDP growth. However, forecasts do not improve, which indicates there is a clear distinction between the forecasting and estimation results of the model. The exercise also sheds some light on the dependence of the model's results on the underlying data set. In particular, the amount of business cycle information present in a certain indicator seems to depend highly on the data set. It should therefore be borne in mind that the conclusions only hold with respect to a certain data set. Although this is a feature of many econometric models, a more profound understanding of the relation between the GDFM's outcome and the composition of the data set would be desirable. Further research could therefore focus on the exploration of this relationship. Apart form this, it could also further highlight the richness of the model by evaluating the variables with respect to each other and not only with respect to the reference cycle. This would provide additional insights into the relationships between different variables and address several economic issues.

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Appendix

In this Appendix we provide a brief outline of the technical details underlying the GDFM used in this paper. We show how the spectral density matrix of x_{nt} can be estimated and the way it is decomposed in an idiosyncratic and common part through a dynamic principal component procedure. This procedure allows to calculate the common and idiosyncratic covariances through an inverse Fourier transform and to estimate the common component. For the latter, we present the one-sided estimation technique based on static factors in Appendix A.2 using the estimated covariances from A.1, which solves the estimation problems for the common component caused by the two-sidedness of the filter applied in Appendix A.1.

A.1 Estimating the spectral density, covariances, common and idiosyncratic components

An estimate of the spectral density matrix $\sum_n(\theta)$ can be obtained by applying a discrete Fourier transform to the sample covariance matrices Γ_k^T of x_{nt}. The spectral density allows to decompose the auto and cross-covariance matrices into periodic components, fruitful for the dynamic analysis in this paper. To allow estimation, the number of cross covariance matrices has to be truncated. For a fixed integer $M(T)$, we compute the sample covariance matrices $\Gamma_k^T = x_{nt}x'_{nt-k}$ with $k=-M,...,M$. The estimation of $\sum_n(\theta)$ is then obtained by multiplying the sample covariance matrices by Barlett-lag window estimator weights $\omega_k = 1-\left(\,|K|\,/\,M+1\right)$ and applying the discrete Fourier transform:

$$\sum\nolimits_n^T(\theta_h) = \frac{1}{2\pi}\sum_{k=-M}^{M}\omega_k\Gamma_k^T e^{-ik\theta_h}$$

The Barlett-weights are needed to avoid biases caused by the truncation. Consistent estimates are ensured, provided that $M(T)\to\infty$ and $M(T)/T\to 0$ as $T\to\infty$. FHLR (2000b) show that a rule of $M=\text{round}\left(\sqrt{T}/4\right)$ performs well. Here, with $T=55$ this delivers $M=2$, we conducted estimates with $M=2$ and $M=3$ and decided in favour of $M=3$ because the rule of $M=2$ is too restrictive in the sense that only a small part of the dynamic information would be considered. In the Fourier transformation, the spectra are evaluated at $2M+1=7$ equal spaced frequencies (including the zero-frequency) in the interval $[-\pi,\pi]$ as suggested by FHLR (2000b), namely at the frequencies $\theta_h = (2\pi h)/6, h=-3,...,3$.

The estimated spectral density matrix is then decomposed in orthogonal components by a dynamic principal component decomposition, in analogy with standard static principal component analysis, albeit at different frequencies. Following Brillinger (1975), we compute the eigenvectors and eigenvalues of $\sum_n^T(\theta_h)$ for each frequency θ_h. Ordering the eigenvalues in descending order for each frequency, the eigenvalue and eigenvector functions $\lambda_{nj}(\theta)$ and $p_{nj}(\theta), j=1,...,n$ are obtained. The dynamic eigenvectors $p_{nj}(\theta)$ are expanded in Fourier series as:

$$p_{nj}(\theta)=\frac{1}{2\pi}\sum_{k=-M}^{M}\left[\int_{-\pi}^{\pi}p_{nj}(\theta)\,e^{ik\theta}d\theta\right]e^{-ik\theta}$$

and can be suitably transferred to the time domain by applying an inverse Fourier transform:

$$\underline{p}_{nj}(L)=\frac{1}{2\pi}\sum_{k=-M}^{M}\left[\int_{-\pi}^{\pi}p_{nj}(\theta)\,e^{ik\theta}d\theta\right]L^{k}$$

The dynamic eigenvalue function $\lambda_{nj}(\theta)$ is equal to the spectral density matrix of the process $\left\{\underline{p}_{nj}(L)\,x_{nt}, t\in Z\right\}$ which is called the j-th dynamic principal component of x_{nt}:

$$p_{nj}(\theta)\Sigma_{n}(\theta)\tilde{p}_{nj}(\theta)=\lambda_{nj}(\theta)$$

The dynamic principal components are mutually orthogonal at any lead or lag and the ratio:

$$c_{j}=\int_{-\pi}^{\pi}\lambda_{nj}(\theta)\,d\theta\,/\sum_{j=1}^{n}\int_{-\pi}^{\pi}\lambda_{nj}(\theta)\,d\theta$$

represents the contribution of the j-th dynamic principal component to the total variance in the system.

Given the fact that the dynamic eigenvectors $p_{nj}(\theta)$ are an orthonormal system of eigenvectors for I_{n} and that the common component χ_{nt} is the projection of x_{nt} on the approximate factor space spanned by the first q diverging dynamic principal components, the common component χ_{nt} can be estimated as:

$$\chi_{nt}=\left[\,\underline{\tilde{p}}_{n1}^{T}(L)\,\underline{p}_{n1}^{T}(L)+...+\underline{\tilde{p}}_{nq}^{T}(L)\,\underline{p}_{nq}^{T}(L)\,\right]x_{nt}$$

and the residual ξ_{nt} as $\xi_{nt}=x_{nt}-\chi_{nt}$.

The spectral density matrix can correspondingly be decomposed in a spectral density matrix of the common component χ_{nt} and idiosyncratic component ξ_{nt}:

$$\Sigma_{n}^{\chi T}(\theta)=\lambda_{n1}^{T}(\theta)\,\tilde{p}_{n1}^{T}(\theta)\,p_{n1}^{T}(\theta)+...+\lambda_{nq}^{T}(\theta)\,\tilde{p}_{nq}^{T}(\theta)\,p_{nq}^{T}(\theta)$$
$$\Sigma_{n}^{\xi T}(\theta)=\lambda_{n,q+1}^{T}(\theta)\,\tilde{p}_{n,q+1}^{T}(\theta)\,p_{n,q+1}^{T}(\theta)+...+\lambda_{nn}^{T}(\theta)\,\tilde{p}_{nn}^{T}(\theta)\,p_{nn}^{T}(\theta)$$

applying an inverse discrete Fourier transform to these matrices delivers the covariance matrices of χ_{nt} and ξ_{nt} at different leads and lags:

$$\Gamma_{nk}^{\chi T} = \int_{-\pi}^{\pi} e^{ik\theta} \Sigma_{n}^{\chi T}(\theta)\, d\theta$$
$$\Gamma_{nk}^{\xi T} = \int_{-\pi}^{\pi} e^{ik\theta} \Sigma_{n}^{\xi T}(\theta)\, d\theta$$

A.2 Estimating and forecasting the common component through static factors

Being based on the spectral density of the data, the filter applied to x_{nt} in A.1 to estimate the common component χ_{nt} is two-sided. This causes problems at the end of the sample to estimate and forecast the common component since no future values are available. To solve this problem the factor space can alternatively be represented by the use of r static factors u_{jt-k}, $j = 1, ..., q$, $k = 1, ..., s$, instead of q dynamic factors u_{jt}, $j = 1, ..., q$ with $r = q(s+1)$ and s the order of the lag operator in (1). Similar to the dynamic factors, these r static factors need to be approximated. Using the estimated covariances matrices $\Gamma_{n0}^{\chi T}$ and $\Gamma_{n0}^{\xi T}$ of A.1 we are able to construct r contemporaneous averages of the x_{it}'s that minimise the fraction of idiosyncratic variance contained in the aggregates, leading to a better approximation of the common factor space than static principal components. These "efficient" static aggregates are obtained as the solutions of a generalised principal component problem. More precisely, we compute the generalised eigenvalues μ_{nj} of the couple of matrices $\left(\Gamma_{n0}^{\chi T}, \Gamma_{n0}^{\xi T}\right)$, i.e. the n complex numbers solving $\det\left(\Gamma_{n0}^{\chi T} - z\,\Gamma_{n0}^{\xi T}\right) = 0$, along with the corresponding generalised eigenvectors $\boldsymbol{V}_{nj}$, $j = 1, ..., n$, i.e. the vectors satisfying:

$$\boldsymbol{V}_{nj}\,\Gamma_{n0}^{\chi T} = \mu_{nj}\,\boldsymbol{V}_{nj}\,\Gamma_{n0}^{\xi T}$$

and the normalising condition:

$$\boldsymbol{V}_{nj}\,\Gamma_{n0}^{\xi T}\,\boldsymbol{V}'_{ni} = \begin{cases} 0 \; for \; j \neq i, \\ 1 \; for \; j = i. \end{cases}$$

Ordering the eigenvalues μ_{nj} in descending order and taking the eigenvectors corresponding to the r largest ones, our estimated static factors are the generalised principal components:

$$\boldsymbol{W}_{nt}^{j} = \boldsymbol{V}'_{nj} x_{nt}, \; j = 1, ..., r\,.$$

These generalised principal components are the contemporaneous linear combinations of the x_{it}'s, with the smallest idiosyncratic/common variance ratio and allow efficient estimates and forecasts of χ_{nt} without the need of future values. Precisely, setting

$\boldsymbol{V}_n = (\boldsymbol{V}_{n1}, ..., \boldsymbol{V}_{nr})$ and $\boldsymbol{W}_{nt} = (\boldsymbol{W}_{nt}^1, ..., \boldsymbol{W}_{nt}^r)' = \boldsymbol{V}_n' x_{nt}$, our estimate of $\chi_{t+h}, h = 0, ..., s$, given the information available at time t, is:

$$\begin{aligned} \chi_{t+h}^T &= \Gamma_{nh}^{\chi T} \boldsymbol{V}_n \left(\boldsymbol{V}_n' \Gamma_0^T \boldsymbol{V}_n \right)^{-1} \boldsymbol{W}_{nt} \\ &= \Gamma_{nh}^{\chi T} \boldsymbol{V}_n \left(\boldsymbol{V}_n' \Gamma_0^T \boldsymbol{V}_n \right)^{-1} \boldsymbol{V}' x_{nt} \end{aligned}$$

In FHLR(2005) it is shown that, as both n and $T \to \infty$ in a proper way the estimated χ_{nt}^T and $\chi_{nt,t+h}^T$ converges to the theoretical χ_{nt}^T and theoretical projection of $\chi_{n,t+h}$ on the present and the past of $u_{1t}, ..., u_{qt}$.

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