Spatial and Temporal Time Series Conversion

A Consistent Estimator of the Error Variance-Covariance Matrix

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Abstract

We focus on the problem of time series conversion from low to high frequency satisfying the twofold temporal and spatial constraint. We offer a simple solution to variance-covariance matrix estimation of the error terms. Since the residuals of high frequency equations of the indicator based regression model are not observable, we inferred the characteristics of their stochastic process through both a specific hypothesis (VAR 1 process) and estimation of the related annual model. We derive a consistent estimator of the variance-covariance matrix and we prove that Di Fonzo's (1990) estimator based on this matrix is asymptotically equivalent to a GLS estimator.

Keywords: Spatial and temporal disaggregation, VAR

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Résumé

Les auteurs se penchent sur le problème de la conversion de basse en haute fréquence de séries chronologiques sous la double contrainte temporelle et spatiale. Ils proposent une solution simple pour l'estimation des matrices de variance-covariance des erreurs. Etant donné que les résidus des équations sur données haute fréquence du modèle d'indicateurs ne sont pas observables, ils s'appuient sur une hypothèse spécifique (processus VAR 1) et sur l'estimation du modèle à partir de données annuelles pour déterminer leurs caractéristiques stochastiques. Ils calculent ensuite un estimateur convergent de la matrice de variance-covariance et démontrent que l'estimateur de Di Fonzo 1990) basé sur cette matrice est asymptotiquement équivalent à un estimateur des MCG.

1 Introduction

During the 1990s, in Italy the occurrence of relatively more frequent and brief cyclical fluctuations with respect to past experience – as in most European economies – together with structural geographical differences in terms of economic development reawakened the interest of economists in business cycle analysis at regional level. The increasing need for timely analysis of economic phenomena at sub-national level has been raised during the present Italian business cycles by local governments, due to both the recent importance of regions in promoting economic development following the process of fiscal federalism as well as central government's role in monitoring the effectiveness of policy interventions.

Business cycle analysis at sub-national level is being undermined by the limited statistical information available. In Europe, regional economic accounts, consistent with the methodology of the European System of National Accounts (ESA95), are released with an annual frequency and a long time delay with respect to the reference period of the last available data.[1] This information has little use in monitoring and evaluating the conjunctural phases of a region or a larger geographical area or in setting an efficient program of economic policy actions.

The limited number of high frequency official statistics has led to a diversity of research to elaborate new methods of producing disaggregated regional account data on a quarterly basis using qualitative and quantitative business cycle indicators. These indicators are very useful because they are promptly available. Moreover, due to their high correlation to economic time series they allow rapid updating of official statistics, thereby reducing the distance between reference and release period. Furthermore, the complete set of national aggregates of the same series at regional level are usually available on a quarterly basis and with a time delay of about one quarter.

We discuss the methodological problem of estimating high frequency (say quarterly and regional) time series, using the relevant low frequency (say annual) series, a set of high frequency related indicators[2] and the aggregates (say national) of the series to be estimated, for each intra-annual period. Di Fonzo (1990) derived a generalized least squares estimator, following and extending Chow and Lin's (1971) approach, that fulfills both temporal (i.e. the sum of quarterly flows must be equal to the respective annual data) and spatial aggregation constraints (i.e. the sum of regional flows must be equal to the respective data aggregated at national level).

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1 In Italy, ISTAT (Istituto Nazionale di Statistica) released in October 2005 the complete version of regional economic account data for 2003 and in December 2005 the preliminary regional data for 2004.

2 We assume that each basic series satisfies a multiple regression relationship with a number of known related indicators available at the required time frequency.

The basic hypothesis for carrying out this exercise is that there exists a set of stochastic linear relationships between the dependent variables to be disaggregated and the related indicators available at the required time as well as at regional frequency. The parameters of these multivariate relations can be consistently estimated by least squares procedure, using annual data. On the contrary, this is not possible for quarterly error terms since the disturbances are latent variables, the residuals being not observable.[3] In Di Fonzo (1990), the unknown quarterly errors are estimated by means of linear projection on annual residuals. This projection is a function of the variance-covariance matrix of the quarterly errors (see equations 8 and 11 in Di Fonzo, 1990). Furthermore, this matrix is also necessary in order to have an efficient estimate of parameters in the generalized least squares approach (see equations 9 and 12 in Di Fonzo, 1990).

Now complex correlations exist, with respect to both time and space, between economic phenomena in different geographical areas. Although joint relations to the same set of homogeneous indicators can be sufficient to show the effect of fundamentals on the co-movements of the variables to be estimated, there are also relations of dynamic externalities between the economic areas that must be taken into account in the estimation procedure. In the quarterly model based on indicators, since the linear relationships have to be necessarily parsimonious as well as preferably static,[4] obviously many of the dynamic covariations between territorial series are embodied in the temporal and spatial covariances of the unknown quarterly errors. These disturbances cannot be considered as white noise. In other words, the methodology we propose follows the track traced by Di Fonzo (1994) and Cabrer and Pavia (1999), because of the explicit consideration that the variance-covariance matrix of the estimated quarterly disturbances is not diagonal. But the present paper has an innovative element: a more general pattern for the autocovariance structure of the model, allowing both autocorrelations within the territorial series and complex intertemporal correlations among them. We also allow a null hypothesis of no spatial correlation in the dynamic structure of economic phenomena at regional level. In Di Fonzo (1990) this assumption is not excluded but it is not explicitly discussed. Furthermore, if we do not take into account this aspect in the procedure, the estimations will not be efficient.

The problem is how to get a consistent estimate of the variance-covariance matrix of the unknown quarterly errors. Even in the univariate case according to Chow and Lin (1971) it is important to know the autocorrelation structure of the errors, implied by a quarterly indicator model (see also Fernández, 1981; Litterman, 1983; Wei and Stram, 1990; Barcellan and Di

3 For calculating residuals of the quarterly equations we should have quarterly time series as dependent variables, but just these variables are the resulting estimates we are discussing.

4 Actually, Santos Silva and Cardoso (2001), Di Fonzo (2003) and Mitchell and al. (2005) apply a dynamic extension taking into account the univariate case, developing iterative procedures to estimate jointly all the coefficients (also the coefficient of the lagged endogenous variable to be estimated at high frequency). In a multivariate approach, because of the high dimensionality of the parameter space and convergence problems of iterative processes, we focus on a two stage approach.

Fonzo, 1994). Nevertheless, in the univariate case, the stochastic process can be parameterized in a relatively parsimonious way, and the estimate can be obtained iteratively or by means of grid search methods due to the limited size of the parametric space. On the contrary, in the multivariate spatial case, entailing the error dynamics of a number of contemporaneous relationships on a territorial basis, even very simple hypotheses (i.e. a VAR process of order one) imply the estimation of a great number of parameters of the error process. This estimation cannot be easily carried out with a maximum likelihood approach.

We define a two step procedure. Unlike the Parks (1967) procedure, however, since we cannot estimate the quarterly error terms consistently, we have to derive indirectly their covariance characteristics. Our estimates will be conditional on yearly error disturbances (consistently estimated) and specific assumptions about the multivariate spatial stochastic process generating the unknown quarterly errors. Given these assumptions we need to derive the process resulting from the yearly aggregation of time series. Quarterly model parameters can be identified if, and only if, we obtain an injective function mapping each point of the parameter space of the yearly model to at most one point of the parameter space of the quarterly model.

The present paper aims to derive a solution, which is both consistent and computationally simple, to estimate the variance-covariance matrix of quarterly error terms resulting from the indicator model used by Di Fonzo (1990). The paper is organized as follows. Section 2 sets out the basics of Di Fonzo's approach using his notation. In Section 3, we derive the statistical model of yearly aggregated errors based on the hypothesis that the quarterly error terms process follows a simple multivariate VAR(1). Then we show the relations linking yearly model parameters to quarterly model parameters in order to prove that we can derive a consistent estimate of quarterly stochastic process parameters from maximum likelihood estimation of yearly model parameters. In Section 4, from the estimation of the quarterly errors generating process we get a consistent estimate of autocovariance matrices. The elements of these matrices can be used, in a second step, to estimate quarterly time series following the generalized least squares approach. We show that this estimator is asymptotically equivalent to a GLS estimator. In Section 5 we propose a parsimonious parameterization for the quarterly error term process, that helps the practical computation of iterative maximum likelihood estimates, and we use it in Section 6 to study the statistical properties of the proposed procedure in a set of Montecarlo experiments. In Section 7 we carry out a real data application with the Italian series of value added at basic prices in the industrial sector, comparing the results of the Di Fonzo procedure and ours. Section 8 provides some brief concluding remarks discussing the main results obtained and the research prospects.

2 Temporal Disaggregation of Time Series Following the Di Fonzo Method

Let $M\ (>1)$ be the number of high frequency series to be estimated. Let q be the number of annual sub-periods (4 in the quarterly case) required for the M time series to be estimated and n the number of annual observations for each series. The number of quarters we need to estimate using the annual information is given by $N = n \cdot q$. The quarterly model satisfying the M multiple regression relationship with a number of known related indicators[5] is given by:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u}, \quad \mathrm{E}(\boldsymbol{u} \mid \boldsymbol{X}) = \boldsymbol{0} \tag{2.1}$$

where $\boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_1' & \boldsymbol{y}_2' & \cdots & \boldsymbol{y}_M' \end{bmatrix}'$ is the (MN) column vector consisting of quarterly time series we want to estimate, $\boldsymbol{X}$ is the ($MN \times Mp$) block-diagonal matrix of p indicators available for the regression of each time series[6], $\boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{\beta}_1' & \boldsymbol{\beta}_2' & \cdots & \boldsymbol{\beta}_M' \end{bmatrix}'$ is the (Mp) vector of parameters and $\boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}_1' & \boldsymbol{u}_2' & \cdots & \boldsymbol{u}_M' \end{bmatrix}'$ is the ($MN \times 1$) vector of quarterly random errors, with zero mean and covariance matrix $\boldsymbol{V}$.

Quarterly series are denoted by a double index (year, quarter), $y_{i,s}$, while annual series are denoted by only one index. The $(N + nM) \times MN$ matrix of temporal and contemporaneous aggregation constraints is the following:

$$\boldsymbol{H} = \begin{bmatrix} \multicolumn{2}{c}{\boldsymbol{i}_M' \otimes \boldsymbol{I}_N} \\ \boldsymbol{I}_{M-1} \otimes \boldsymbol{B} & | \quad \boldsymbol{0} \\ \hline 0 & | \quad \boldsymbol{B} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_w \\ -- \\ \boldsymbol{H}_M \end{bmatrix}, \tag{2.2}$$

where $(\boldsymbol{i}_M' \otimes \boldsymbol{I}_N)\boldsymbol{y} = \boldsymbol{z}$ represents the known quarterly series at national level, that is the aggregation of the M unknown quarterly regional series, and $(\boldsymbol{I}_M \otimes \boldsymbol{B})\boldsymbol{y} = \boldsymbol{y}_0$ is the known vector of yearly regional data. In this paper, we discuss only the case of quarterly disaggregation, hence the matrix representing the aggregation of infra-annual data is given by $\boldsymbol{B} = \boldsymbol{I}_n \otimes \boldsymbol{i}_4'$.

Following Di Fonzo (1990), only $r = N + M(n-1)$ rows of the matrix $\boldsymbol{H}$ are linearly independent, because the spatial aggregation constraints imply temporal constraints on the M th series. Hence, when we define least squares estimators, it may be useful to focus only on the constraints of the matrix $\boldsymbol{H}_w$, to avoid the Moore-Penrose generalized inverse calculation.

5 We call this function "index-model".

6 The results of the paper do not depend on the hypothesis that each of the M time series has the same number of indicators (p).

Premultiplying (2.1) by $\boldsymbol{H}_w$, we obtain:

$$\boldsymbol{H}_w \boldsymbol{y} = \boldsymbol{H}_w \boldsymbol{X\beta} + \boldsymbol{H}_w \boldsymbol{u} \ ; \quad \boldsymbol{y}_w = \boldsymbol{X}_w \boldsymbol{\beta} + \boldsymbol{u}_w , \tag{2.3}$$

where $\boldsymbol{y}_w = \begin{bmatrix} \boldsymbol{z}' & \boldsymbol{y}'_{01} & \cdots & \boldsymbol{y}'_{0M-1} \end{bmatrix}'$. We derive Di Fonzo's estimator (see equations 11 and 12 of the cited paper) applying least squares to the linear relationship (3):

$$\hat{\boldsymbol{y}} = \boldsymbol{X}\hat{\boldsymbol{\beta}} + \hat{\boldsymbol{u}} \quad \text{with} \quad \hat{\boldsymbol{u}} = \boldsymbol{V}\boldsymbol{H}'_w \boldsymbol{V}_w^{-1} \left(\boldsymbol{y}_w - \boldsymbol{X}_w \hat{\boldsymbol{\beta}} \right), \tag{2.4}$$

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'_w \boldsymbol{V}_w^{-1} \boldsymbol{X}_w)^{-1} \boldsymbol{X}'_w \boldsymbol{V}_w^{-1} \boldsymbol{y}_w . \tag{2.5}$$

Equation (2.4) is defined by two orthogonal terms: the linear projection of $\boldsymbol{y}$ on the space generated by $\boldsymbol{X}$ indicators and the projection of quarterly random error on the space generated by annual residuals. This second term is not a consistent estimator of quarterly random errors. Indeed, even if we know that $(\boldsymbol{y}_w - \boldsymbol{X}_w \hat{\boldsymbol{\beta}}) = \hat{\boldsymbol{u}}_w \xrightarrow{p} \boldsymbol{u}_w = \boldsymbol{H}_w \boldsymbol{u}$, nevertheless:

$$\hat{\boldsymbol{u}} = \mathrm{E}(\boldsymbol{u}\boldsymbol{u}'_w)\,\mathrm{E}(\boldsymbol{u}_w \boldsymbol{u}'_w)^{-1} \hat{\boldsymbol{u}}_w = \boldsymbol{V}\boldsymbol{H}'_w \boldsymbol{V}_w^{-1} \hat{\boldsymbol{u}}_w \xrightarrow{p} \boldsymbol{V}\boldsymbol{H}'_w \left(\boldsymbol{H}_w \boldsymbol{V} \boldsymbol{H}'_w \right)^{-1} \boldsymbol{H}_w \boldsymbol{u} \neq \boldsymbol{u} . \tag{2.6}$$

In other words the second term of (2.4) cannot be used to estimate consistently the stochastic process generating the unknown quarterly disturbances and, consequently, the variance-covariance matrix $\boldsymbol{V}$. To estimate $\boldsymbol{V}$ we assume a restrictive hypothesis on the stochastic process generating the unknown quarterly errors. Then we need to derive accordingly the characteristics of the stochastic process of the aggregated yearly errors which can be consistently estimated.

Di Fonzo (1990) also shows the equations to be used when an extrapolation problem occurs. This problem arises in a twofold sense: when estimates of the basic series are desired outside the sample period and the relevant temporal aggregates are unavailable. On the one hand, we can have a *constrained* extrapolation (see equations 19 and 20 of the cited article) when only quarterly spatial aggregate values are known and hence the extrapolated data should be consistent with them. On the other hand, in the case of *pure* extrapolation, no aggregation constraint should have to be fulfilled. Fundamental elements of *constrained* extrapolation solution are both the matrix $\boldsymbol{V}$, and the following covariance matrices:

$$\boldsymbol{V}_e = \mathrm{E}(\boldsymbol{u}_e \boldsymbol{u}'_e); \quad \boldsymbol{\Omega} = \mathrm{E}(\boldsymbol{u}_e \boldsymbol{u}') \quad \text{where} \quad \boldsymbol{u}_e = \begin{bmatrix} \boldsymbol{u}'_{e1} & \boldsymbol{u}'_{e2} & \cdots & \boldsymbol{u}'_{eM} \end{bmatrix}', \tag{2.7}$$

where $\boldsymbol{u}_e$ collects the vectors of the quarterly errors in the prediction sample for the M equations between basic series and indicators. The elements of these matrices can be consistently estimated together with the matrix $\boldsymbol{V}$, as we will show in the next sections.

3 Identification of the Stochastic Process Generating the Unknown Quarterly Errors

In the literature on temporal disaggregation of stationary processes several major contributions have shown the conditional hypotheses used to derive the related unknown parameters of the underlying quarterly process from yearly observed data (Wei and Stram, 1990; Cainelli and Lupi, 1999; Bordignon and Di Fonzo, 1992; Marcellino, 1997). Broadly speaking, these papers take into account the univariate case, when a temporal disaggregation of only one time series at low frequency is required.

When we discuss the case of both temporal and spatial aggregation constraints, the hypothesis about the generating process of unknown quarterly errors must take into account the dynamic covariation of spatially disaggregated series. Hence the hypothesis regards a multivariate process.

H1: the quarterly errors generating process is a stationary vector autoregressive model of order 1, VAR(1),

$$\boldsymbol{u}_{t,s} = \begin{bmatrix} u_{t,s}^{1} & u_{t,s}^{2} & \cdots & u_{t,s}^{M} \end{bmatrix}'; \quad \boldsymbol{u}_{t,s} = \boldsymbol{A}\boldsymbol{u}_{t,s-1} + \boldsymbol{\varepsilon}_{t,s}\,, \tag{3.1}$$

$$\left|\boldsymbol{I}-\boldsymbol{A}\right| \neq 0; \quad \left|\boldsymbol{I}-\boldsymbol{A}s\right| = 0 \Rightarrow |\,s\,| > 1; \quad \mathrm{E}(\boldsymbol{\varepsilon}_{t,s}) = \boldsymbol{0}, \quad \mathrm{E}(\boldsymbol{\varepsilon}_{t,s}\boldsymbol{\varepsilon}_{t,s}') = \boldsymbol{\Sigma}_{\varepsilon}\,, \tag{3.2}$$

furthermore, the parameter matrix $\boldsymbol{A}$ has full rank and real positive eigenvalues. In other words, the error vector does not have a degenerate probability density function with singular variance-covariance matrix, and the impulse-response functions to shocks originating in whatever region monotonically tend to zero.

From hypothesis 1 we can derive the statistical model of yearly disturbances. We consider only the case based upon flow variables where the yearly aggregate is the sum of 4 related quarters:

$$\boldsymbol{u}_{t,4} + \boldsymbol{u}_{t,3} + \boldsymbol{u}_{t,2} + \boldsymbol{u}_{t,1} = \boldsymbol{A}\boldsymbol{u}_{t,3} + \boldsymbol{A}\boldsymbol{u}_{t,2} + \boldsymbol{A}\boldsymbol{u}_{t,1} + \boldsymbol{A}\boldsymbol{u}_{t-1,4} + \boldsymbol{\varepsilon}_{t,4} + \boldsymbol{\varepsilon}_{t,3} + \boldsymbol{\varepsilon}_{t,2} + \boldsymbol{\varepsilon}_{t,1}\,. \tag{3.3}$$

Now, since

$$\begin{cases} \boldsymbol{u}_{t,4} = \boldsymbol{A}^4\boldsymbol{u}_{t-1,4} + \boldsymbol{\varepsilon}_{t,4} + \boldsymbol{A}\boldsymbol{\varepsilon}_{t,3} + \boldsymbol{A}^2\boldsymbol{\varepsilon}_{t,2} + \boldsymbol{A}^3\boldsymbol{\varepsilon}_{t,1} \\ \boldsymbol{u}_{t,3} = \boldsymbol{A}^4\boldsymbol{u}_{t-1,3} + \boldsymbol{\varepsilon}_{t,3} + \boldsymbol{A}\boldsymbol{\varepsilon}_{t,2} + \boldsymbol{A}^2\boldsymbol{\varepsilon}_{t,1} + \boldsymbol{A}^3\boldsymbol{\varepsilon}_{t-1,4} \\ \boldsymbol{u}_{t,2} = \boldsymbol{A}^4\boldsymbol{u}_{t-1,2} + \boldsymbol{\varepsilon}_{t,2} + \boldsymbol{A}\boldsymbol{\varepsilon}_{t,1} + \boldsymbol{A}^2\boldsymbol{\varepsilon}_{t-1,4} + \boldsymbol{A}^3\boldsymbol{\varepsilon}_{t-1,3} \\ \boldsymbol{u}_{t,1} = \boldsymbol{A}^4\boldsymbol{u}_{t-1,1} + \boldsymbol{\varepsilon}_{t,1} + \boldsymbol{A}\boldsymbol{\varepsilon}_{t-1,4} + \boldsymbol{A}^2\boldsymbol{\varepsilon}_{t-1,3} + \boldsymbol{A}^3\boldsymbol{\varepsilon}_{t-1,2} \end{cases} \tag{3.4}$$

substituting (3.4) into the right hand side of (3.3) we obtain

$$\begin{aligned}\boldsymbol{u}_{t,4}+\boldsymbol{u}_{t,3}+\boldsymbol{u}_{t,2}+\boldsymbol{u}_{t,1} &= \boldsymbol{A}^4(\boldsymbol{u}_{t-1,4}+\boldsymbol{u}_{t-1,3}+\boldsymbol{u}_{t-1,2}+\boldsymbol{u}_{t-1,1})\\ &+\boldsymbol{\varepsilon}_{t,4}+(\boldsymbol{I}+\boldsymbol{A})\boldsymbol{\varepsilon}_{t,3}+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\boldsymbol{\varepsilon}_{t,2}+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\varepsilon}_{t,1}\\ &+(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\varepsilon}_{t-1,4}+(\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\varepsilon}_{t-1,3}+\boldsymbol{A}^3\boldsymbol{\varepsilon}_{t-1,2},\end{aligned} \tag{3.5}$$

that is, in terms of the aggregate process,

$$\boldsymbol{u}_t=\boldsymbol{A}^4\boldsymbol{u}_{t-1}+\boldsymbol{v}_t, \tag{3.6}$$

where

$$\begin{aligned}\boldsymbol{v}_t &= \boldsymbol{\varepsilon}_{t,4}+(\boldsymbol{I}+\boldsymbol{A})\boldsymbol{\varepsilon}_{t,3}+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\boldsymbol{\varepsilon}_{t,2}+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\varepsilon}_{t,1}\\ &+(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\varepsilon}_{t-1,4}+(\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\varepsilon}_{t-1,3}+\boldsymbol{A}^3\boldsymbol{\varepsilon}_{t-1,2}.\end{aligned} \tag{3.7}$$

A closer look at equation (3.6) shows that the errors of the autoregressive process have a moving average structure of order one, VMA(1). In fact by the computation of autocovariance matrices:

$$\begin{aligned}\mathrm{E}(\boldsymbol{v}_t\boldsymbol{v}_t') &= \boldsymbol{\Sigma}_\varepsilon+(\boldsymbol{I}+\boldsymbol{A})\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A})'+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)'\\ &+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)'+(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)'\\ &+(\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{A}^2+\boldsymbol{A}^3)'+\boldsymbol{A}^3\boldsymbol{\Sigma}_\varepsilon\boldsymbol{A}'^3;\end{aligned} \tag{3.8}$$

$$\mathrm{E}(\boldsymbol{v}_t\boldsymbol{v}_{t-1}')=(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon+(\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A})'+\boldsymbol{A}^3\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)'; \tag{3.9}$$

$$\mathrm{E}(\boldsymbol{v}_t\boldsymbol{v}_{t-k}')=\boldsymbol{0}\quad \forall k>1,$$

hence,

$$\boldsymbol{u}_t=\boldsymbol{A}^4\boldsymbol{u}_{t-1}+\boldsymbol{\xi}_t+\boldsymbol{B}\boldsymbol{\xi}_{t-1}, \tag{3.10}$$

where

$$\boldsymbol{v}_t=\boldsymbol{\xi}_t+\boldsymbol{B}\boldsymbol{\xi}_{t-1}.$$

The process (3.10) is stationary because of the hypothesis (3.2). Let us assume it is also identifiable (see Granger and Newbold, 1977, Section 7.3):

H2: the stochastic process (3.10) is identifiable, that is: (i) order 1 of MA part is the lowest among all other VARMA equivalent representations; (ii) order 1 of AR part is the lowest among all other VARMA equivalent representations; (iii) the matrix $[\boldsymbol{A}^4\boldsymbol{B}]$ has full rank.

Hypothesis 2 implies that the stochastic process (3.10) can be consistently estimated by a maximum likelihood algorithm, that is:

$$[\hat{\boldsymbol{A}}^4\hat{\boldsymbol{B}}\,\hat{\boldsymbol{\Sigma}}_\xi]_{ml}(\boldsymbol{u}_0)\xrightarrow{p}[\boldsymbol{A}^4\boldsymbol{B}\boldsymbol{\Sigma}_\xi] \quad \text{where} \quad \boldsymbol{u}_0=(\boldsymbol{i}'_M\otimes\boldsymbol{I}_N)\boldsymbol{u} \tag{3.11}$$

and where $[\hat{\boldsymbol{A}}^4\hat{\boldsymbol{B}}\hat{\boldsymbol{\Sigma}}_\xi]_{ml}(\boldsymbol{u}_0)$ represents the maximum likelihood estimator of the parameters as a function of the true annual errors.

In what follows we argue that maximum likelihood estimates of the model (3.10) parameters, based on the consistently estimated yearly errors, are consistent themselves. The yearly error term $\boldsymbol{u}_0$, actually, can be consistently estimated by least squares,

$$\hat{\boldsymbol{u}}_0=\boldsymbol{y}_0-\boldsymbol{X}_0\hat{\boldsymbol{\beta}}_{LS}\,;\quad \hat{\boldsymbol{\beta}}_{LS}=\left(\boldsymbol{X}'_w\left(\boldsymbol{H}_w\boldsymbol{H}'_w\right)^{-1}\boldsymbol{X}_w\right)^{-1}\boldsymbol{X}'_w\left(\boldsymbol{H}_w\boldsymbol{H}'_w\right)^{-1}\boldsymbol{y}_w\,. \tag{3.12}$$

Since, a) it can be easily demonstrated that under hypothesis (2.1) $\hat{\boldsymbol{u}}_0\xrightarrow{p}\boldsymbol{u}_0$, and b) the maximum likelihood estimator is a continuous function of the sample observations,

$$\text{plim}\,[\hat{\hat{\boldsymbol{A}}}^4\hat{\hat{\boldsymbol{B}}}\,\hat{\hat{\boldsymbol{\Sigma}}}_\xi]_{ml}(\hat{\boldsymbol{u}}_0)=[\hat{\hat{\boldsymbol{A}}}^4\hat{\hat{\boldsymbol{B}}}\hat{\hat{\boldsymbol{\Sigma}}}_\xi]_{ml}(\text{plim}\,\hat{\boldsymbol{u}}_0)=[\hat{\boldsymbol{A}}^4\hat{\boldsymbol{B}}\hat{\boldsymbol{\Sigma}}_\xi]_{ml}(\boldsymbol{u}_0)\,,$$

where $[\hat{\hat{\boldsymbol{A}}}^4\hat{\hat{\boldsymbol{B}}}\hat{\hat{\boldsymbol{\Sigma}}}_\xi]_{ml}(\hat{\boldsymbol{u}}_0)$ represents the maximum likelihood estimator of the parameters as a function of the estimated annual residuals, and hence, alternatively:

$$[\hat{\hat{\boldsymbol{A}}}^4\hat{\hat{\boldsymbol{B}}}\hat{\hat{\boldsymbol{\Sigma}}}_\xi]_{ml}(\hat{\boldsymbol{u}}_0)\xrightarrow{p}[\boldsymbol{A}^4\boldsymbol{B}\boldsymbol{\Sigma}_\xi]\,. \tag{3.13}$$

If we could show that there are the conditions for identifying the parameters of the quarterly error model, on the basis of known parameters of the yearly model, then we could easily solve the problem of consistently estimating the parameters sub (3.1). First of all, we note that matrix $\boldsymbol{A}$ is nonsingular, by assumption, hence it has *one and only one inverse* $\boldsymbol{A}^{-1}$. Therefore, for any couple ($\boldsymbol{A}$; -$\boldsymbol{A}$) also $\boldsymbol{A}^4$ is unique and has full rank. Since, by hypothesis 1, we know that all the eigenvalues of $\boldsymbol{A}$ are real and positive, we can estimate $\boldsymbol{A}^4=T\Lambda^4T^{-1}$ under the constraint that all the elements of the diagonal matrix $\boldsymbol{\Lambda}$ are real and positive. So, we can choose the unique fourth root of $\boldsymbol{A}^4$ that has all the eigenvalues positive, $\boldsymbol{A}$. In this way, the function $\boldsymbol{A}^4\rightarrow\boldsymbol{A}$ is injective, and the parameters of (3.1) are identified.

If we want to show that also the covariance matrix $\boldsymbol{\Sigma}_\varepsilon$ is identified we need to recall the autocovariance functions of VMA(1) processes: $\text{E}(\boldsymbol{v}_t\boldsymbol{v}'_t)=\boldsymbol{\Sigma}_\xi+\boldsymbol{B}\boldsymbol{\Sigma}_\xi\boldsymbol{B}'$ and $\text{E}(\boldsymbol{v}_t\boldsymbol{v}'_{t-1})=\boldsymbol{B}\boldsymbol{\Sigma}_\xi$

(see Lütkepohl, 1993). By combining these latter two with equations (3.8) and (3.9) and applying the properties of the vec operator, we obtain:

$$\begin{aligned}\left[\boldsymbol{I}\otimes\boldsymbol{I}+\boldsymbol{B}\otimes\boldsymbol{B}\right]\cdot\mathrm{vec}\,\boldsymbol{\Sigma}_{\xi}=&\left[\boldsymbol{I}\otimes\boldsymbol{I}+(\boldsymbol{I}+\boldsymbol{A})\otimes(\boldsymbol{I}+\boldsymbol{A})+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\otimes(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\right.\\ &+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)+(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\\ &\left.+(\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes(\boldsymbol{A}^2+\boldsymbol{A}^3)+\boldsymbol{A}^3\otimes\boldsymbol{A}^3\right]\cdot\mathrm{vec}\,\boldsymbol{\Sigma}_{\varepsilon};\end{aligned}\qquad(3.14)$$

$$\left[\boldsymbol{I}\otimes\boldsymbol{B}\right]\cdot\mathrm{vec}\,\boldsymbol{\Sigma}_{\xi}=\left[\boldsymbol{I}\otimes(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)+(\boldsymbol{I}+\boldsymbol{A})\otimes(\boldsymbol{A}^2+\boldsymbol{A}^3)+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\otimes\boldsymbol{A}^3\right]\cdot\mathrm{vec}\,\boldsymbol{\Sigma}_{\varepsilon}\,,\qquad(3.15)$$

which are two linear systems of M^2 equations in M^2 unknown elements. Identification of the covariance matrix $\boldsymbol{\Sigma}_{\varepsilon}$ depends on the rank of matrices sub (3.14) and (3.15):

$$\begin{aligned}\boldsymbol{C}=&\left[\boldsymbol{I}\otimes\boldsymbol{I}+(\boldsymbol{I}+\boldsymbol{A})\otimes(\boldsymbol{I}+\boldsymbol{A})+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\otimes(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\right.\\ &+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)+(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\\ &\left.+(\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes(\boldsymbol{A}^2+\boldsymbol{A}^3)+\boldsymbol{A}^3\otimes\boldsymbol{A}^3\right];\end{aligned}$$

$$\boldsymbol{D}=\left[\boldsymbol{I}\otimes(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)+(\boldsymbol{I}+\boldsymbol{A})\otimes(\boldsymbol{A}^2+\boldsymbol{A}^3)+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\otimes\boldsymbol{A}^3\right].$$

If M^2 linearly independent vectors exist among the rows of $\boldsymbol{C}$ and $\boldsymbol{D}$, then we can prove the identification of covariances. According to this condition, we carried out a series of Montecarlo experiments (each experiment consisting of 10,000 extractions) to calculate the rank of matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ built as functions of the nonsingular matrix $\boldsymbol{A}$, whose elements are trials of a uniform random variable defined on [-1, +1]. For each dimension of $\boldsymbol{A}$ that we considered in each experiment, full rank matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ resulted. Hence, we can make the following assumption:

H3: the matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ of systems (3.14) and (3.15) are invertible.

Subject to hypothesis 3, the quarterly model is overidentified. The assumption of a vector autoregressive process of order 1 for quarterly errors means that we are implying M^2 constraints on the parameters of matrix $\boldsymbol{B}$. In fact, solving equation (3.14) by the covariance vector, we obtain

$$\boldsymbol{C}^{-1}\left[\boldsymbol{I}\otimes\boldsymbol{I}+\boldsymbol{B}\otimes\boldsymbol{B}\right]\cdot\mathrm{vec}\,\boldsymbol{\Sigma}_{\xi}=\mathrm{vec}\,\boldsymbol{\Sigma}_{\varepsilon}\,.\qquad(3.16)$$

Substituting (3.16) into (3.15), gives:

$$\left[\boldsymbol{I}\otimes\boldsymbol{B}\right]\cdot\mathrm{vec}\,\boldsymbol{\Sigma}_{\xi}=\boldsymbol{D}\cdot\boldsymbol{C}^{-1}\left[\boldsymbol{I}\otimes\boldsymbol{I}+\boldsymbol{B}\otimes\boldsymbol{B}\right]\cdot\mathrm{vec}\,\boldsymbol{\Sigma}_{\xi}\qquad(3.17)$$

If the matrix $\left[\boldsymbol{I} \otimes \boldsymbol{I} + \boldsymbol{B} \otimes \boldsymbol{B}\right]$ is invertible, equation (3.17) shows exactly the M^2 independent constraints allowing calculation of all elements of the matrix $\boldsymbol{B}$. But the invertibility of matrix $\left[\boldsymbol{I} \otimes \boldsymbol{I} + \boldsymbol{B} \otimes \boldsymbol{B}\right]$ is strictly dependent on the eigenvalues of $\boldsymbol{B} \otimes \boldsymbol{B}$ being internal to the unit circle. The eigenvalues are given by the products $\lambda_i \lambda_j$, where λ_i and λ_j are both eigenvalues of $\boldsymbol{B}$. This means that the invertibility condition is equivalent to invertibility of MA terms of the stochastic process of the yearly disturbances. This comes as no surprise if we think of the well known uniqueness condition of the MA representation of stochastic processes. Uniqueness is obtained by means of the invertibility condition of the process.

In this section, we showed that the parameters of the unknown quarterly error terms model are a continuous and injective function of yearly model parameters (which we can consistently estimate). Hence, estimating a VARMA(1,1) model from (3.10) and using equations (3.14) and (3.15), we can get consistent estimates of parameters for (3.1) and (3.2):

$$[\hat{\hat{\boldsymbol{A}}}\hat{\hat{\boldsymbol{\Sigma}}}_{\varepsilon}]_{mv}(\hat{\boldsymbol{u}}_0) \xrightarrow{p} [\boldsymbol{A}\boldsymbol{\Sigma}_{\varepsilon}]. \tag{3.18}$$

4 Consistent Estimator of the Variance-Covariance Matrix of Quarterly Disturbances

From the consistent estimate of the parameters of model (3.1) we can easily work out the autocovariance matrices of the related stochastic process VAR(1):

$$\operatorname{vec} \hat{\boldsymbol{\Gamma}}_0 = \left[\boldsymbol{I} - \hat{\boldsymbol{A}} \otimes \hat{\boldsymbol{A}}\right]^{-1} \operatorname{vec} \hat{\boldsymbol{\Sigma}}_{\varepsilon} \xrightarrow{p} \operatorname{vec} \mathrm{E}(\boldsymbol{u}_{t,s}\boldsymbol{u}'_{t,s}) = \operatorname{vec} \boldsymbol{\Gamma}_0 ; \tag{4.1}$$

$$\hat{\boldsymbol{\Gamma}}_1 = \hat{\boldsymbol{A}}\hat{\boldsymbol{\Gamma}}_0 \xrightarrow{p} \mathrm{E}(\boldsymbol{u}_{t,s}\boldsymbol{u}'_{t,s-1}) = \boldsymbol{\Gamma}_1 ; \quad \hat{\boldsymbol{\Gamma}}_k = \hat{\boldsymbol{A}}^k \hat{\boldsymbol{\Gamma}}_0 \xrightarrow{p} \boldsymbol{\Gamma}_k . \tag{4.2}$$

From (4.1) and (4.2) we can calculate the elements of the matrix $\boldsymbol{V}$, using the following formulas:

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{V}_{11} & \boldsymbol{V}_{12} & \cdots & \boldsymbol{V}_{1M} \\ \boldsymbol{V}_{21} & \boldsymbol{V}_{22} & \cdots & \boldsymbol{V}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{V}_{M1} & \boldsymbol{V}_{M2} & \cdots & \boldsymbol{V}_{MM} \end{bmatrix}; \ \boldsymbol{V}_{is} = \mathrm{E}(\boldsymbol{u}_i\boldsymbol{u}'_s) = \left[\boldsymbol{\sigma}_{jr}^{is}\right]; \ \boldsymbol{\sigma}_{jr}^{is} = \begin{cases} \text{if } j \geq r: \ \boldsymbol{\Gamma}_{|j-r|}(i,s) \\ \text{if } j < r: \ \boldsymbol{\Gamma}_{|j-r|}(s,i) \end{cases} \tag{4.3}$$

where $\boldsymbol{\Gamma}_k(i,s)$ is the i-s th element of the autocovariance matrix of order k, in the equation (4.2).

In the same way, we calculate the other matrices as being part of regression-extrapolation equations:

$$\boldsymbol{V}_e = \begin{bmatrix} \boldsymbol{V}_{e11} & \boldsymbol{V}_{e12} & \cdots & \boldsymbol{V}_{e1M} \\ \boldsymbol{V}_{e21} & \boldsymbol{V}_{e22} & \cdots & \boldsymbol{V}_{e2M} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{V}_{eM1} & \boldsymbol{V}_{eM2} & \cdots & \boldsymbol{V}_{eMM} \end{bmatrix}; \ \boldsymbol{V}_{eis} = \mathrm{E}(\boldsymbol{u}_{ei}\boldsymbol{u}'_{es}) = \left[\boldsymbol{\sigma}_{jr}^{eis}\right]; \ \boldsymbol{\sigma}_{jr}^{eis} = \begin{cases} \text{if } j \geq r: \ \boldsymbol{\Gamma}_{|j-r|}(i,s) \\ \text{if } j < r: \ \boldsymbol{\Gamma}_{|j-r|}(s,i) \end{cases} \tag{4.4}$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \cdots & \boldsymbol{\Omega}_{1M} \\ \boldsymbol{\Omega}_{21} & \boldsymbol{\Omega}_{22} & \cdots & \boldsymbol{\Omega}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Omega}_{M1} & \boldsymbol{\Omega}_{M2} & \cdots & \boldsymbol{\Omega}_{MM} \end{bmatrix}; \ \boldsymbol{\Omega}_{is} = \mathrm{E}(\boldsymbol{u}_{ei}\boldsymbol{u}'_{s}) = \left[\boldsymbol{\omega}_{jr}^{is}\right]; \ \boldsymbol{\omega}_{jr}^{is} = \boldsymbol{\Gamma}_{|N+j-r|}(i,s). \tag{4.5}$$

The estimator (2.5), based on the estimated $\hat{\boldsymbol{V}}$ (EGLS), is asymptotically equivalent to the generalized least squares estimator when we know the variance-covariance matrix (GLS). In fact

$$\begin{aligned} &\sqrt{r}\left(\hat{\boldsymbol{\beta}}_{EGLS} - \hat{\boldsymbol{\beta}}_{GLS}\right) = \sqrt{r}\left[\left(\hat{\boldsymbol{\beta}}_{EGLS} - \hat{\boldsymbol{\beta}}\right) - \left(\hat{\boldsymbol{\beta}}_{GLS} - \hat{\boldsymbol{\beta}}\right)\right] \\ &= \left(\frac{1}{r}\boldsymbol{X}'_w\hat{\boldsymbol{V}}_w^{-1}\boldsymbol{X}_w\right)^{-1}\frac{1}{\sqrt{r}}\boldsymbol{X}'_w\hat{\boldsymbol{V}}_w^{-1}\boldsymbol{u}_w - \left(\frac{1}{r}\boldsymbol{X}'_w\boldsymbol{V}_w^{-1}\boldsymbol{X}_w\right)^{-1}\frac{1}{\sqrt{r}}\boldsymbol{X}'_w\boldsymbol{V}_w^{-1}u_w \\ &= \left(\frac{1}{r}\boldsymbol{X}'_w\hat{\boldsymbol{V}}_w^{-1}\boldsymbol{X}_w\right)^{-1}\frac{1}{\sqrt{r}}\boldsymbol{X}'_w\left(\hat{\boldsymbol{V}}_w^{-1} - \boldsymbol{V}_w^{-1}\right)\boldsymbol{u}_w \\ &\quad + \left[\left(\frac{1}{r}\boldsymbol{X}'_w\hat{\boldsymbol{V}}_w^{-1}\boldsymbol{X}_w\right)^{-1} - \left(\frac{1}{r}\boldsymbol{X}'_w\boldsymbol{V}_w^{-1}\boldsymbol{X}_w\right)^{-1}\right]\frac{1}{\sqrt{r}}\boldsymbol{X}'_w\boldsymbol{V}_w^{-1}\boldsymbol{u}_w. \end{aligned} \tag{4.6}$$

Now since we have shown that $\hat{\boldsymbol{V}}_w = \boldsymbol{H}_w\hat{\boldsymbol{V}}\boldsymbol{H}'_w \xrightarrow{p} \boldsymbol{H}_w\boldsymbol{V}\boldsymbol{H}'_w = \boldsymbol{V}_w$, and under the hypothesis

$$\left(\frac{1}{r}\boldsymbol{X}'_w\boldsymbol{V}_w^{-1}\boldsymbol{X}_w\right)^{-1} \xrightarrow{p} \boldsymbol{Q} \ \textit{nonstochastic, uniformely limited}, \tag{4.7}$$

taking the probability limit of (4.6) we get

$$\operatorname{plim}\left(\frac{1}{r}\boldsymbol{X}'_w\hat{\boldsymbol{V}}_w^{-1}\boldsymbol{X}_w\right)^{-1}\operatorname{plim}\frac{1}{\sqrt{r}}\boldsymbol{X}'_w\left(\hat{\boldsymbol{V}}_w^{-1} - \boldsymbol{V}_w^{-1}\right)\boldsymbol{u}_w = \boldsymbol{Q}\cdot\boldsymbol{0} = \boldsymbol{0};$$

$$\operatorname{plim}\left[\left(\frac{1}{r}\boldsymbol{X}'_w\hat{\boldsymbol{V}}_w^{-1}\boldsymbol{X}_w\right)^{-1} - \left(\frac{1}{r}\boldsymbol{X}'_w\boldsymbol{V}_w^{-1}\boldsymbol{X}_w\right)^{-1}\right]\operatorname{plim}\frac{1}{\sqrt{r}}\boldsymbol{X}'_w\boldsymbol{V}_w^{-1}\boldsymbol{u}_w = [\boldsymbol{Q} - \boldsymbol{Q}]\operatorname{plim}\frac{1}{\sqrt{r}}\boldsymbol{X}'_w\boldsymbol{V}_w^{-1}\boldsymbol{u}_w = \boldsymbol{0}$$

.

Hence also $\sqrt{r}\left(\hat{\boldsymbol{\beta}}_{EGLS} - \hat{\boldsymbol{\beta}}_{GLS}\right) \xrightarrow{p} \boldsymbol{0}$, that is both estimators are asymptotically equivalent.

5 Parsimonious Parameterization for Empirical Applications

In empirical works, estimating a VARMA like the process given in equation (3.10) from annual residual with a maximum likelihood algorithm is not an easy task. Most of the time, homogeneous as well as territorially disaggregated observations are not available for more than 20 years. The paucity of time series sample data, even when the number of region is low, poses estimation problems.

However, a parsimonious parameterization of the equations system in terms of constraints in the elements of matrix $\boldsymbol{A}$, based either on the geographical pattern of intra-regional trade or on an *ad hoc* hypothesis,[7] can be found.

In what follows, we show a specific parameterization reducing problems in iterative estimation procedure both through a limitation of parametric space, as well as allowing single equation estimate of M models of quarterly errors.

H4: parameters matrix $\boldsymbol{A}$ is diagonal.

Hypothesis 4 permits a contemporaneous territorial dispersion of regional *shocks* as the variance-covariance matrix of error terms $\boldsymbol{\varepsilon}_{t,s}$ is non-diagonal, but, at the same time, it constrains the shape of the time pattern of these spreading shocks. We make the explicit assumption of an equal intra-regional time pattern following an exponential decay depending only on an autoregressive parameter which differs in each region.

Under hypothesis 4, the i-s th equation of model (3.10) is given by:

$$\boldsymbol{u}_{it} = \boldsymbol{a}_i^4 \boldsymbol{u}_{it-1} + (\boldsymbol{I} + \boldsymbol{B}_i \boldsymbol{L})\xi_t = \boldsymbol{a}_i^4 \boldsymbol{u}_{it-1} + \boldsymbol{v}_{it} , \tag{5.1}$$

where $\boldsymbol{a}_i^4$ is the fourth power of the i-s th element on the $\boldsymbol{A}$ diagonal and $\boldsymbol{B}_i$ is the i-s th row of matrix $\boldsymbol{B}$. It is simple to show that the errors of process (5.1) follow an MA(1) structure:

$$\mathrm{E}(\boldsymbol{v}_{it}^2) = \mathrm{E}\left\{(\boldsymbol{\xi}_{it} + \boldsymbol{B}_i \boldsymbol{\xi}_t)(\boldsymbol{\xi}_{it} + \boldsymbol{B}_i \boldsymbol{\xi}_t)'\right\} = \mathrm{Var}\left(\boldsymbol{\xi}_{it}\right) + \boldsymbol{B}_i \boldsymbol{\Sigma}_{\xi} \boldsymbol{B}_i' ; \tag{5.2}$$

$$\mathrm{E}(\boldsymbol{v}_{it} \boldsymbol{v}_{it-1}) = \boldsymbol{B}_i' \boldsymbol{\Sigma}_{\xi} ; \tag{5.3}$$

$$\mathrm{E}(\boldsymbol{v}_{it} \boldsymbol{v}_{it-k}) = \boldsymbol{0} \quad \forall k > 1 . \tag{5.4}$$

7 These constraints reduce the system flexibility in shaping the dynamic relations of economic phenomena on a territorial basis, but at the same time allow parameter estimates.

Hence, the i-s th equation (5.1) follows a simple ARMA(1,1) structure and it can be estimated consistently using a maximum likelihood algorithm on the i-s th series of annual errors.

After estimating the parameters of diagonal $\boldsymbol{A}$ (equation by equation), we can obtain a consistent estimator of matrix $\boldsymbol{\Sigma}_\varepsilon$ recalling equation (3.6) and computing the variance of annual errors, $\boldsymbol{u}_t$:

$$\mathrm{E}(\boldsymbol{u}_t\boldsymbol{u}_t') = \mathrm{E}(\boldsymbol{A}^4\boldsymbol{u}_{t-1}+\boldsymbol{v}_t)(\boldsymbol{A}^4\boldsymbol{u}_{t-1}+\boldsymbol{v}_t)' = \mathrm{E}\left[\boldsymbol{A}^4\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}'\boldsymbol{A}'^4+\boldsymbol{v}_t\boldsymbol{v}_t'+\boldsymbol{A}^4\boldsymbol{u}_{t-1}\boldsymbol{v}_t'+\boldsymbol{v}_t\boldsymbol{u}_{t-1}'\boldsymbol{A}'^4\right]. \quad (5.5)$$

The expected value of the first term can be easily calculated, while equation (3.8) can be used for the second term. As far as the third and fourth terms are concerned, their expected values can be found recalling equations (3.5) and (3.7):

$$\mathrm{E}(\boldsymbol{v}_t\boldsymbol{u}_{t-1}') = (\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon+(\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A})'+\boldsymbol{A}^3\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)'. \quad (5.6)$$

Hence:

$$\begin{aligned}\mathrm{E}(\boldsymbol{u}_t\boldsymbol{u}_t') &= \boldsymbol{\Sigma}_0 = \boldsymbol{A}^4\boldsymbol{\Sigma}_0\boldsymbol{A}'^4\\ &+\boldsymbol{\Sigma}_\varepsilon+(\boldsymbol{I}+\boldsymbol{A})\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A})'+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)'\\ &+(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)'+(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)'\\ &+(\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{A}^2+\boldsymbol{A}^3)'+\boldsymbol{A}^3\boldsymbol{\Sigma}_\varepsilon\boldsymbol{A}'^3\\ &+\boldsymbol{A}^4\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)'+\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A})\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{A}^2+\boldsymbol{A}^3)'+\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\boldsymbol{\Sigma}_\varepsilon\boldsymbol{A}'^3\\ &+(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon\boldsymbol{A}'^4+(\boldsymbol{A}^2+\boldsymbol{A}^3)\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A})'\boldsymbol{A}'^4+\boldsymbol{A}^3\boldsymbol{\Sigma}_\varepsilon(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)'\boldsymbol{A}'^4;\end{aligned} \quad (5.7)$$

$$\begin{aligned}(\boldsymbol{I}-\boldsymbol{A}^4\otimes\boldsymbol{A}^4)\,\mathrm{vec}\,\boldsymbol{\Sigma}_0 &= \boldsymbol{C}\,\mathrm{vec}\,\boldsymbol{\Sigma}_\varepsilon\\ &+\left[(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes\boldsymbol{A}^4+(\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A})+\boldsymbol{A}^3\otimes\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\right]\mathrm{vec}\,\boldsymbol{\Sigma}_\varepsilon\\ &+\left[\boldsymbol{A}^4\otimes(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)+\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A})\otimes(\boldsymbol{A}^2+\boldsymbol{A}^3)+\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\otimes\boldsymbol{A}^3\right]\mathrm{vec}\,\boldsymbol{\Sigma}_\varepsilon.\end{aligned} \quad (5.8)$$

Equation (5.8) shows a relationship between matrices $\boldsymbol{\Sigma}_\varepsilon$ and $\boldsymbol{\Sigma}_0$. Moreover, the matrix $\boldsymbol{\Sigma}_0$ can be estimated using errors of annual regression. Hence, substituting for parameters their consistent estimates, we obtain the following estimator:

$$\mathrm{vec}\,\hat{\boldsymbol{\Sigma}}_\varepsilon = (\hat{\boldsymbol{C}}+\hat{\boldsymbol{E}}+\hat{\boldsymbol{F}})^{-1}(\boldsymbol{I}-\hat{\boldsymbol{A}}^4\otimes\hat{\boldsymbol{A}}^4)\,\mathrm{vec}\,\hat{\boldsymbol{\Sigma}}_0, \quad (5.9)$$

$$\boldsymbol{E} = (\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes\boldsymbol{A}^4+(\boldsymbol{A}^2+\boldsymbol{A}^3)\otimes\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A})+\boldsymbol{A}^3\otimes\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2), \quad (5.10)$$

$$\boldsymbol{F} = \boldsymbol{A}^4\otimes(\boldsymbol{A}+\boldsymbol{A}^2+\boldsymbol{A}^3)+\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A})\otimes(\boldsymbol{A}^2+\boldsymbol{A}^3)+\boldsymbol{A}^4(\boldsymbol{I}+\boldsymbol{A}+\boldsymbol{A}^2)\otimes\boldsymbol{A}^3. \quad (5.11)$$

6 Montecarlo experiments

The performance of the proposed estimator (denoted by GM) was analyzed carrying out Montecarlo experiments depicted in such a way that we could explore some of the most widespread cases in applied works. We also show a comparison with results obtained using a specific interpretation[8] of the Di Fonzo (1990) procedure (denoted by DF). In the following simulations, the estimation procedure used is based on the parsimonious parameterization discussed in Section 5.

The first step to carry out the Montecarlo experiments is the generation of quarterly stationary series in 4 regions, summing, in each region, an "index component", that is $\boldsymbol{X\beta}$ of equation (2.4), and an "error component", that is $\boldsymbol{u}$ of (2.4).

Moreover, the "index component" of each region, is generated from the aggregation of 4 stationary autoregressive processes.[9] Contributions to total variance of each process are depicted in Table 1.

Table 1 **Contribution of the four stationary components to the regional indexes**

	Region 1	Region 2	Region 3	Region 4
Component 1: $y_t = 1.535 y_{t-1} - 0.9 y_{t-2} + \upsilon_t$	80%	50%	60%	40%
Component 2: $y_t = 0.9 y_{t-1} + \upsilon_t$	20%	50%	20%	40%
Component 3: $y_t = 0.5 y_{t-1} + \upsilon_t$			20%	
Component 4: $y_t = 0.5 y_{t-1} + \upsilon_t$				20%

The "error component" is generated consistently with hypothesis 1.[10] The first 6 simulations[11] can be divided into two classes.

In simulations 1-4 an $\boldsymbol{A}$ diagonal matrix was used (the parameters of regions 1 – 4 are respectively equal to 0.9, 0.8, 0.7, 0.6), following hypothesis 4, consistent with the GM estimation procedure. Simulations 1-4 differ from one another due to the changing weight of

8 In our Montecarlo experiments the DF estimator treats the quarterly errors of indicator based regressions like independent white noises (time and space independence). Hence, the estimates of variances of these errors can be calculated dividing by 4 the sample variance of OLS annual residuals. Covariances and autocovariances are equal to zero.

9 The first process has strongly persistent cyclical characteristics, with a time period of 10 quarters. The second component is similar to a random walk. The third and fourth components are AR(1) processes with very low persistency.

10 See Section 3.

11 Each simulation is based on 1000 repetitions.

"index" and "error" components in determining the total variance of generated series and the changing relationship among standard deviations of the 4 regional series. Table 2 is a scheme of different simulations. For example, following this scheme, case 4 simulation in characterized by four regional series which, as a whole, show very different standard deviations (as indicated by the inter regional ratios of total standard deviations 8, 6, 3, 1 reported in columns labels) and, at the same time, every regional series is additively formed by an index component whose volatility is three times the error component one (as indicated by ratios of components standard deviations 3, 1 reported in rows labels).

Table 2 **Characteristics of the first four simulations**

		Relationship among standard deviations of the regional series	
		identical among regions: 1, 1, 1, 1	different among regions: 8, 6, 3, 1
Relationship among standard deviations of the component series	1 = index component 1 = error component	Case 1	Case 3
	3 = index component 1 = error component	Case 2	Case 4

In simulations 5 and 6 the model of quarterly errors that we used to generate the data (nondiagonal matrix $\boldsymbol{A}$) is more general with respect to the restriction considered within the proposed estimator.[12] In other words, in simulations 5 and 6, the proposed parsimonious parameterization is incorrect and the estimation procedure we have used cannot be considered a maximum likelihood algorithm. Nevertheless, it is worth monitoring the behavior of the GM estimator and analyzing its performance compared to the DF estimator, even when interactions between regional series are characterized by feedback effects with time variant lags.

The sample period taken into account for the first 6 cases of Montecarlo experiments is 400 quarterly data which can be considered quite an unusually long time period in applied works, but it can shed some useful insights on consistency properties of the GM estimator. The sample period used for the seventh simulation (whose statistical characteristics are the same as case 1 previously described) is made by 80 observations (20 years). This simulation aims to analyze the performance of the GM estimator in a more realistic case for applied works.

12 In simulation 5 $\boldsymbol{A}=\begin{bmatrix}0.5 & 0.2 & 0.1 & 0.1\\ 0.2 & 0.5 & 0.1 & 0.1\\ 0.2 & 0.2 & 0.5 & 0\\ 0.2 & 0.2 & 0 & 0.5\end{bmatrix}$; in simulation 6 $\boldsymbol{A}=\begin{bmatrix}0.7 & 0.1 & 0.05 & 0.05\\ 0.1 & 0.6 & 0.05 & 0.05\\ 0.05 & 0.05 & 0.6 & 0\\ 0.1 & 0.1 & 0 & 0.5\end{bmatrix}$.

Tables 3, 4 and 5 show the performance of DF and GM estimators in each region, respectively for the total series, the index and the error components.

As regards the first four simulations, the following five aspects seem to be particularly important.

- Statistical diagnostics show that the GM estimator fits the generated quarterly series better than the DF estimator, the average improvement being between 2 and more than 7 per cent of standard deviation of the generated series. This improvement stems from greater accuracy in estimates of both the index (between 1 and 3 per cent) and the error component (between 6 and 11 per cent).
- Quite unsurprisingly, the main contribution to discrepancy between generated and estimated series comes from the error component (the RMSE is about 30-40% of the standard deviation of the component) for both the GM and DF estimators. A pretty good advantage of the GM over the DF procedure appears to be concentrated precisely in the error component, given the explicit way of modeling the stochastic links of both time and spatial disturbances. All the estimators in cases 2 and 4 show a better performance than in cases 1 and 3, while the gain of the GM over DF estimator increases by around 2% and 4% of the standard deviation of the original series.
- As regards the error component (henceforth also the total series) the overall accuracy of estimations, for both the analyzed procedures, is an increasing function of the persistency degree of quarterly estimates as well as of the stochastic process we choose to generate the error component of each region (the autoregressive coefficients vary between 0.9 and 0.6, from first to fourth region). The reason for this result lies in the decomposition process from annual to quarterly residuals: the higher the positive relation between errors of contiguous quarters, the easier is the decomposition process. A random scheme for residuals makes the decomposition process more difficult.
- The closer the autoregressive coefficient of regional errors generating process either to the upper or to the lower bound of the admissible range, the greater is the overall improvement in the GM estimator over the DF estimator. Indeed, in the neighborhood of the upper bound the function, mapping from the parameters of the annual process (5.1) to the parameters of the quarterly process, becomes flatter

$$\frac{\mathrm{d}}{\mathrm{d}a_i}\left[a_i^4\right]=\frac{1}{4}a_i^{-3/4};\quad \frac{\mathrm{d}^2}{\mathrm{d}a_i^2}\left[a_i^4\right]<0\,,$$

and hence an error in the estimation of the autoregressive parameter at annual level generally causes a smaller error at quarterly level. For the autoregressive parameter approaching zero, the slope of the mapping function tends to infinity and we have a discontinuity point in which we cannot identify the quarterly parameters from the annual estimates. In the neighborhood of the lower bound the slope of the mapping function is very high, so the GM estimator uses a lower constraint (0.05 at annual level, that is 0.473 on a quarterly basis) for the maximum likelihood iterative procedure creating at 0.473 a mass probability point in the distribution of the estimates. Therefore, when the true value of the autoregressive parameter approaches the constraint, the mean RMSE of the estimator improves, as does the performance of the estimated quarterly series.

- The results of simulations in cases 3 and 4 show a substantial invariance of the performance of the GM estimator, with respect to cases 1 and 2. On the contrary, application of the DF estimator to cases with a difference in the variance of the regional series produces a major loss of accuracy in quarterly estimates, so the gain of the GM estimator over the DF increases (from 6 to 10 percent of the standard deviation of the error component). Here the main advantage of the GM procedure lies in its better use of the spatial constraint to spread the contemporary error term at national level among the regions, due to more accurate estimation of the variances of the regional quarterly error terms. Indeed, while the DF procedure estimates the quarterly error variances as the annual sample variance divided by 4, the GM estimator takes into account explicitly the effect of time stochastic dependence shaping the relationship between annual and quarterly error variance.[13]

In cases 5 and 6, identical to case 1 except for the characteristics of the autoregressive matrix $\boldsymbol{A}$, we study the behavior of the estimators when the restriction implied by the parsimonious parameterization of hypothesis 4 is false. Surprisingly, the GM and DF estimators both perform better when the matrix is nondiagonal. This result is probably related to the positive correlation scheme among regional errors, sketched by matrix $\boldsymbol{A}$ (see footnote 12), which helps create a more predictable distribution of error terms at national level among regions (the spatial constraint). On average, the gain of the GM over the DF estimator decreases, depending also on the incorrectness of the simplified parameterization.

Up to now, we have examined simulations of 100 observation samples, while in applied works such long time series are usually not available. Case 7 analyses the performance of GM and DF estimators on generated 20-year samples. Also in this case the GM performs better than the DF estimator, the gain being about 2 percent on a regional average. Moving from case 7 to 1, increasing the number of observations, we argue that the convergence of the quarterly estimates to the true series is faster using the GM rather than the DF estimator. The average gain of the GM over the DF estimator increases to about 4 percent. This result

13 The variance of the annual aggregate of a quarterly time series is equal to four times the variance of the quarterly series only if the process is serially uncorrelated. For an AR(1) serially correlated quarterly process the annual aggregate shows a variance that is directly proportional to the quarterly variance, with a proportionality factor that is a positive function of the autoregressive parameter.

stems mainly from the estimate of the error component as the GM estimator relies heavily on the consistency property of the error variance-covariance matrix in splitting the residuals temporally and spatially.

As regards the estimates of the parameters of the stochastic process, we can highlight the following arguments:

- The absolute performance of the GM estimator of the index-model coefficients (see 2.1) seems quite good (see Table 6): the bias appears fairly irrelevant if compared to the true value of the parameters, and the RMSE shows minor discrepancies (about 6% of the true value in the worst simulation case with a large sample, and 20% with a sample of 20 years). At any rate the GM estimator outperforms the DF estimator (see also Figure 1 for case 1 simulation): the higher the regional error autoregressive parameters, the higher the gains of the GM over the DF estimator.
- Comparing case 7 (20-year sample) to case 1 (100-year sample) we can argue the consistency of the GM estimator for the index-model coefficients (see Table 6 and Figure 2).
- On the estimation of the error variance-covariance matrix, Table 8 shows a fairly sizeable bias and RMSE (from about 5 to 25 percent of the true value) of the estimator of the quarterly error autoregressive parameters: the former decreases when the autoregressive parameters get closer to either the upper or lower bound, while the latter varies inversely with the true value of the parameter. The consistency is apparent in comparing case 1 to case 7 (see also Figure 3), although convergence to the true value does not proceed at a fast rate (the higher the true value of the autoregressive parameter, the faster the rate).

7 Real Data Application

Since Montecarlo experiments can be supposed to provide only limited insights, because of the ideal nature of the experiment which based on a perfect knowledge of the data generating process, in this section we illustrate the properties of the GM estimator in a real data application.

We carried out a temporal decomposition of the real value added at basis prices of industry in Italy. The regional dimension refers to the four main traditional geographical aggregates published by ISTAT: North-West, North-East, Center and South.[14] We used the simplest version of an index model for all the four regions: a trend indicator, represented by the cumulated value of the short term demand expectation, and a cyclical indicator,

[14] The mentioned geographical areas are aggregations of NUTS2 regions. North-West: Piemonte, Val d'Aosta, Lombardia and Liguria. North-East: Trentino Alto Adige, Friuli Venezia Giulia, Veneto and Emilia-Romagna. Center: Toscana, Marche, Umbria and Lazio. South: Campania, Puglia, Basilicata, Calabria, Sicilia and Sardegna.

represented by the balance of orders; the source of both indicators are the ISAE business surveys. The sample period is from 1986 to 2004 (18 annual observations). Since the Italian aggregate value added series we take into account is seasonally and working day adjusted, we have filtered the business survey data in a similar way to get homogeneous information in the estimation process.

The DF method has been applied maintaining the stochastic independence of the innovations belonging to different regions and different time periods. The GM estimator takes into account correlations both along time and between regions, by estimating univariate ARMA models on the annual residuals under the constraint implied by the relation (3.17). The small sample size of the regional statistics, and the volatility of results produced by the iterative process of the maximum likelihood estimation, make this mathematical a-priori very useful in real data applications to obtain a quick convergence to ML results consistent to the statistical model proposed. The ARMA estimation of the innovation process produces the following quarterly autoregressive parameters: 0.98 for North-West, 0.99 for North-East, 0.84 for Center and 0.88 for South. These estimates appear consistent to the stylized facts of business cycles of industrialized countries where the leading regions are usually characterized by self sustained economic dynamics.

The quarterly decomposed series are shown in Figures 4-7. The comparison between the two methods has been carried out within the traditional framework of cyclical analysis. The use of the Bry Boschan routine shed some light on the different cyclical characteristics of the produced GM and DF series. In Table 9 the turning points of the aggregate series (the real value added in Italy) are displayed for the four main regions. Broadly speaking the GM estimates show a profile more closely related to the widespread knowledge of the cyclical behaviour of Italian regions. This is particularly true during the most recent business cycle.

First of all, the application of the automatic routine on the DF series cannot detect some of the turning points of the value added in Center (the last peak of the beginning of 2001) and North-East regions (the entire phase between the first quarter of 1992 and the end of 1993). This is probably due to the absence of an autocorrelation structure preventing from the arising of a clear cut view of peak/trough points.

Table 9 **Turning point analysis**

	Peak	Trough	Peak	Trough	Peak
Italy	Q1-1992	Q3-1993	*Q1-1996*	*Q4-1996*	Q1-2001
Center DF	Q1-1992	Q1-1993	Q2-1995	Q1-1997	*Q4-2002*
Center GM	Q1-1992	Q3-1993	Q1-1996	Q1-1997	Q1-2001
North-East DF	*Q1-1992*	*Q3-1993*	*Q1-1996*	*Q4-1996*	Q1-2001
North-East GM	Q1-1992	Q3-1993	*Q1-1996*	*Q4-1996*	Q1-2000
North-West DF	Q2-1990	Q3-1993	*Q1-1996*	*Q4-1996*	Q1-2001
North-West GM	Q2-1990	Q3-1993	*Q1-1996*	*Q4-1996*	Q1-2001
South DF	Q1-1992	Q1-1993	Q3-1995	Q4-1996	Q4-2003
South GM	Q1-1992	Q3-1993	Q1-1996	Q4-1996	Q3-2002

Furthermore, the leading/lagging characteristics of the DF series present some anomalies with respect to stylized facts of the Italian business cycle. Firstly, the South and Center DF series lead somewhat (two quarters) the recovery of industrial activity of the end of 1993, anticipating the through points of both the North-West and the North-East regions. This result is strikingly strange, since we well know how the recovery during the post '93 period was largely based on the depreciation of the Italian Lira, that boosted the northern export led economic regions. Secondly, South and Center DF series unusually lead a little recession which is detectable only by visual inspection of Italian series in 1996. All these anomalies are not present in the series estimated by the GM method.

During the second half of '90, the last expansionary phase is detected by the South DF series with an unlikely lag (eleven quarters) with respect to the Italian series. The lag showed by the GM series (six quarters) appears more realistic, even when we give a cursory glance to all the other cyclical indicators of the region.

All the issues we pointed out can be probably explained by the absence, in the DF estimator, of an autocorrelation structure between regions which can make consistent the dynamics of the quarterly residuals in the different areas. In case of an index model which poorly summarizes the statistical information on the cyclical behaviour of the series, the distribution of the residual between time bear a major role in detecting turning points. The constraint of a well defined and reasonable autocorrelation structure between regional dynamics can be of a great help in distributing residuals and producing a true consistency among the cyclical properties of the regional estimated series.

8 Conclusions

Within the literature about methods to be used for converting (and hence forecasting) items of regional accounts from observed annual to estimated quarterly frequency, optimal (in the least squares sense) estimators which fulfill both temporal and spatial aggregation constraints have gained great importance. Following Di Fonzo's (1990) approach, the present article shows a possible solution to the variance-covariance matrix estimation of the unknown quarterly errors derived from the multiequational model where each basic series to be disaggregated is a function of related indicators. The proposed solution is consistent and easy to calculate.

Explicit consideration of the variance-covariance matrix of the error terms is crucial when we carry on a temporal disaggregating exercise. Indeed, under different hypotheses of time and regional transmission paths of disturbance errors as well as statistical characteristics of time series to be disaggregated, a set of Montecarlo experiments was carried out to test properties of the proposed estimator (GM estimator). The results proved that the GM estimator performed better than a procedure (Di Fonzo estimator) usually used in applied works both in terms of precision and robustness of fitted data. The set of Montecarlo simulations also confirmed the consistency property of the GM procedure in estimating the variance-covariance matrix of the unknown quarterly errors. This matrix bears very important information under both economic and statistical aspects.

The comparison between the DF and the GM estimates has been carried out in a real data application. The GM quarterly decomposition of the industrial value added seems to outperform the DF estimates in a business cycle analysis framework.

On the one hand, the correct temporal and spatial distribution of the estimated errors depends upon the variance-covariance matrix. Furthermore, explicit consideration of this matrix as nondiagonal means that we are seeking to analyze intra-regional correlations (temporal and spatial dynamics) of economic phenomena which are important structural characteristics of a country's economic development. For example, production dynamics in a country usually imply intra-regional exchanges (these exchanges are embodied in the "net import" item of regional account data) which explain in specific industries most bottlenecks for regional endogenous growth. Another example of the economic relevance of the topic is offered by the geographical distribution of so-called "industrial districts". Such aggregations of production activities spread over geographical areas usually differing from the administrative areas and the positive externalities to the economy affect both adjacent and remote geographical areas.

In a more strictly statistical sense an inconsistent estimate of the variance-covariance matrix affects the efficiency of the econometric method used. In the model based on indicators, since the linear relationships have to be necessarily parsimonious as well as static, many dynamic covariations between territorial series are embodied in the temporal and spatial covariances of the unknown quarterly errors. These disturbances cannot be considered as white noise. Furthermore, our hypothesis about a VAR(1) structure of the stochastic process seems to be both fairly simple and general, such that the derived estimator can be applied to a broad map of economic time series at regional level.

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Appendix

Table 3 **Error measures of quarterly series estimators – total series**

		Region 1		Region 2		Region 3		Region 4		DF-GM			
		DF	GM	DF	GM	DF	GM	DF	GM	Region 1	Region 2	Region 3	Region 4
Case 1	StD	1.414		1.414		1.414		1.414					
	M(RMSE)	0.313	0.188	0.305	0.281	0.337	0.310	0.404	0.344	0.1252	0.0238	0.0268	0.0598
	StD (RMSE)	0.052	0.018	0.020	0.030	0.020	0.025	0.028	0.036	3.9%	-1.9%	-1.3%	-0.3%
	M(MAE)	0.247	0.149	0.241	0.224	0.266	0.247	0.320	0.273	0.0981	0.0177	0.0199	0.0468
	StD (MAE)	0.041	0.014	0.016	0.024	0.016	0.020	0.022	0.029	3.0%	-1.6%	-1.1%	-0.3%
	M(RMSE)/StD	22.1%	13.3%	21.6%	19.9%	23.8%	21.9%	28.5%	24.3%	8.8%	1.7%	1.9%	4.2%
	M(MAE)/StD	17.5%	10.6%	17.1%	15.8%	18.8%	17.4%	22.6%	19.3%	6.9%	1.3%	1.4%	3.3%
Case 2	StD	1.581		1.581		1.581		1.581					
	M(RMSE)	0.156	0.094	0.153	0.140	0.169	0.155	0.202	0.171	0.0626	0.0132	0.0137	0.0305
	StD (RMSE)	0.028	0.010	0.010	0.013	0.011	0.013	0.014	0.018	1.6%	-0.6%	-0.6%	-0.1%
	M(MAE)	0.124	0.074	0.121	0.111	0.133	0.123	0.160	0.136	0.0491	0.0098	0.0103	0.0238
	StD (MAE)	0.022	0.008	0.008	0.011	0.009	0.010	0.011	0.015	1.2%	-0.6%	-0.5%	-0.1%
	M(RMSE)/StD	9.9%	5.9%	9.7%	8.8%	10.7%	9.8%	12.8%	10.8%	4.0%	0.8%	0.9%	1.9%
	M(MAE)/StD	7.8%	4.7%	7.6%	7.0%	8.4%	7.8%	10.1%	8.6%	3.1%	0.6%	0.7%	1.5%
Case 3	StD	5.657		4.243		2.121		0.707					
	M(RMSE)	1.101	0.613	0.683	0.569	0.708	0.558	0.305	0.227	0.4878	0.1147	0.1497	0.0782
	StD (RMSE)	0.145	0.050	0.066	0.047	0.042	0.042	0.014	0.017	5.2%	0.0%	3.1%	6.7%
	M(MAE)	0.869	0.487	0.539	0.452	0.560	0.444	0.242	0.180	0.3822	0.0869	0.1166	0.0615
	StD (MAE)	0.115	0.039	0.052	0.038	0.034	0.033	0.011	0.014	4.0%	-0.1%	2.3%	5.2%
	M(RMSE)/StD	19.5%	10.8%	16.1%	13.4%	33.4%	26.3%	43.1%	32.1%	8.6%	2.7%	7.1%	11.1%
	M(MAE)/StD	15.4%	8.6%	12.7%	10.6%	26.4%	20.9%	34.2%	25.5%	6.8%	2.0%	5.5%	8.7%
Case 4	StD	6.325		4.743		2.372		0.791					
	M(RMSE)	0.542	0.305	0.338	0.282	0.351	0.278	0.152	0.113	0.2368	0.0563	0.0729	0.0390
	StD (RMSE)	0.067	0.023	0.032	0.022	0.021	0.020	0.007	0.008	2.3%	0.1%	1.4%	3.0%
	M(MAE)	0.428	0.242	0.267	0.224	0.278	0.221	0.120	0.090	0.1859	0.0427	0.0570	0.0307
	StD (MAE)	0.053	0.018	0.025	0.018	0.017	0.016	0.006	0.007	1.8%	0.0%	1.0%	2.3%
	M(RMSE)/StD	8.6%	4.8%	7.1%	5.9%	14.8%	11.7%	19.2%	14.3%	3.7%	1.2%	3.1%	4.9%
	M(MAE)/StD	6.8%	3.8%	5.6%	4.7%	11.7%	9.3%	15.2%	11.3%	2.9%	0.9%	2.4%	3.9%

Table 3 **Error measures of quarterly series estimators – total series (continued)**

		Region 1		Region 2		Region 3		Region 4		DF-GM			
		DF	GM	DF	GM	DF	GM	DF	GM	Region 1	Region 2	Region 3	Region 4
Case 5	StD	1.414		1.414		1.414		1.414					
	M(RMSE)	0.216	0.215	0.235	0.233	0.252	0.246	0.235	0.227	0.0014	0.0020	0.0063	0.0072
	StD (RMSE)	0.009	0.009	0.011	0.011	0.012	0.012	0.012	0.011	-1.2%	-1.4%	-1.2%	-1.1%
	M(MAE)	0.172	0.171	0.187	0.186	0.200	0.196	0.187	0.181	0.0011	0.0016	0.0049	0.0056
	StD (MAE)	0.008	0.008	0.009	0.009	0.010	0.010	0.010	0.009	-1.0%	-1.2%	-1.0%	-0.9%
	M(RMSE)/StD	15.3%	15.2%	16.6%	16.5%	17.8%	17.4%	16.6%	16.1%	0.1%	0.1%	0.4%	0.5%
	M(MAE)/StD	12.2%	12.1%	13.2%	13.1%	14.2%	13.8%	13.2%	12.8%	0.1%	0.1%	0.3%	0.4%
Case 6	StD	1.414		1.414		1.414		1.414					
	M(RMSE)	0.280	0.254	0.309	0.306	0.352	0.340	0.337	0.331	0.0259	0.0031	0.0115	0.0063
	StD (RMSE)	0.019	0.028	0.014	0.023	0.018	0.037	0.017	0.024	-1.5%	-2.4%	-3.1%	-2.4%
	M(MAE)	0.223	0.202	0.246	0.244	0.279	0.270	0.268	0.263	0.0203	0.0022	0.0087	0.0048
	StD (MAE)	0.015	0.021	0.012	0.018	0.015	0.028	0.014	0.019	-1.1%	-1.9%	-2.4%	-2.0%
	M(RMSE)/StD	19.8%	18.0%	21.9%	21.7%	24.9%	24.0%	23.8%	23.4%	1.8%	0.2%	0.8%	0.4%
	M(MAE)/StD	15.7%	14.3%	17.4%	17.2%	19.7%	19.1%	18.9%	18.6%	1.4%	0.2%	0.6%	0.3%
Case 7	StD	1.414		1.414		1.414		1.414					
	M(RMSE)	0.314	0.229	0.324	0.322	0.352	0.346	0.411	0.385	0.0845	0.0017	0.0052	0.0257
	StD (RMSE)	0.101	0.079	0.050	0.085	0.047	0.079	0.061	0.086	-6.7%	-9.5%	-8.5%	-8.6%
	M(MAE)	0.249	0.182	0.258	0.257	0.279	0.276	0.327	0.307	0.0666	0.0010	0.0026	0.0200
	StD (MAE)	0.081	0.062	0.041	0.067	0.037	0.063	0.049	0.067	-5.4%	-7.6%	-6.9%	-6.8%
	M(RMSE)/StD	22.2%	16.2%	22.9%	22.8%	24.9%	24.5%	29.1%	27.2%	6.0%	0.1%	0.4%	1.8%
	M(MAE)/StD	17.6%	12.9%	18.2%	18.1%	19.7%	19.5%	23.1%	21.7%	4.7%	0.1%	0.2%	1.4%

StD = standard deviation of the original series if not differently specified

M = average value

Table 4 **Error measures of quarterly series estimators – index component**

		Region 1		Region 2		Region 3		Region 4		DF-GM			
		DF	GM	DF	GM	DF	GM	DF	GM	Region 1	Region 2	Region 3	Region 4
Case 1	StD	1.000		1.000		1.000		1.000					
	M(RMSE)	0.074	0.029	0.087	0.053	0.070	0.052	0.072	0.052	0.0456	0.0340	0.0184	0.0202
	StD (RMSE)	0.056	0.022	0.067	0.041	0.054	0.039	0.055	0.039	-3.2%	-7.4%	-7.5%	-7.4%
	M(MAE)	0.060	0.023	0.070	0.043	0.056	0.042	0.058	0.041	0.0366	0.0272	0.0147	0.0161
	StD (MAE)	0.045	0.017	0.053	0.033	0.044	0.031	0.044	0.031	-2.6%	-5.9%	-6.0%	-6.0%
	M(RMSE)/StD	7.4%	2.9%	8.7%	5.3%	7.0%	5.2%	7.2%	5.2%	4.6%	3.4%	1.8%	2.0%
	M(MAE)/StD	6.0%	2.3%	7.0%	4.3%	5.6%	4.2%	5.8%	4.1%	3.7%	2.7%	1.5%	1.6%
Case 2	StD	1.500		1.500		1.500		1.500					
	M(RMSE)	0.039	0.015	0.043	0.027	0.037	0.028	0.036	0.040	0.0244	0.0159	0.0089	-0.0039
	StD (RMSE)	0.030	0.022	0.034	0.028	0.029	0.046	0.027	0.346	-1.9%	-3.0%	-4.4%	-32.9%
	M(MAE)	0.031	0.012	0.034	0.022	0.030	0.022	0.029	0.032	0.0196	0.0127	0.0072	-0.0030
	StD (MAE)	0.024	0.017	0.027	0.022	0.023	0.036	0.022	0.367	-1.4%	-2.4%	-3.4%	-26.2%
	M(RMSE)/StD	2.6%	1.0%	2.9%	1.8%	2.5%	1.9%	2.4%	2.7%	1.6%	1.1%	0.6%	-0.3%
	M(MAE)/StD	2.1%	0.8%	2.3%	1.4%	2.0%	1.5%	1.9%	2.1%	1.3%	0.8%	0.5%	-0.2%
Case 3	StD	4.000		3.000		1.500		0.500					
	M(RMSE)	0.279	0.111	0.258	0.144	0.115	0.080	0.041	0.028	0.1683	0.1139	0.0349	0.0133
	StD (RMSE)	0.215	0.087	0.199	0.109	0.088	0.062	0.030	0.021	-3.3%	-6.5%	-7.7%	-7.7%
	M(MAE)	0.223	0.089	0.206	0.115	0.092	0.064	0.033	0.023	0.1348	0.0912	0.0280	0.0106
	StD (MAE)	0.172	0.069	0.159	0.087	0.071	0.049	0.024	0.017	-2.7%	-5.2%	-6.1%	-6.2%
	M(RMSE)/StD	7.0%	2.8%	8.6%	4.8%	7.7%	5.3%	8.3%	5.6%	4.2%	3.8%	2.3%	2.7%
	M(MAE)/StD	5.6%	2.2%	6.9%	3.8%	6.1%	4.3%	6.6%	4.5%	3.4%	3.0%	1.9%	2.1%
Case 4	StD	6.000		4.500		2.250		0.750					
	M(RMSE)	0.136	0.057	0.133	0.080	0.057	0.040	0.020	0.014	0.0799	0.0531	0.0177	0.0058
	StD (RMSE)	0.101	0.043	0.103	0.059	0.043	0.030	0.016	0.010	-1.1%	-2.4%	-2.5%	-2.7%
	M(MAE)	0.109	0.045	0.107	0.064	0.046	0.032	0.016	0.011	0.0640	0.0426	0.0142	0.0046
	StD (MAE)	0.081	0.035	0.083	0.047	0.035	0.024	0.012	0.008	-0.9%	-1.9%	-2.0%	-2.2%
	M(RMSE)/StD	2.3%	0.9%	3.0%	1.8%	2.5%	1.8%	2.6%	1.9%	1.3%	1.2%	0.8%	0.8%
	M(MAE)/StD	1.8%	0.8%	2.4%	1.4%	2.0%	1.4%	2.1%	1.5%	1.1%	0.9%	0.6%	0.6%

Table 4 **Error measures of quarterly series estimators – index component (continued)**

		Region 1		Region 2		Region 3		Region 4		DF-GM			
		DF	GM	DF	GM	DF	GM	DF	GM	Region 1	Region 2	Region 3	Region 4
Case 5	StD	1.000		1.000		1.000		1.000					
	M(RMSE)	0.067	0.032	0.097	0.039	0.077	0.038	0.097	0.038	0.0353	0.0574	0.0382	0.0595
	StD (RMSE)	0.054	0.023	0.076	0.030	0.058	0.028	0.077	0.028	-4.2%	-4.8%	-4.8%	-4.5%
	M(MAE)	0.054	0.026	0.078	0.032	0.061	0.031	0.078	0.030	0.0282	0.0459	0.0306	0.0476
	StD (MAE)	0.043	0.019	0.061	0.024	0.047	0.022	0.062	0.022	-3.4%	-3.9%	-3.8%	-3.6%
	M(RMSE)/StD	6.7%	3.2%	9.7%	3.9%	7.7%	3.8%	9.7%	3.8%	3.5%	5.7%	3.8%	5.9%
	M(MAE)/StD	5.4%	2.6%	7.8%	3.2%	6.1%	3.1%	7.8%	3.0%	2.8%	4.6%	3.1%	4.8%
Case 6	StD	1.000		1.000		1.000		1.000					
	M(RMSE)	0.069	0.039	0.084	0.052	0.069	0.051	0.082	0.043	0.0300	0.0323	0.0181	0.0389
	StD (RMSE)	0.054	0.031	0.064	0.038	0.054	0.039	0.061	0.034	-5.5%	-7.0%	-7.5%	-5.6%
	M(MAE)	0.055	0.031	0.067	0.041	0.055	0.040	0.066	0.035	0.0240	0.0258	0.0144	0.0311
	StD (MAE)	0.043	0.025	0.052	0.030	0.043	0.031	0.048	0.027	-4.4%	-5.6%	-6.0%	-4.5%
	M(RMSE)/StD	6.9%	3.9%	8.4%	5.2%	6.9%	5.1%	8.2%	4.3%	3.0%	3.2%	1.8%	3.9%
	M(MAE)/StD	5.5%	3.1%	6.7%	4.1%	5.5%	4.0%	6.6%	3.5%	2.4%	2.6%	1.4%	3.1%
Case 7	StD	1.000		1.000		1.000		1.000					
	M(RMSE)	0.171	0.099	0.182	0.142	0.164	0.150	0.159	0.141	0.0717	0.0399	0.0143	0.0178
	StD (RMSE)	0.144	0.090	0.148	0.121	0.122	0.122	0.121	0.117	-16.2%	-22.9%	-22.9%	-22.0%
	M(MAE)	0.139	0.080	0.147	0.115	0.132	0.121	0.129	0.114	0.0584	0.0325	0.0115	0.0145
	StD (MAE)	0.117	0.073	0.121	0.098	0.098	0.098	0.098	0.094	-13.1%	-18.6%	-18.5%	-17.8%
	M(RMSE)/StD	17.1%	9.9%	18.2%	14.2%	16.4%	15.0%	15.9%	14.1%	7.2%	4.0%	1.4%	1.8%
	M(MAE)/StD	13.9%	8.0%	14.7%	11.5%	13.2%	12.1%	12.9%	11.4%	5.8%	3.3%	1.1%	1.4%

StD = standard deviation of the original series if not differently specified

M = average value

Table 5 **Error measures of quarterly series estimators – error component**

		Region 1		Region 2		Region 3		Region 4		DF-GM			
		DF	GM	DF	GM	DF	GM	DF	GM	Region 1	Region 2	Region 3	Region 4
Case 1	StD	1.000		1.000		1.000		1.000					
	M(RMSE)	0.322	0.190	0.319	0.287	0.344	0.315	0.410	0.348	0.1314	0.0316	0.0293	0.0622
	StD (RMSE)	0.054	0.018	0.028	0.032	0.023	0.026	0.029	0.037	5.9%	-2.8%	-2.0%	-0.4%
	M(MAE)	0.205	0.255	0.298	0.253	0.320	0.272	0.353	0.326	-0.0500	0.0456	0.0475	0.0278
	StD (MAE)	0.028	0.043	0.037	0.023	0.028	0.019	0.038	0.023	-12.1%	-1.4%	0.0%	-3.3%
	M(RMSE)/StD	32.2%	19.0%	31.9%	28.7%	34.4%	31.5%	41.0%	34.8%	13.1%	3.2%	2.9%	6.2%
	M(MAE)/StD	20.5%	25.5%	29.8%	25.3%	32.0%	27.2%	35.3%	32.6%	-5.0%	4.6%	4.7%	2.8%
Case 2	StD	0.500		0.500		0.500		0.500					
	M(RMSE)	0.161	0.095	0.160	0.143	0.173	0.157	0.205	0.174	0.0662	0.0171	0.0153	0.0318
	StD (RMSE)	0.030	0.010	0.014	0.014	0.013	0.014	0.015	0.018	5.3%	-2.2%	-2.2%	-0.4%
	M(MAE)	0.103	0.128	0.148	0.126	0.160	0.137	0.176	0.163	-0.0245	0.0216	0.0236	0.0134
	StD (MAE)	0.016	0.024	0.018	0.011	0.015	0.010	0.019	0.012	-12.8%	-1.5%	-0.4%	-3.6%
	M(RMSE)/StD	32.2%	19.0%	31.9%	28.5%	34.5%	31.4%	41.1%	34.7%	13.2%	3.4%	3.1%	6.4%
	M(MAE)/StD	20.6%	25.5%	29.6%	25.3%	32.0%	27.3%	35.3%	32.6%	-4.9%	4.3%	4.7%	2.7%
Case 3	StD	4.000		3.000		1.500		0.500					
	M(RMSE)	1.146	0.626	0.739	0.593	0.716	0.564	0.307	0.228	0.5203	0.1470	0.1514	0.0788
	StD (RMSE)	0.159	0.054	0.100	0.058	0.044	0.043	0.014	0.017	7.7%	-0.4%	4.3%	9.5%
	M(MAE)	0.688	0.907	0.644	0.585	0.573	0.567	0.231	0.244	-0.2190	-0.0584	0.0064	-0.0126
	StD (MAE)	0.107	0.127	0.104	0.081	0.046	0.035	0.017	0.012	-11.3%	-4.2%	-5.0%	-8.3%
	M(RMSE)/StD	28.6%	15.6%	24.6%	19.8%	47.7%	37.6%	61.5%	45.7%	13.0%	4.9%	10.1%	15.8%
	M(MAE)/StD	17.2%	22.7%	21.5%	19.5%	38.2%	37.8%	46.2%	48.7%	-5.5%	1.9%	0.4%	-2.5%
Case 4	StD	2.000		1.500		0.750		0.250					
	M(RMSE)	0.564	0.312	0.368	0.296	0.354	0.281	0.153	0.114	0.2517	0.0717	0.0736	0.0392
	StD (RMSE)	0.075	0.025	0.052	0.030	0.021	0.021	0.007	0.009	7.6%	-0.7%	4.2%	9.4%
	M(MAE)	0.341	0.446	0.321	0.291	0.285	0.281	0.115	0.121	-0.1050	0.0298	0.0044	-0.0064
	StD (MAE)	0.049	0.060	0.055	0.042	0.022	0.017	0.009	0.006	-10.7%	-4.5%	-4.6%	-8.5%
	M(RMSE)/StD	28.2%	15.6%	24.5%	19.7%	47.3%	37.4%	61.1%	45.4%	12.6%	4.8%	9.8%	15.7%
	M(MAE)/StD	17.1%	22.3%	21.4%	19.4%	38.0%	37.4%	45.9%	48.5%	-5.3%	2.0%	0.6%	-2.5%
Case 5	StD	1.000		1.000		1.000		1.000					
	M(RMSE)	0.230	0.218	0.258	0.237	0.264	0.249	0.255	0.231	0.0123	0.0207	0.0155	0.0243
	StD (RMSE)	0.023	0.010	0.035	0.012	0.023	0.012	0.034	0.012	-2.1%	-2.7%	-2.0%	-2.2%
	M(MAE)	0.230	0.183	0.259	0.206	0.261	0.210	0.252	0.203	0.0466	0.0528	0.0503	0.0489
	StD (MAE)	0.023	0.018	0.037	0.029	0.023	0.018	0.035	0.027	0.5%	-1.3%	0.9%	-1.4%
	M(RMSE)/StD	23.0%	21.8%	25.8%	23.7%	26.4%	24.9%	25.5%	23.1%	1.2%	2.1%	1.5%	2.4%
	M(MAE)/StD	23.0%	18.3%	25.9%	20.6%	26.1%	21.0%	25.2%	20.3%	4.7%	5.3%	5.0%	4.9%

Table 5 **Error measures of quarterly series estimators – error component (continued)**

		Region 1		Region 2		Region 3		Region 4		DF-GM			
		DF	GM	DF	GM	DF	GM	DF	GM	Region 1	Region 2	Region 3	Region 4
Case 6	StD	1.000		1.000		1.000		1.000					
	M(RMSE)	0.291	0.258	0.323	0.311	0.359	0.344	0.347	0.334	0.0326	0.0117	0.0145	0.0135
	StD (RMSE)	0.024	0.028	0.025	0.024	0.021	0.038	0.023	0.025	-2.0%	-3.7%	-4.4%	-3.4%
	M(MAE)	0.267	0.231	0.321	0.257	0.349	0.284	0.343	0.276	0.0358	0.0641	0.0648	0.0670
	StD (MAE)	0.033	0.020	0.031	0.020	0.040	0.017	0.029	0.019	-1.7%	1.3%	0.8%	1.9%
	M(RMSE)/StD	29.1%	25.8%	32.3%	31.1%	35.9%	34.4%	34.7%	33.4%	3.3%	1.2%	1.5%	1.3%
	M(MAE)/StD	26.7%	23.1%	32.1%	25.7%	34.9%	28.4%	34.3%	27.6%	3.6%	6.4%	6.5%	6.7%
Case 7	StD	1.000		1.000		1.000		1.000					
	M(RMSE)	0.356	0.251	0.376	0.357	0.383	0.377	0.440	0.411	0.1050	0.0187	0.0057	0.0285
	StD (RMSE)	0.123	0.090	0.089	0.104	0.064	0.096	0.073	0.099	-10.7%	-17.5%	-15.5%	-14.3%
	M(MAE)	0.293	0.284	0.382	0.300	0.388	0.305	0.422	0.350	0.0090	0.0823	0.0831	0.0714
	StD (MAE)	0.111	0.099	0.115	0.074	0.091	0.052	0.097	0.059	-20.1%	-10.6%	-6.0%	-8.5%
	M(RMSE)/StD	35.6%	25.1%	37.6%	35.7%	38.3%	37.7%	44.0%	41.1%	10.5%	1.9%	0.6%	2.8%
	M(MAE)/StD	29.3%	28.4%	38.2%	30.0%	38.8%	30.5%	42.2%	35.0%	0.9%	8.2%	8.3%	7.1%

StD = standard deviation of the original series if not differently specified

M = average value

Table 6 **Distribution of the GLS estimators of the beta coefficients**

		Region 1		Region 2		Region 3		Region 4		DF-GM			
		DF	GM	DF	GM	DF	GM	DF	GM	Region 1	Region 2	Region 3	Region 4
Case 1	BIAS	0.001	0.000	0.003	0.002	-0.002	0.000	-0.004	0.000	0.0008	0.0013	0.0018	0.0042
	RMSE	0.095	0.037	0.111	0.068	0.090	0.066	0.092	0.066	0.0579	0.0432	0.0245	0.0259
Case 2	BIAS	-0.001	0.001	-0.001	0.000	0.000	0.000	0.000	-0.001	0.0002	0.0004	-0.0002	-0.0006
	RMSE	0.034	0.012	0.037	0.022	0.032	0.023	0.031	0.022	0.0216	0.0146	0.0090	0.0090
Case 3	BIAS	-0.001	0.001	0.003	0.000	0.004	0.001	0.001	0.002	-0.0002	0.0031	0.0032	-0.0012
	RMSE	0.090	0.036	0.110	0.060	0.098	0.068	0.103	0.071	0.0543	0.0493	0.0299	0.0318
Case 4	BIAS	0.000	-0.001	0.000	-0.001	0.001	0.001	0.002	0.001	-0.0005	-0.0008	0.0001	0.0008
	RMSE	0.029	0.012	0.038	0.022	0.032	0.022	0.034	0.023	0.0172	0.0154	0.0101	0.0104
Case 5	BIAS	-0.004	-0.002	-0.003	-0.002	-0.004	-0.004	-0.005	-0.003	0.0019	0.0009	-0.0005	0.0024
	RMSE	0.089	0.040	0.125	0.050	0.098	0.047	0.125	0.047	0.0485	0.0745	0.0510	0.0781
Case 6	BIAS	-0.001	0.000	-0.001	0.000	-0.003	-0.001	-0.001	0.001	0.0010	0.0006	0.0022	-0.0003
	RMSE	0.089	0.051	0.106	0.065	0.088	0.064	0.103	0.056	0.0386	0.0418	0.0241	0.0473
Case 7	BIAS	-0.013	-0.007	0.000	-0.009	0.016	-0.001	0.011	0.009	0.0062	-0.0085	0.0146	0.0021
	RMSE	0.245	0.143	0.246	0.193	0.214	0.202	0.205	0.188	0.1027	0.0530	0.0114	0.0171

Table 7 **Distribution of the estimators of the beta coefficients estimates variance**

		Region 1		Region 2		Region 3		Region 4		DF-GM			
		DF	GM	DF	GM	DF	GM	DF	GM	Region 1	Region 2	Region 3	Region 4
Case 1	BIAS	0.003	0.001	-0.003	0.000	0.001	0.000	0.000	-0.001	0.0020	0.0022	0.0011	-0.0002
	RMSE	0.004	0.001	0.003	0.001	0.002	0.001	0.002	0.001	0.0029	0.0015	0.0010	0.0009
Case 2	BIAS	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.0010	0.0006	0.0006	0.0005
	RMSE	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.0003	0.0002	0.0001	0.0001
Case 3	BIAS	0.010	0.002	0.010	0.004	0.010	0.004	0.009	0.004	0.0083	0.0057	0.0061	0.0046
	RMSE	0.003	0.001	0.002	0.001	0.002	0.001	0.002	0.001	0.0024	0.0015	0.0013	0.0009
Case 4	BIAS	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.0009	0.0006	0.0007	0.0005
	RMSE	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.0003	0.0002	0.0001	0.0001
Case 5	BIAS	0.012	0.001	0.011	0.001	0.012	0.001	0.011	0.001	0.0112	0.0099	0.0108	0.0103
	RMSE	0.004	0.000	0.003	0.000	0.003	0.000	0.003	0.000	0.0035	0.0031	0.0033	0.0032
Case 6	BIAS	0.011	0.001	0.010	0.002	0.010	0.002	0.009	0.002	0.0097	0.0073	0.0070	0.0074
	RMSE	0.003	0.001	0.003	0.001	0.002	0.001	0.002	0.000	0.0025	0.0019	0.0014	0.0018
Case 7	BIAS	0.001	-0.007	-0.009	-0.018	0.005	-0.024	0.000	-0.019	-0.0062	-0.0093	-0.0188	-0.0183
	RMSE	0.040	0.010	0.031	0.011	0.027	0.010	0.021	0.010	0.0306	0.0202	0.0165	0.0109

Table 8 **Distribution of autoregressive coefficient estimators of quarterly errors using GM procedure**

		A1	A2	A3	A4
Case 1	BIAS	-0.017	-0.048	-0.046	0.012
	RMSE	0.045	0.113	0.141	0.150
Case 2	BIAS	-0.019	-0.047	-0.050	0.004
	RMSE	0.045	0.107	0.139	0.149
Case 3	BIAS	-0.016	-0.048	-0.053	0.009
	RMSE	0.043	0.110	0.139	0.152
Case 4	BIAS	-0.019	-0.044	-0.049	0.009
	RMSE	0.046	0.106	0.138	0.151
Case 5	BIAS	0.377	0.376	0.374	0.372
	RMSE	0.049	0.049	0.055	0.056
Case 6	BIAS	0.086	0.153	0.099	0.237
	RMSE	0.091	0.117	0.139	0.124
Case 7	BIAS	-0.134	-0.115	-0.049	0.036
	RMSE	0.167	0.175	0.176	0.179

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