Benchmarking Systems of Seasonally Adjusted Time Series

Tommaso Di Fonzo* and Marco Marini**

Abstract

When a system of time series is seasonally adjusted, generally the accounting constraints originally linking the series are not fulfilled. To overcome this problem, we discuss an extension to a system of series linked by an accounting constraint of the classical univariate benchmarking procedure due to Denton (1971), which is founded on a movement preservation principle that is very relevant in this case. The presence of linear dependence between the variables makes it necessary to deal with the whole set of contemporaneous and temporal aggregation relationships. The cases of one-way classified (e.g., by regions or by industries) and of two-way classified (e.g., by regions and by industries) systems of series are studied. An empirical application to the Canadian retail trade series by province (12 series) and trade groups (18 series) is considered to show the capability of the proposed procedures.

Key Words: Benchmarking, Systems of time series, Accounting constraints, Denton's movement preservation principle

JEL Classification: C32, C63

* Dipartimento di Scienze Statistiche, Università degli Studi di Padova, Italy; difonzo@stat.unipd.it

** Direzione della Contabilità Nazionale, ISTAT, Rome, Italy; marco.marini@istat.it

We would like to thank Benoit Quenneville and an anonymous referee for their valuable comments on an earlier version of this paper. Financing from MIUR is also gratefully acknowledged.

Résumé

Lorsqu'on opère une correction des variations saisonnières sur un système de séries chronologiques, les contraintes comptables reliant à l'origine ces séries entre elles ne sont plus satisfaites. Afin de remédier à ce problème, nous examinons la possibilité d'étendre à un système de séries reliées par une contrainte comptable la procédure de calage univariée classique proposée par Denton (1971) se fondant sur un principe de préservation du mouvement tout à fait pertinent dans ce cas d'espèce. L'existence d'une dépendance linéaire entre les variables oblige à traiter l'ensemble des relations d'agrégation contemporaines et temporelles. Nous étudions le cas de systèmes de séries reposant sur une classification par rapport à un seul critère (région ou secteur d'activité, par exemple) ou par rapport à deux critères (région et secteur d'activité, par exemple). Nous appliquons ensuite la procédure proposée aux séries canadiennes du commerce de détail par province (12 séries) et par catégorie de produits (18 séries) afin de montrer ce qu'elle permet de faire.

1 Introduction

In this paper we deal with methods of producing high-frequency (e.g. monthly or quarterly) time series consistent with low-frequency (e.g. annual) data. The statistical procedure which combines a series of high-frequency data with a series of less frequent data for a certain variable into a consistent time series is generally known as benchmarking of time series (Bloem *et al*., 2001, p. 82). However, rather than on a single time series (Hillmer and Trabelsi, 1987, Cholette and Dagum, 1994, Durbin and Quenneville, 1997, Dagum *et al*., 1998, Quenneville *et al*., 2003), we focus on systems of time series to be reconciled in a coherent accounting framework. In other words, it is expected that the benchmarked series fulfil both temporal (for the same variable observed at different time frequencies) and contemporaneous (for different variables at the same time) constraints. Building on the least squares adjustment procedure by Stone *et al*. (1942),[1] we present statistical procedures grounded on a "movement preservation principle" (Denton, 1971), according to which the temporal profiles of the benchmarked series should be as similar as possible to those of the unbenchmarked counterparts. At least three distinctive features should be mentioned: (i) the proposed benchmarking procedure does not require any reliability measurement (e.g., standard errors) for the unbenchmarked series; (ii) the nature of the links for the series within and between the systems introduces some redundancy in the constraints that has to be taken into account when formulating a feasible solution; (iii) depending on the data characteristics and needs of the user, some high-frequency series in the systems can be considered as either binding (in other words, to be left unchanged by the benchmarking procedure) or unbinding. Moreover, in practical situations the dimension of the problem (which is related to the number of the series in the systems and to the covered time span) can be very large, making it difficult to handle the general benchmarking formula. In such cases, simplified expressions which exploit the constraint sparsity and the structure of the involved matrices can significantly reduce computer time and store requirements. All these characteristics are rather attractive, for example, when the user needs to restore the additivity (at both temporal and contemporaneous level) of one or more systems of time series individually seasonally adjusted through either X11 or X12, wishing at the same time to maintain their temporal profiles (Taillon, 1988, Quenneville and Rancourt, 2005).

Benchmarking systems of time series is a typical problem for data producers. Different classifications for the same phenomenon often lead to different total aggregates, so that an adjustment to the estimates is needed to give a coherent picture to users. In this context, Laniel and Fyfe (1989, pp. 464-65) write:

> *Most economic sub-annual surveys produce series of estimates for a number of industrial activities within a number of geographical regions. These are published sub-annually in the form of tables, where the cells as well as the marginals and the*

1 The literature on least-squares reconciliation of economic data is rather wide. We recall, amongst others, Byron (1978), van der Ploeg (1982, 1984), Barker *et al.* (1984), Weale (1992), Solomou and Weale (1993), Sefton and Weale (1995) and Smith *et al.* (1998).

grand totals need to be benchmarked. If one applies a benchmarking method independently on each cell series, each marginal series and the grand total series, the results will be a series of benchmarked sub-annual estimates where the sums of the cell totals are not equal to the marginal totals, and the sum of the marginal totals are not equal to the grand total. In other words, a series of inconsistent tables will be produced. To avoid this problem, a number of strategies can be adopted. Amongst these strategies, the first that comes to mind is the following simple approach. First, the cell series are independently benchmarked. Then, the benchmarked cell totals are summed up to get the benchmarked marginal totals and benchmarked grand totals. With this method one might get benchmarked margins and grand totals with chronological patterns which look more noisy than if they were directly benchmarked (this is a problem well known in seasonal adjustment). If this is the case one would be better to use the following method:

i) First benchmark the series of grand totals.

ii) Then, independently benchmark each series of marginal totals and then for each sub-annual period separately adjust the benchmarked margins by a constant factor so that they add up to the benchmarked grand totals.

iii) Finally, independently benchmark each series of cell totals and then for each sub-annual period separately adjust the benchmarked cells using the raking ratio algorithm (also called iterative proportional fitting, see Deming and Stephan, 1940) so that they add up to the adjusted benchmarked margins.

This method assumes that the series of grand totals is the most important series of the table in terms of preserving month-to-month trends, the series of marginal totals are the second most important and the series of cell totals are the least important. An inconvenient with this method is that the month-to-month trends of the cells can be very much disturbed (…) One can also think of benchmarking simultaneously the cell series with the margin series and the grand total series. Then the problem can become very large in terms of the number of parameters to estimate and even difficult to handle with a computer. This has been addressed by Cholette (1988b) in the case where series are to be benchmarked with Denton's method.

Our starting point is the latter reference quoted by Laniel and Fyfe, that is the paper by Cholette (1988), which presents a wide review of the problems that a researcher faces when systems of time series are to be benchmarked in order to simultaneously fulfil temporal and contemporaneous (longitudinal, geographical) aggregation constraints. In what follows we will refer to benchmarking systems of (or tables of two-way classified) individually seasonally adjusted (SA) series, but the results are more general and apply to any context in which accounting constraints hold (Chen and Dagum, 1997, Eurostat, 1999, Di Fonzo, 2002). We will consider benchmarking according to Denton's movement preservation principle, dealing with both binding and unbinding constraints.[2] Particular attention should be given to computational issues, which until now have represented a major obstacle to large-scale application of these types of procedures.

The paper is organized as follows. In Section 2 we set the notation and present the general solution for benchmarking one system of series when a binding accounting constraint is available. Two classic variants (additive and proportional) of Denton's movement preservation principle are discussed in Section 3, where we consider also the case of a system of series with an unbinding accounting constraint. The benchmarking of two systems of series linked by the same accounting constraint (either binding or unbinding) is described in Section 4. Then we deal with two enlargements of the simultaneous benchmarking problem. Firstly (Section 5) we develop a benchmarking procedure according to a movement preservation principle explicitly operating on period-to-period rates of change, based on suitable log-transformations of the variables. Secondly (Section 6), we discuss the more general problem of benchmarking a two-way classified table of time series, giving a solution in two cases generally encountered in practice. Finally, in Section 7 the proposed procedures have been used in benchmarking individually seasonally adjusted monthly Canadian retail trade series, classified by province and industry (trade groups).

2 Statement of the Problem and some Notation

We assume that the available information is given by (i) $M+1$ monthly raw series $\{x_{it}\}$, $i=1,...,M+1$, $t=1,...,n$, linked by the accounting constraints $\sum_{i=1}^{M} x_{it} = x_{M+1,t}$, $t=1,...,n$, and (ii) by $M+1$ monthly unbenchmarked SA series $\{y_{it}\}$, $i=1,...,M+1$, $t=1,...,n$, such that, in general, neither contemporaneous (between SA series) nor temporal (with the annual sums of the raw series) aggregation links are fulfilled:

$$\sum_{i=1}^{M} y_{it} \neq y_{M+1,t}\,,\ t=1,...,n\,; \qquad \sum_{s=1}^{12} y_{i,(T-1)12+s} \neq \sum_{s=1}^{12} x_{i,(T-1)12+s}\,,\ i=1,...,M+1\,,\ T=1,...,N\,.$$

We want to estimate benchmarked SA monthly series, say $\{y_{i,t}^{*}\}$, $i=1,...,M+1$, $t=1,...,n$, such that all the aggregation constraints are simultaneously fulfilled:

$$\sum_{i=1}^{M} y_{i,t}^{*} = y_{M+1,t}^{*}\,,\ t=1,...,n\,; \qquad \sum_{s=1}^{12} y_{i,(T-1)12+s}^{*} = \sum_{s=1}^{12} x_{i,(T-1)12+s}\,,\ i=1,...,M+1\,,\ T=1,...,N\,.$$

In what follows we consider the situation, often encountered in practice, in which one series – generally the one representing the total – constitutes a *binding* constraint ($y_{M+1,t} \equiv y_{M+1,t}^{*}$), and thus it must not be benchmarked. Then we present the general least squares solution to the benchmarking problem, which has to be worked out in order to deal

2 Benchmarking one system of time series with a binding contemporaneous constraint has been dealt with in Eurostat (1999) and Di Fonzo (2002).

with redundant constraints. In the next sections, we deal with two other cases: (i) one in which the whole set of $M+1$ series must be benchmarked (*unbinding* constraint) and (ii) the contemporaneous benchmarking of two systems of time series linked by an accounting constraint, either binding or not.

2.1 One system of series with a binding accounting constraint

Let us denote by $\boldsymbol{x}_i$, $i=1,...,M$, the $(n\times 1)$ vectors of raw data for the M component series, $\boldsymbol{y}_i$ the SA unbenchmarked counterparts, $\boldsymbol{y}_i^*$ the benchmarked vectors to be derived and, finally, $\boldsymbol{z}=\boldsymbol{y}_{M+1}=\boldsymbol{y}_{M+1}^*$ the aggregate series to be assumed as a binding constraint to be fulfilled by the benchmarked series. Denoting $\boldsymbol{I}_M=(1,\ldots,1,\ldots1)'$ and $\boldsymbol{J}$ the $(N\times n)$ matrix performing temporal aggregation of a single monthly vector,[3] the whole set of aggregation constraints can be expressed in matrix form as follows:

$$\begin{bmatrix} \boldsymbol{I}_M'\otimes\boldsymbol{I}_n \\ \boldsymbol{I}_M\otimes\boldsymbol{J}\end{bmatrix}\boldsymbol{y}^*=\begin{bmatrix}\boldsymbol{z}\\ \boldsymbol{x}_0\end{bmatrix}, \tag{1}$$

where $\boldsymbol{y}^*=\left(\boldsymbol{y}_1^{*\prime},\ldots,\boldsymbol{y}_i^{*\prime},\ldots,\boldsymbol{y}_M^{*\prime}\right)'$, $\boldsymbol{x}_0=(\boldsymbol{I}_M\otimes\boldsymbol{J})\boldsymbol{x}$ is the $((M\cdot N)\times 1)$ vector containing the yearly sums of the M component raw series, and $\boldsymbol{x}=\left(\boldsymbol{x}_1',\ldots,\boldsymbol{x}_i',\ldots,\boldsymbol{x}_M'\right)'$. The accounting constraints (1) can be written in compact form as $\boldsymbol{H}\boldsymbol{y}^*=\boldsymbol{x}_a$, where the matrix

$$\boldsymbol{H}=\begin{bmatrix} \boldsymbol{I}_M'\otimes\boldsymbol{I}_n \\ \boldsymbol{I}_M\otimes\boldsymbol{J}\end{bmatrix} \quad \text{has dimension } \left((n+M\cdot N)\times M\cdot n\right)$$

and $\boldsymbol{x}_a=[\boldsymbol{z}' \quad \boldsymbol{x}_0']'$ is a $((n+M\cdot N)\times 1)$ vector containing the contemporaneous and temporal aggregates. Notice that a contemporaneous accounting constraint holds for the yearly sums too, $\sum_{i=1}^{M}x_{0i,T}=x_{0M+1,T}=\sum_{s=1}^{12}z_{(T-1)12+s}$, $T=1,...,N$. That is, in matrix form,

$$\left[\boldsymbol{J} \quad \vdots \quad -\left(\boldsymbol{I}_M'\otimes\boldsymbol{I}_N\right)\right]\boldsymbol{x}_a=\boldsymbol{0}. \tag{2}$$

Relationship (2) states that N linear restrictions of the $n+M\cdot N$ established by expression (1) are superfluous, so that matrix $\boldsymbol{H}$ is not of full row rank. To clarify this fact, partition $\boldsymbol{H}$ in such a way as to distinguish the temporal aggregation constraints linking $\boldsymbol{y}_M^*$ to $\boldsymbol{x}_{0,M}=\boldsymbol{J}\boldsymbol{x}_M$ from the remainder: $\boldsymbol{H}=[\boldsymbol{H}_w' \quad \boldsymbol{H}_M']'$, where

$$\boldsymbol{H}_w=\begin{bmatrix} \boldsymbol{I}_{M-1}'\otimes\boldsymbol{I}_n & \vdots & \boldsymbol{I}_n \\ \boldsymbol{I}_{M-1}\otimes\boldsymbol{J} & \vdots & \boldsymbol{0}\end{bmatrix} \quad \text{and} \quad \boldsymbol{H}_M=[\boldsymbol{0} \quad \vdots \quad \boldsymbol{J}]$$

---

3 For example, assuming that the first monthly observation of the series falls on January, and assuming $n=12\cdot N$, $\boldsymbol{J}$ takes the form $\boldsymbol{J}=\boldsymbol{I}_N\otimes\boldsymbol{I}_{12}'$.

are matrices $(r \times M \cdot n)$ and $(N \times M \cdot n)$, respectively, with $r = n + (M-1) \cdot N$, such that

$$\boldsymbol{H}_w \boldsymbol{y}^* = \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{J}\boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{J}\boldsymbol{x}_{M-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{x}_{01} \\ \vdots \\ \boldsymbol{x}_{0M-1} \end{bmatrix} = \boldsymbol{w} \quad \text{and} \quad \boldsymbol{H}_M \boldsymbol{y}^* = \boldsymbol{J}\boldsymbol{x}_M = \boldsymbol{x}_{0M} .$$

Denoting $\boldsymbol{W}$ the $(N \times r)$ matrix $\boldsymbol{W} = \left[\boldsymbol{J} \;\vdots\; -\left(\boldsymbol{I}'_{M-1} \otimes \boldsymbol{I}_N \right) \right]$ and $\boldsymbol{R}$ the $((r+N) \times r)$ matrix $\boldsymbol{R} = \left[\boldsymbol{I}_r \quad \boldsymbol{W}' \right]'$, the following relationships hold:

$$\boldsymbol{H}_M = \boldsymbol{W}\boldsymbol{H}_w, \qquad \boldsymbol{H} = \boldsymbol{R}\boldsymbol{H}_w, \tag{3}$$

$$\boldsymbol{R}\boldsymbol{w} = \begin{bmatrix} \boldsymbol{I}_r \\ \boldsymbol{W} \end{bmatrix} \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{x}_{01} \\ \vdots \\ \boldsymbol{x}_{0M-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{x}_{01} \\ \vdots \\ \boldsymbol{x}_{0M-1} \\ \boldsymbol{J}\boldsymbol{z} - \sum_{i=1}^{M-1} \boldsymbol{x}_{0i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{w} \\ \boldsymbol{x}_{0M} \end{bmatrix} = \boldsymbol{x}_a .$$

Thus, constraints (1) can be expressed as $\boldsymbol{R}\boldsymbol{H}_w \boldsymbol{y}^* = \boldsymbol{R}\boldsymbol{w}$.

2.2 Benchmarking as a least-squares estimation problem

Assume that the available SA data $\boldsymbol{y}_i$ are distributed without bias around the "true" benchmarked SA series $\boldsymbol{y}_i^*$ according to the model

$$\boldsymbol{y}_i = \boldsymbol{y}_i^* + \boldsymbol{e}_i , \quad i = 1, \ldots, M , \tag{4}$$

where $\boldsymbol{e}_i$ are $(n \times 1)$ zero-mean random disturbances, with $E\left(\boldsymbol{e}_i \boldsymbol{e}_j' \right) = \boldsymbol{\Omega}_{ij}$, $i, j = 1, \ldots, M$, and $\boldsymbol{\Omega}_{ij}$ are $(n \times n)$ known matrices.[4] Putting together the *M* relationships (4) we have the complete model $\boldsymbol{y} = \boldsymbol{y}^* + \boldsymbol{e}$, with $E(\boldsymbol{e}) = \boldsymbol{0}$ and $E(\boldsymbol{e}\boldsymbol{e}') = \boldsymbol{\Omega}$. The simultaneously benchmarked series can be obtained as solution of the least squares problem

$$\min_{\boldsymbol{y}^*} \left(\boldsymbol{y} - \boldsymbol{y}^* \right)' \boldsymbol{\Omega}^{-1} \left(\boldsymbol{y} - \boldsymbol{y}^* \right) \qquad \text{subject to} \quad \boldsymbol{H}\boldsymbol{y}^* = \boldsymbol{x}_a .$$

4 For the time being, we assume that these matrices have full rank (see footnote 6).

However, the constraints (1) being linearly dependent, an extension of the classical result by Stone *et al.* (1942) is needed due to the rank of the matrices involved in the procedure. It can be shown (Di Fonzo and Marini, 2003) that the benchmarked estimates can be expressed as

$$\hat{\boldsymbol{y}}^* = \boldsymbol{y} + \boldsymbol{\Omega H}'\left(\boldsymbol{H\Omega H}'\right)^{-}\left(\boldsymbol{x}_a - \boldsymbol{Hy}\right), \tag{5}$$

where $\left(\boldsymbol{H\Omega H}'\right)^{-}$ denotes the Moore-Penrose generalized inverse of $\boldsymbol{\Omega}_a = \boldsymbol{H\Omega H}'$. A solution, equivalent to (5), and which does not involve singular matrices to be inverted, can be expressed in terms of r "free" observations. In fact, using expression (3), the singular matrix $\boldsymbol{H\Omega H}'$ can be written as $\boldsymbol{RH}_w\boldsymbol{\Omega H}'_w\boldsymbol{R}' = \boldsymbol{R\Omega}_w\boldsymbol{R}'$, where $\boldsymbol{\Omega}_w = \boldsymbol{H}_w\boldsymbol{\Omega H}'_w$ is a full rank $(r \times r)$ matrix. Furthermore, it can be readily checked (Di Fonzo and Marini, 2003) that $\boldsymbol{\Omega}_a^{-}$ is univocally given by

$$\boldsymbol{\Omega}_a^{-} = \left(\boldsymbol{H\Omega H}'\right)^{-} = \boldsymbol{R}\left(\boldsymbol{R}'\boldsymbol{R}\right)^{-1}\boldsymbol{\Omega}_w^{-1}\left(\boldsymbol{R}'\boldsymbol{R}\right)^{-1}\boldsymbol{R}'. \tag{6}$$

By substituting (6) into (5), and taking into account that $\boldsymbol{R}'\boldsymbol{H} = \boldsymbol{R}'\boldsymbol{RH}_w$, after some algebra we find the more feasible benchmarking formula

$$\hat{\boldsymbol{y}}^* = \boldsymbol{y} + \boldsymbol{\Omega H}'_w\boldsymbol{\Omega}_w^{-1}\left(\boldsymbol{w} - \boldsymbol{H}_w\boldsymbol{y}\right). \tag{7}$$

The benchmarked estimates are thus obtained by distributing a linear combination of the discrepancies pertaining to r unconstrained observations of the aggregated vector $\boldsymbol{x}_a$ over the original unbenchmarked data. It should be noted that expression (7) involves the inversion only of full rank matrices and fulfils both temporal and contemporaneous constraints.

3 The Choice of $\boldsymbol{\Omega}$: Denton's Movement Preservation Principle

A natural way of proceeding in the choice of $\boldsymbol{\Omega}$ is to reconsider the "movement preservation principle" stated by Denton (1971).[5] According to that principle, additive and proportional variants of the multivariate extension of Denton's benchmarking procedure should operate by focusing on, respectively, the simple period-to-period change,

$$\left(y_{i,t}^* - y_{i,t-1}^*\right) - \left(y_{i,t} - y_{i,t-1}\right) \equiv \left(y_{i,t}^* - y_{i,t}\right) - \left(y_{i,t-1}^* - y_{i,t-1}\right), \quad i = 1, \dots, M \,,$$

or the proportional period-to-period change,

$$\frac{y_{i,t}^* - y_{i,t}}{y_{i,t}} - \frac{y_{i,t-1}^* - y_{i,t-1}}{y_{i,t-1}} \equiv \frac{y_{i,t}^*}{y_{i,t}} - \frac{y_{i,t-1}^*}{y_{i,t-1}}, \quad i = 1, \dots, M \,.$$

5 See also Cholette (1988).

The objective functions to be minimized are thus given by

$$\sum_{i=1}^{M}\sum_{t=2}^{n}\left[\left(y_{i,t}^{*}-y_{i,t}\right)-\left(y_{i,t-1}^{*}-y_{i,t-1}\right)\right]^{2} \quad \text{and}$$

$$\sum_{i=1}^{M}\sum_{t=2}^{n}\left(\frac{y_{i,t}^{*}}{y_{i,t}}-\frac{y_{i,t-1}^{*}}{y_{i,t-1}}\right)^{2} \quad \text{respectively.}$$

Using matrix notation, $\boldsymbol{\Omega}$ can be expressed as (Eurostat, 1999, Di Fonzo, 2002):

$$\left[\boldsymbol{I}_M \otimes \left(\boldsymbol{D}'\boldsymbol{D}\right)\right]^{-1} = \boldsymbol{I}_M \otimes \left(\boldsymbol{D}'\boldsymbol{D}\right)^{-1} \qquad \text{Additive First Differences (AFD)}$$

$$\hat{\boldsymbol{Y}}\left(\boldsymbol{I}_M \otimes \boldsymbol{D}'\boldsymbol{D}\right)^{-1}\hat{\boldsymbol{Y}} = \hat{\boldsymbol{Y}}\left[\boldsymbol{I}_M \otimes \left(\boldsymbol{D}'\boldsymbol{D}\right)^{-1}\right]\hat{\boldsymbol{Y}} \qquad \text{Proportional First Differences (PFD)}$$

where $\boldsymbol{D}$ is the $(n \times n)$ matrix[6] performing first differences:

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix} \quad \text{and}$$

$$\hat{\boldsymbol{Y}} = \begin{bmatrix} diag(\boldsymbol{y}_1) & \boldsymbol{0} & \cdots & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & diag(\boldsymbol{y}_2) & \cdots & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & diag(\boldsymbol{y}_i) & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \cdots & diag(\boldsymbol{y}_M) \end{bmatrix}.$$

The dimensions of the matrices involved in the calculations can be considerable in practical situations, giving rise to long computational times. However, it is possible to obtain a valuable saving of time by exploiting the partitioned form of the matrices (Di Fonzo and Marini, 2003).

---

6 For computational convenience, we present the original difference matrix used by Denton (1971). As Cholette (1984) pointed out, at univariate level the computational reasons behind this specification, which involves $y_{i,0}^{*}-y_{i,0}=0$, have become obsolete. We are currently engaged in working out simplified multivariate benchmarking formulae, which correctly deal with the first observation of each series.

3.1 One system of series and an unbinding accounting constraint

In this case the series y_{M+1} has to be benchmarked together with the other M component series. As a consequence, the aggregation constraints, expressed so as to distinguish the "free" links from the redundant ones, are given by

$$\begin{bmatrix} \boldsymbol{1}'_M \otimes \boldsymbol{I}_n & \vdots - \boldsymbol{I}_n \\ \boldsymbol{I}_M \otimes \boldsymbol{J} & \vdots \;\; \boldsymbol{0} \\ \boldsymbol{0} & \vdots \;\; \boldsymbol{J} \end{bmatrix} \boldsymbol{y}^* = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{x}_0 \end{bmatrix}, \tag{8}$$

where, with a slight change of notation with respect to the previous sections, $\boldsymbol{y}^* = \left(\boldsymbol{y}_1^{*\prime},\ldots,\boldsymbol{y}_i^{*\prime},\ldots,\boldsymbol{y}_M^{*\prime},\boldsymbol{y}_{M+1}^{*\prime}\right)'$, $\boldsymbol{x}_0 = \left(\boldsymbol{I}_{M+1} \otimes \boldsymbol{J}\right)\boldsymbol{x}$ and $\boldsymbol{x} = \left(\boldsymbol{x}'_1,\ldots,\boldsymbol{x}'_i,\ldots,\boldsymbol{x}'_M,\boldsymbol{x}'_{M+1}\right)'$.

We can easily deal with this case by a re-parameterisation which transforms the problem in hand into standard benchmarking with a binding constraint, as considered in Section 2. In fact, it is sufficient to consider $\boldsymbol{g} = \left(\boldsymbol{y}'_1,\ldots,\boldsymbol{y}'_i,\ldots,\boldsymbol{y}'_M,-\boldsymbol{y}'_{M+1}\right)'$, $\boldsymbol{g}^* = \left(\boldsymbol{y}_1^{*\prime},\ldots,\boldsymbol{y}_i^{*\prime},\ldots,\boldsymbol{y}_M^{*\prime},-\boldsymbol{y}_{M+1}^{*\prime}\right)'$, $\boldsymbol{v} = \left(\boldsymbol{x}'_1,\ldots,\boldsymbol{x}'_i,\ldots,\boldsymbol{x}'_M,-\boldsymbol{x}'_{M+1}\right)'$, $\boldsymbol{v}_0 = \left(\boldsymbol{x}'_{01},\ldots,\boldsymbol{x}'_{0M},-\boldsymbol{x}'_{0M+1}\right)'$ and re-write (8) in the equivalent form

$$\boldsymbol{H}\boldsymbol{g}^* = \boldsymbol{v}_a, \quad \text{where, in this case,} \quad \boldsymbol{H} = \begin{bmatrix} \boldsymbol{1}'_{M+1} \otimes \boldsymbol{I}_n \\ \boldsymbol{I}_{M+1} \otimes \boldsymbol{J} \end{bmatrix}$$

has dimension $(n+(M+1)N)\times(M+1)n)$, and $\boldsymbol{v}_a = \begin{bmatrix} \boldsymbol{0}' & \boldsymbol{v}'_0 \end{bmatrix}'$. The benchmarked estimates are thus given by

$$\hat{\boldsymbol{g}}^* = \boldsymbol{g} + \boldsymbol{\Omega}\boldsymbol{H}'_u\,\boldsymbol{\Omega}_u^{-1}\left(\boldsymbol{u} - \boldsymbol{H}_u\boldsymbol{g}\right), \quad \text{where}$$

$$\boldsymbol{H}_u = \begin{bmatrix} \boldsymbol{1}'_M \otimes \boldsymbol{I}_n & \vdots & \boldsymbol{I}_n \\ \boldsymbol{I}_M \otimes \boldsymbol{J} & \vdots & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{\Omega}_u = \boldsymbol{H}_u\boldsymbol{\Omega}\boldsymbol{H}'_u \quad \text{and} \quad \boldsymbol{u} = \boldsymbol{H}_u\boldsymbol{g}^* = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{J}\boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{J}\boldsymbol{x}_M \end{bmatrix}.$$

The final benchmarked estimates are thus given by

$$\hat{\boldsymbol{y}}_i^* = \hat{\boldsymbol{g}}_i^*, \quad i = 1,\ldots,M \quad \text{and} \quad \hat{\boldsymbol{y}}_{M+1}^* = -\hat{\boldsymbol{g}}_{M+1}^*.$$

4 Benchmarking Two Systems of Time Series Linked by an Accounting Constraint

Consider the situation in which the same aggregate can be broken down according to two different single classification schemes[7] (e.g., the former by regions and the latter by economic activity sectors). If the component series of both systems have been seasonally adjusted individually, it is likely that the "geographically" aggregated series is different from the aggregate obtainable from the "sector side", and possibly from a directly seasonally adjusted aggregate series too. In the following we will consider three cases, corresponding respectively to situations in which:

1. a binding contemporaneous constraint is available;
2. no preliminary estimate of the constraint series is available;
3. a preliminary estimate of the constraint series is available and has to be benchmarked together (and coherently) with the component series of both systems.

4.1 Binding constraint

In this case the available data are given by

$\boldsymbol{a}_i$, $i=1,...,R$:	component unbenchmarked SA series of the first system;
$\boldsymbol{b}_j$, $j=1,...,S$:	component unbenchmarked SA series of the second system;
$\boldsymbol{a}_{0i}$, $i=1,...,R$:	annual sums of the first system's component series;
$\boldsymbol{b}_{0j}$, $j=1,...,S$:	annual sums of the second system's component series;
$\boldsymbol{z}$:	binding constraint series.

The temporally aggregated component series must satisfy the accounting relationship $\sum_{i=1}^{R}\boldsymbol{a}_{0i}=\sum_{j=1}^{S}\boldsymbol{b}_{0j}$. Moreover, the estimated benchmarked series $\hat{\boldsymbol{a}}_i^*$ and $\hat{\boldsymbol{b}}_j^*$ must be such that $\sum_{i=1}^{R}\hat{\boldsymbol{a}}_i^*=\boldsymbol{z}$, $\sum_{j=1}^{S}\hat{\boldsymbol{b}}_j^*=\boldsymbol{z}$ (that is $\sum_{i=1}^{R}\hat{\boldsymbol{a}}_i^*-\sum_{j=1}^{S}\hat{\boldsymbol{b}}_j^*=\boldsymbol{0}$), $\boldsymbol{J}\hat{\boldsymbol{a}}_i^*=\boldsymbol{a}_{0i}$, $i=1,...,R$, and $\boldsymbol{J}\hat{\boldsymbol{b}}_j^*=\boldsymbol{b}_{0j}$, $j=1,...,S$. The vector containing all the available aggregated data, arranged in a way which helps in writing the aggregation matrix valid for the "free" information, is

$$\boldsymbol{y}_a=\begin{bmatrix}\boldsymbol{z}' & \boldsymbol{a}'_{01} & \cdots & \boldsymbol{a}'_{0R-1} & \boldsymbol{z}' & \boldsymbol{b}'_{01} & \cdots & \boldsymbol{b}'_{0S-1} & \boldsymbol{a}'_{0R} & \boldsymbol{b}'_{0S}\end{bmatrix}'.$$

In fact, $2N$ observations are "redundant", in the sense that for each system the N annually sums for a variable are automatically determined by the difference between the annually sums of the binding series and the sums of the remaining $M-1$ annually component series. The whole set of aggregation constraints can thus be expressed as

[7] Notice that we are not dealing with a two-way classification scheme (see Cholette, 1988, and Section 6).

$\boldsymbol{H}_w \boldsymbol{y}^* = \boldsymbol{y}_w$, where

$$\boldsymbol{H}_w = \begin{bmatrix} \boldsymbol{1}'_{R-1} \otimes \boldsymbol{I}_n & \boldsymbol{I}_n & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I}_{R-1} \otimes \boldsymbol{J} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1}'_{S-1} \otimes \boldsymbol{I}_n & \boldsymbol{I}_n \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_{S-1} \otimes \boldsymbol{J} & \boldsymbol{0} \end{bmatrix},$$

$$\boldsymbol{y}^* = \left[\boldsymbol{a}_1^{*\prime} \quad \cdots \quad \boldsymbol{a}_R^{*\prime} \quad \boldsymbol{b}_1^{*\prime} \quad \cdots \quad \boldsymbol{b}_S^{*\prime} \right]' \text{ and } \boldsymbol{y}_w = \left[\boldsymbol{z}' \quad \boldsymbol{a}'_{01} \quad \cdots \quad \boldsymbol{a}'_{0R-1} \quad \boldsymbol{z}' \quad \boldsymbol{b}'_{01} \quad \cdots \quad \boldsymbol{b}'_{0S-1} \right]'.$$

Now, remember that for the additive variant of the benchmarking procedure (see Section 3), $\boldsymbol{\Omega} = \boldsymbol{I}_{R+S} \otimes \boldsymbol{Q}$, $\boldsymbol{Q} = \left(\boldsymbol{D}'\boldsymbol{D}\right)^{-1}$. Intuitively, the block-diagonal structure of both $\boldsymbol{H}_w$ and $\boldsymbol{\Omega}$ should notably simplify the benchmarking formulae. In fact, it can be easily checked that

$$\boldsymbol{\Omega}_w = \begin{bmatrix} \boldsymbol{\Omega}_w^a & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Omega}_w^b \end{bmatrix}, \text{ with}$$

$$\boldsymbol{\Omega}_w^a = \begin{bmatrix} R \cdot \boldsymbol{Q} \vdots & \boldsymbol{1}'_{R-1} \otimes \boldsymbol{QJ}' \\ \boldsymbol{1}_{R-1} \otimes \boldsymbol{JQ} \vdots & \boldsymbol{I}_{R-1} \otimes \boldsymbol{JQJ}' \end{bmatrix} \text{ and } \boldsymbol{\Omega}_w^b = \begin{bmatrix} S \cdot \boldsymbol{Q} \vdots & \boldsymbol{1}'_{S-1} \otimes \boldsymbol{QJ}' \\ \boldsymbol{1}_{S-1} \otimes \boldsymbol{JQ} \vdots & \boldsymbol{I}_{S-1} \otimes \boldsymbol{JQJ}' \end{bmatrix}.$$

Denoting $\hat{\boldsymbol{y}}^* = \left[\hat{\boldsymbol{a}}^{*\prime}, \hat{\boldsymbol{b}}^{*\prime} \right]'$ and $\boldsymbol{y} = \left[\boldsymbol{a}', \boldsymbol{b}' \right]'$, the benchmarking formula (7) reduces to

$$\begin{aligned} \hat{\boldsymbol{a}}^* &= \boldsymbol{a} + \boldsymbol{\Omega}^a \boldsymbol{H}_w^{a\prime} \left(\boldsymbol{\Omega}_w^a\right)^{-1} \left(\boldsymbol{a}_w - \boldsymbol{H}_w^a \boldsymbol{a}\right) \\ \hat{\boldsymbol{b}}^* &= \boldsymbol{b} + \boldsymbol{\Omega}^b \boldsymbol{H}_w^{b\prime} \left(\boldsymbol{\Omega}_w^b\right)^{-1} \left(\boldsymbol{b}_w - \boldsymbol{H}_w^b \boldsymbol{b}\right) \end{aligned}, \text{ where, with obvious notation,}$$

$$\boldsymbol{\Omega}^a = \boldsymbol{I}_R \otimes \boldsymbol{Q}, \ \boldsymbol{\Omega}^b = \boldsymbol{I}_S \otimes \boldsymbol{Q}, \ \boldsymbol{H}_w^a = \begin{bmatrix} \boldsymbol{1}'_{R-1} \otimes \boldsymbol{I}_n & \vdots & \boldsymbol{I}_n \\ \boldsymbol{I}_{R-1} \otimes \boldsymbol{J} & \vdots & \boldsymbol{0} \end{bmatrix}, \ \boldsymbol{H}_w^b = \begin{bmatrix} \boldsymbol{1}'_{S-1} \otimes \boldsymbol{I}_n & \vdots & \boldsymbol{I}_n \\ \boldsymbol{I}_{S-1} \otimes \boldsymbol{J} & \vdots & \boldsymbol{0} \end{bmatrix} \text{ and}$$

$$\boldsymbol{a}_w = \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{a}_{01} \\ \vdots \\ \boldsymbol{a}_{0R-1} \end{bmatrix}, \quad \boldsymbol{b}_w = \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{b}_{01} \\ \vdots \\ \boldsymbol{b}_{0S-1} \end{bmatrix}.$$

In conclusion, when a contemporaneous binding constraint is available, the benchmarked estimates for the component series of both systems are exactly those obtained by separately benchmarking each system of series with a binding constraint. Given the form of the matrices involved, this result holds for the proportional variant of the benchmarking procedure too.

4.2 Unbinding (and unavailable preliminary estimate) constraint

In this case the available data are given by

$\boldsymbol{a}_i$, $i = 1,...,R$: component unbenchmarked SA series of the first system;

$\boldsymbol{b}_j$, $j = 1,...,S$: component unbenchmarked SA series of the second system;

$\boldsymbol{a}_{0i}$, $i = 1,...,R$: annual sums of the first system's component series;

$\boldsymbol{b}_{0j}$, $j = 1,...,S$: annual sums of the second system's component series.

As regards the accounting constraints, the estimated benchmarked series $\hat{\boldsymbol{a}}_i^*$ and $\hat{\boldsymbol{b}}_j^*$ must be such that $\sum_{i=1}^{R} \hat{\boldsymbol{a}}_i^* - \sum_{j=1}^{S} \hat{\boldsymbol{b}}_j^* = \boldsymbol{0}$, $\boldsymbol{J}\hat{\boldsymbol{a}}_i^* = \boldsymbol{a}_{0i}$, $i = 1,...,R$, and $\boldsymbol{J}\hat{\boldsymbol{b}}_j^* = \boldsymbol{b}_{0j}$, $j = 1,...,S$. It is immediate to recognize that we can turn to a standard benchmarking of a system with a binding constraint (see Section 2) by means of the following re-parameterization:

$$M = R + S, \qquad \boldsymbol{z} = \boldsymbol{0}, \quad \boldsymbol{y} = \begin{bmatrix} \boldsymbol{a} \\ -\boldsymbol{b} \end{bmatrix}, \quad \boldsymbol{x}_a = \left[\boldsymbol{0}', \boldsymbol{a}'_{01}, \ldots, \boldsymbol{a}'_{0R}, \boldsymbol{b}'_{01}, \ldots, \boldsymbol{b}'_{0S}\right]'.$$

It should be noted that the benchmarked estimates of the second system's component series are obtained after a change of sign.

4.3 Unbinding constraint with an available preliminary estimate

In this case, besides the aggregated data considered in the previous sub-section, we assume that a $(n \times 1)$ vector $\boldsymbol{v}$ is available, which is the unbenchmarked SA series for the aggregate. In general, for this new information no aggregation constraint is fulfilled:

$$\sum_{i=1}^{R} \boldsymbol{a}_i \neq \boldsymbol{v}, \quad \sum_{j=1}^{S} \boldsymbol{b}_j \neq \boldsymbol{v}, \quad \sum_{i=1}^{R} \boldsymbol{a}_{0i} \neq \boldsymbol{J}\boldsymbol{v} \quad \text{and} \quad \sum_{j=1}^{S} \boldsymbol{b}_{0j} \neq \boldsymbol{J}\boldsymbol{v}.$$

Again, we wish to get benchmarked estimates that preserve as much as possible the dynamic profiles of the original unbenchmarked series while fulfilling all the aggregation constraints.

Let $\boldsymbol{y}^* = \left[\boldsymbol{a}^{*\prime}, \boldsymbol{b}^{*\prime}, \boldsymbol{v}^{*\prime}\right]'$ be the $((R+S+1)n \times 1)$ vector of unknown benchmarked SA series and $\hat{\boldsymbol{y}}^* = \left[\hat{\boldsymbol{a}}^{*\prime}, \hat{\boldsymbol{b}}^{*\prime}, \hat{\boldsymbol{v}}^{*\prime}\right]'$ the benchmarked estimates, which have to fulfil the constraints $\sum_{i=1}^{R} \hat{\boldsymbol{a}}_i^* - \hat{\boldsymbol{v}}^* = \boldsymbol{0}$, $\sum_{j=1}^{S} \hat{\boldsymbol{b}}_j^* - \hat{\boldsymbol{v}}^* = \boldsymbol{0}$ (that is, $\sum_{i=1}^{R} \hat{\boldsymbol{a}}_i^* - \sum_{j=1}^{S} \hat{\boldsymbol{b}}_j^* = \boldsymbol{0}$), $\boldsymbol{J}\hat{\boldsymbol{a}}_i^* = \boldsymbol{a}_{0i}$, $i = 1,...,R$ and $\boldsymbol{J}\hat{\boldsymbol{b}}_j^* = \boldsymbol{b}_{0j}$, $j = 1,...,S$. As done previously, it is convenient to express the vector containing all the available aggregated data in such a way as to easily write the aggregation matrix working only on the "free" information:

$$\boldsymbol{y}_a = \left[\boldsymbol{0}' \quad \boldsymbol{a}'_{01} \quad \cdots \quad \boldsymbol{a}'_{0R-1} \quad \boldsymbol{0}' \quad \boldsymbol{b}'_{01} \quad \cdots \quad \boldsymbol{b}'_{0S-1} \quad \boldsymbol{a}'_{0R} \quad \boldsymbol{b}'_{0S}\right]'.$$

Also in this case, $2N$ observations are “redundant”, in the sense that for each system the N annual sums for a variable are automatically determined by the difference between the yearly sums of $R-1$ variables of the first system and the yearly sums of $S-1$ variables of the second system. The whole set of aggregation constraints can thus be expressed as

$\boldsymbol{H}_w \boldsymbol{y}^* = \boldsymbol{y}_w$, where now

$$\boldsymbol{H}_w = \begin{bmatrix} \boldsymbol{1}'_{R-1} \otimes \boldsymbol{I}_n & \boldsymbol{I}_n & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I}_n \\ \boldsymbol{I}_{R-1} \otimes \boldsymbol{J} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1}'_{S-1} \otimes \boldsymbol{I}_n & \boldsymbol{I}_n & -\boldsymbol{I}_n \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_{S-1} \otimes \boldsymbol{J} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \quad \text{and}$$

$$\boldsymbol{y}_w = \begin{bmatrix} \boldsymbol{0}' & \boldsymbol{a}'_{01} & \cdots & \boldsymbol{a}'_{0R-1} & \boldsymbol{0}' & \boldsymbol{b}'_{01} & \cdots & \boldsymbol{b}'_{0S-1} \end{bmatrix}'.$$

Denoting $\boldsymbol{y} = [\boldsymbol{a}', \boldsymbol{b}', \boldsymbol{v}']'$ the vector containing all the unbenchmarked SA series, the benchmarked estimates are then given by

$$\hat{\boldsymbol{y}}^* = \boldsymbol{y} + \boldsymbol{\Omega}\boldsymbol{H}'_w \boldsymbol{\Omega}_w^{-1} \left(\boldsymbol{y}_w - \boldsymbol{H}_w \boldsymbol{y} \right).$$

In this case time for calculations can be reduced considerably by using simplified expressions which exploit the partitioned nature of the involved matrices (Di Fonzo and Marini, 2003).

5 Benchmarking while Preserving Period-to-period Growth Rates

The proportional adjustment of a system of time series mostly alters those component series having greater magnitude. As we will see in Section 7, the ranges of corrections present (quasi) perfect correlation with the ranking (by mean) of the variables. Such a result might be in contrast with the fact that the most reliable series of a survey are generally the larger ones (and *viceversa*). An attempt to overcome this kind of problem could be made by considering benchmarking according to a movement preservation principle referred to the growth rates and in presence of a binding constraint. In this case, the criterion to be minimised would be

$$\sum_{i=1}^{M} \left[\sum_{t=2}^{n} \left(\frac{y_{it}^* - y_{it-1}^*}{y_{it-1}^*} - \frac{y_{it} - y_{it-1}}{y_{it-1}} \right)^2 \right] \equiv \sum_{i=1}^{M} \left[\sum_{t=2}^{n} \left(\frac{y_{it}^*}{y_{it-1}^*} - \frac{y_{it}}{y_{it-1}} \right)^2 \right]. \quad (9)$$

At the univariate level, many authors (Helfand *et al.*, 1977, Bozik and Otto, 1988, Bloem *et al.*, 2001) consider this criterion as "the ideal objective formulation" (Bloem *et al.*, 2001, p. 100), but it is not generally pursued[8] because of the inherent nonlinearity of the problem and because the proportional variant of Denton's procedure has been generally considered a good approximation (Helfand *et al.*, 1977). In what follows we work out[9] a solution to the minimization problem defined so far, moving from an approximation of (9) through the log-transformed expression

$$\sum_{i=1}^{M}\left\{\sum_{t=2}^{n}\left[\left(\ln y_{it}^{*}-\ln y_{it-1}^{*}\right)-\left(\ln y_{it}-\ln y_{it-1}\right)\right]^{2}\right\}.$$

For notational convenience, let us denote by $y_{iT,s}^{*}$, $i=1,...,M$, $T=1,...,N$, $s=1,...,12$, the benchmarked series to be estimated, $x_{iT}=\sum_{s=1}^{12}y_{iT,s}^{*}=\sum_{s=1}^{12}x_{iT,s}$ the available temporal aggregates (flow variable) and $z_{T,s}=\sum_{i=1}^{M}y_{iT,s}^{*}$ the high-frequency contemporaneous aggregate (which is a binding constraint). Let us consider the Taylor series expansion (truncated at the first order term) of $\ln y_{iT,s}^{*}$ around its low-frequency-period-average, $\bar{x}_{iT}=1/12\sum_{s=1}^{12}y_{iT,s}^{*}=x_{iT}/12$:

$$\ln y_{iT,s}^{*}\simeq g_{iT,s}=\ln\bar{x}_{iT}+\frac{1}{\bar{x}_{iT}}\left(y_{iT,s}^{*}-\bar{x}_{iT}\right)=\ln x_{iT}-\ln 12+\frac{12y_{iT,s}^{*}}{x_{iT}}-1.$$

Summing over $s=1,...,12$ and $i=1,...,M$, respectively, we have

$$\sum_{s=1}^{12}\ln y_{iT,s}^{*}\simeq g_{iT}=\sum_{s=1}^{12}g_{iT,s}=12\ln x_{iT}-12\ln 12+\frac{12\sum_{s=1}^{12}y_{iT,s}^{*}}{x_{iT}}-12=12\ln x_{iT}-12\ln 12, \qquad (10)$$

$$\sum_{i=1}^{M}\ln y_{iT,s}^{*}\simeq\sum_{i=1}^{M}g_{iT,h}=\sum_{i=1}^{M}\ln x_{iT}-M\ln 12+12\sum_{i=1}^{M}\frac{y_{iT,s}^{*}}{x_{iT}}-M. \qquad (11)$$

It should be noted that the *rhs* expression in (11) is an unfeasible approximation, because $y_{iT,s}^{*}$ is obviously unknown. However, for our purposes, $y_{iT,s}^{*}$ can be substituted by a temporally benchmarked value, say $\breve{y}_{iT,s}$, such that $\sum_{s=1}^{12}\breve{y}_{iT,s}=x_{iT}$, $i=1,...,M$, $T=1,...,N$. The complete transformation we consider is the following:

---

8 An exception is Bozik and Otto (1988).

9 We extend to a system of M time series linked by a contemporaneous accounting constraint the approximation proposed for a single time series by Salazar *et al.* (1997) and Aadland (2000).

$$\ln y^*_{iT,s} \simeq g_{iT,s} = \ln x_{iT} - \ln 12 + \frac{12\breve{y}_{iT,s}}{x_{iT}} - 1, \quad i = 1,...,M, \; T = 1,...,N, \; s = 1,...,12$$

$$\ln y^*_{it} \simeq g_{it} = \ln \breve{y}_{it}, \quad t < 1 \quad \text{and/or} \quad t = Ns + 1, \; Ns + 2, ...,$$

where the second expression permits to derive benchmarked estimates also for those high-frequency periods in which low-frequency benchmarks are not available (constrained backcalculation/extrapolation or preliminary benchmarking). As can be easily shown, relationship (10) is still valid and we can write

$$\sum_{i=1}^{M} \ln y^*_{iT,s} \simeq \sum_{i=1}^{M} g_{iT,h} = \sum_{i=1}^{M} \ln x_{iT} - M \ln 12 + 12 \sum_{i=1}^{M} \frac{\breve{y}_{iT,s}}{x_{iT}} - M, \quad T = 1,...,N, \; s = 1,...,12.$$

Now, we have

$$\sum_{i=1}^{M} g_{iT} = \sum_{i=1}^{M} \left(12 \ln x_{iT} - 12 \ln 12\right) = 12 \sum_{i=1}^{M} \ln x_{iT} - 12M \ln 12, \quad T = 1,...,N \quad \text{and}$$

$$\sum_{s=1}^{12} \sum_{i=1}^{M} g_{iT,s} = 12 \sum_{i=1}^{M} \ln x_{iT} - 12M \ln 12 + 12 \sum_{s=1}^{12} \sum_{i=1}^{M} \frac{\breve{y}_{iT,s}}{x_{iT}} - 12M = 12 \sum_{i=1}^{M} \ln x_{iT} - 12M \ln 12, \quad T = 1,...,N.$$

In other words, the chosen approximation for $\ln y^*_{iT,s}$ satisfies a low-frequency temporal aggregation constraint analogous to that valid for the variables' levels. Now, denoting $w_{i,Ts} = \ln y_{i,Ts}$, let us minimise the objective function

$$\sum_{i=1}^{M} \left\{ \sum_{t=2}^{n} \left[\left(g_{it} - g_{it-1}\right) - \left(w_{it} - w_{it-1}\right) \right]^2 \right\}$$

constrained by $\sum_{s=1}^{12} g_{iT,s} = g_{i,T}$ and $\sum_{i=1}^{M} g_{iT,s} = z^*_{T,s}$, $T = 1,...,N, \; s = 1,...,12$, where $z^*_{T,s} = \sum_{i=1}^{M} \ln x_{iT} - M \ln 12 + 12 \sum_{i=1}^{M} \left(\breve{y}_{iT,s} / x_{iT}\right) - M$, $T = 1,...,N, \; s = 1,...,12$. Thus, we can get benchmarked estimates of $g_{i,Ts}$, say $\tilde{g}_{i,Ts}$, using Denton's AFD procedure on the log-transformed data. A "natural" estimate of the benchmarked series is given by $\tilde{y}^*_{iT,s} = \exp\left\{\tilde{g}_{iT,s}\right\}$. However, due to the approximations involved in the calculations, the estimates $\tilde{y}^*_{iT,s}$ need to be further adjusted (for example, through Denton's PFD procedure) in order to get the final benchmarked values fulfilling both contemporaneous and temporal constraints.

6 Benchmarking a System of Two-way Classified Series

Let us consider the following table, containing $R \times S$ elementary (and unknown) series $\boldsymbol{y}_{ij}^*$, $i = 1,...,R$, $j = 1,...,S$, classified by (say) R regions and S industries.

Region	Industry 1	$\cdots$	j	$\cdots$	S	Total
1	$\boldsymbol{y}_{11}^*$	$\cdots$	$\boldsymbol{y}_{1j}^*$	$\cdots$	$\boldsymbol{y}_{1S}^*$	$\boldsymbol{a}_1$
$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
i	$\boldsymbol{y}_{i1}^*$	$\cdots$	$\boldsymbol{y}_{ij}^*$	$\cdots$	$\boldsymbol{y}_{iS}^*$	$\boldsymbol{a}_i$
$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
R	$\boldsymbol{y}_{R1}^*$	$\cdots$	$\boldsymbol{y}_{Rj}^*$	$\cdots$	$\boldsymbol{y}_{RS}^*$	$\boldsymbol{a}_R$
Total	$\boldsymbol{b}_1$	$\cdots$	$\boldsymbol{b}_j$	$\cdots$	$\boldsymbol{b}_S$	$\boldsymbol{z}$

Each vector in the table has dimension $(n \times 1)$, and the links between the component series and the totals (by region, by industry and general) are the following:

$$\boldsymbol{a}_i = \sum_{j=1}^{S} \boldsymbol{y}_{ij}^*, \quad i = 1,...,R, \qquad \boldsymbol{b}_j = \sum_{i=1}^{R} \boldsymbol{y}_{ij}^*, \quad j = 1,...,S, \qquad \boldsymbol{z} = \sum_{i=1}^{R} \sum_{j=1}^{S} \boldsymbol{y}_{ij}^* = \sum_{i=1}^{R} \boldsymbol{a}_i = \sum_{j=1}^{S} \boldsymbol{b}_j .$$

Using matrix notation, let us consider the $(Rn \times S)$ matrix

$$\boldsymbol{Y}^* = \begin{bmatrix} \boldsymbol{y}_{11}^* & \cdots & \boldsymbol{y}_{1j}^* & \cdots & \boldsymbol{y}_{1S}^* \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{y}_{i1}^* & \cdots & \boldsymbol{y}_{ij}^* & \cdots & \boldsymbol{y}_{iS}^* \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{y}_{R1}^* & \cdots & \boldsymbol{y}_{Rj}^* & \cdots & \boldsymbol{y}_{RS}^* \end{bmatrix},$$

and define as $\boldsymbol{y}^* = \text{vec}(\boldsymbol{Y}^*)$ the $(RSn \times 1)$ vector containing the observations stacked by column of the $R \times S$ component series. With obvious notation, let us define the vectors containing the totals by region ($\boldsymbol{a}$) and by industry ($\boldsymbol{b}$), respectively:

$$\boldsymbol{a} = [\boldsymbol{a}_1' \ldots \boldsymbol{a}_i' \ldots \boldsymbol{a}_R']' \ (Rn \times 1), \qquad \boldsymbol{b} = [\boldsymbol{b}_1' \ldots \boldsymbol{b}_j' \ldots \boldsymbol{b}_S']' \ (Sn \times 1).$$

The aggregation constraints can thus be expressed as

$$\boldsymbol{a}_i = \sum_{j=1}^{S} \boldsymbol{y}_{ij}^* , \; i=1,\dots,R , \; \Leftrightarrow \; \left(\boldsymbol{1}'_S \otimes \boldsymbol{I}_{Rn}\right)\boldsymbol{y}^* = \boldsymbol{a}$$

$$\boldsymbol{b}_j = \sum_{i=1}^{R} \boldsymbol{y}_{ij}^* , \; j=1,\dots,S , \; \Leftrightarrow \; \left[\boldsymbol{I}_S \otimes \left(\boldsymbol{1}'_R \otimes \boldsymbol{I}_n\right)\right]\boldsymbol{y}^* = \boldsymbol{b}$$

$$\boldsymbol{z} = \sum_{i=1}^{R}\sum_{j=1}^{S} \boldsymbol{y}_{ij}^* = \sum_{i=1}^{R} \boldsymbol{a}_i = \sum_{j=1}^{S} \boldsymbol{b}_j \quad \Leftrightarrow \quad \begin{cases} \left(\boldsymbol{1}'_{RS} \otimes \boldsymbol{I}_n\right)\boldsymbol{y}^* = \boldsymbol{z} \\ \left(\boldsymbol{1}'_R \otimes \boldsymbol{I}_n\right)\boldsymbol{a} = \boldsymbol{z} \\ \left(\boldsymbol{1}'_S \otimes \boldsymbol{I}_n\right)\boldsymbol{b} = \boldsymbol{z} \end{cases} .$$

Moreover, we assume that $R \times S$ temporally aggregated component series are available such that $\boldsymbol{J}\boldsymbol{y}_{ij}^* = \boldsymbol{y}_{0,ij}$, $i=1,\dots,R$, $j=1,\dots,S$, where $\boldsymbol{J}$ is the $(N \times n)$ temporal aggregation matrix defined in Section 2. We assume that $R \times S$ preliminary series $\boldsymbol{y}_{ij}$, $i=1,\dots,R$, $j=1,\dots,S$, are available, which have to be benchmarked in such a way that all the aggregation (both temporal and contemporaneous) constraints are fulfilled by the benchmarked estimates $\hat{\boldsymbol{y}}_{ij}^*$, $i=1,\dots,R$, $j=1,\dots,S$. Moreover, in many practical situations the total series, either by region or by industry, are available in preliminary form (i.e., unbinding constraints to be benchmarked together with the component series) and/or as fully reliable (binding constraints) to be fulfilled by the benchmarked estimates. In general, $3^3 = 27$ different cases are theoretically possible, as shown in Table 1 in Appendix A. In practice, 12 of 27 cases are genuinely interesting, 7 being unrealistic (either $\boldsymbol{a}$ or $\boldsymbol{b}$ available with $\boldsymbol{z}$ unavailable), and 5 practically equivalent (for symmetry between $\boldsymbol{a}$ and $\boldsymbol{b}$) to 5 of the 12 numbered cases in Table 1.

In what follows we consider two polar situations, namely those labeled as “type 1” and “type 12”: in the former, R regional and S industry series are available and considered as (fully reliable) binding constraints, in the latter no total series (neither binding nor preliminary) is available.

6.1 A system of two-way classified series with binding constraints

As previously shown, the crucial point to develop feasible benchmarking formulae according to Denton’s movement preservation principle is to write down the whole set of (temporal and contemporaneous) aggregation constraints in such a way as to distinguish the redundant (superfluous) relations from the remaining ones. In this case, the links between the series to be estimated, $\boldsymbol{y}^*$, and the available benchmarks can be expressed as

$$\boldsymbol{y}_w = \boldsymbol{H}_w \boldsymbol{y}^* , \quad \text{where} \quad \boldsymbol{H}_w = \begin{bmatrix} \boldsymbol{1}'_{S-1} \otimes \boldsymbol{I}_{Rn} & \vdots & \boldsymbol{I}_{Rn} \\ \boldsymbol{I}_{S-1} \otimes \boldsymbol{A} & \vdots & \boldsymbol{0} \end{bmatrix}$$

has $r = Rn + (S-1)n + (S-1)(R-1)N$ rows and RSn columns.

Matrix $\boldsymbol{A}$, given by

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{I}'_{R-1} \otimes \boldsymbol{I}_n & \vdots & \boldsymbol{I}_n \\ \boldsymbol{I}_{R-1} \otimes \boldsymbol{J} & \vdots & \boldsymbol{0} \end{bmatrix}, \text{ has dimension } (n+(r-1)N \times Rn),$$

while $\boldsymbol{y}_w = \left[\boldsymbol{a}', \boldsymbol{b}'_1\ \boldsymbol{y}'_{0,11} \quad \cdots \quad \boldsymbol{y}'_{0,R-1,1} \cdots \boldsymbol{b}'_{S-1}\ \boldsymbol{y}'_{0,1,S-1} \quad \cdots \quad \boldsymbol{y}'_{0,R-1,S-1}\right]'$ is the vector containing the "free" available observations, re-organized in a convenient way for expressing the benchmarking formulae. The components of vector $\boldsymbol{y}_w$ have been chosen as follows:

- given that $\boldsymbol{z} = \sum_{i=1}^{R} \boldsymbol{a}_i = \sum_{j=1}^{S} \boldsymbol{b}_j$, the n observations of $\boldsymbol{z}$ are superfluous (in the sense that they can be derived from either $\boldsymbol{a}_i$ or $\boldsymbol{b}_j$);
- $\sum_{i=1}^{R} \boldsymbol{a}_i = \sum_{j=1}^{S} \boldsymbol{b}_j$ means that another n observations are superfluous; without loss of generality, we do not consider $\boldsymbol{b}_S$ (which can be derived from the remaining $\boldsymbol{a}_i$ and $\boldsymbol{b}_j$ as $\sum_{i=1}^{R} \boldsymbol{a}_i - \sum_{j=1}^{S-1} \boldsymbol{b}_j$);
- given that $\sum_{i=1}^{R} \boldsymbol{y}_{0,ij} = \boldsymbol{J}\boldsymbol{b}_j$, $j = 1,...,S$, there are SN superfluous observations contained, for example, in the vectors $\boldsymbol{y}_{0,Rj}$, $j = 1,...,S$;
- equivalently, as regards "regional" series $\boldsymbol{a}_i$, we have $\sum_{j=1}^{S} \boldsymbol{y}_{0,ij} = \boldsymbol{J}\boldsymbol{a}_i$, $i = 1,...,R$, so that RN observations are redundant, here chosen as those contained in the vectors $\boldsymbol{y}_{0,iS}$, $i = 1,...,R$ (as is obvious, the N observations contained in vector $\boldsymbol{y}_{0,RS}$ must be accounted for only once).

At this point, simultaneous benchmarking of the whole set of $R \times S$ series can be achieved by applying the benchmarking formula $\hat{\boldsymbol{y}}^* = \boldsymbol{y} + \boldsymbol{\Omega}\boldsymbol{H}'_w \boldsymbol{\Omega}_w^{-1}\left(\boldsymbol{y}_w - \boldsymbol{H}_w \boldsymbol{y}\right)$ developed so far. Obviously, the dimensions of the matrices implied in the calculations could be really prohibitive, such that simplified, sub-optimal procedures have been devised to overcome this problem (Quenneville and Rancourt, 2005)[10]. For example, if two-way classified monthly series by 12 regions and 18 industries are considered, 216 elementary monthly series have to be benchmarked. Assuming annual benchmarks available for 10 years, getting simultaneously benchmarked estimates involves the calculation of $\boldsymbol{\Omega}\boldsymbol{H}'_w$, which has 25,920 rows and 5,350 columns, and the inversion of $\boldsymbol{\Omega}_w = \boldsymbol{H}_w \boldsymbol{\Omega}\boldsymbol{H}'_w$, which has dimension (5,350×5,350).[11]

10 Cholette (1987, p. 45) states: "Practical experience with simultaneous benchmarking may show that very similar results can be achieved with some combination of individual benchmarking with raking (…) However, simultaneous benchmarking does provide a standard, i.e. a norm, against which alternative and simple approaches may be assessed".

11 With the modern computation facilities, managing such large matrices is a feasible task, even though still time consuming. For example, the inversion of a (6,000×6,000) positive definite matrix using the command invpd of Gauss for Windows (version 6) takes about 1,500 seconds on a Pentium 4 CPU 2.66GHz with 512 MB of RAM. A significant reduction of computation time can be obtained either using algorithms for sparse matrices or, in line with Di Fonzo and Marini (2003), by exploiting the partitioned nature of the matrices. We are currently involved in this last issue.

6.2 A system of two-way classified series with neither binding nor preliminary constraints

In this case the benchmarked estimates $\hat{\boldsymbol{y}}_{ij}^{*}$ must be such that $\sum_{i=1}^{R}\hat{\boldsymbol{a}}_i=\sum_{j=1}^{S}\hat{\boldsymbol{b}}_j$, where $\hat{\boldsymbol{a}}_i=\sum_{j=1}^{S}\hat{\boldsymbol{y}}_{ij}^{*}$, $i=1,\ldots,R$, and $\hat{\boldsymbol{b}}_j=\sum_{i=1}^{R}\hat{\boldsymbol{y}}_{ij}^{*}$, $j=1,\ldots,S$. The available aggregated "observations" are thus given by the $((n+RSN)\times 1)$ vector $\boldsymbol{y}_w=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{y}_{0,11}' & \cdots & \boldsymbol{y}_{0,RS}'\end{bmatrix}'$, linked to the unknown series to be estimated by the relationship $\boldsymbol{H}_w\boldsymbol{y}=\boldsymbol{y}_w$, where $\boldsymbol{H}_w$ is the $((n+RSN)\times RSn)$ matrix

$$\boldsymbol{H}_w=\begin{bmatrix}(\boldsymbol{1}_R'\otimes\boldsymbol{I}_n)(\boldsymbol{1}_S'\otimes\boldsymbol{I}_{Rn})-(\boldsymbol{1}_S'\otimes\boldsymbol{I}_n)\left[\boldsymbol{I}_S\otimes(\boldsymbol{1}_R'\otimes\boldsymbol{I}_n)\right]\\ \boldsymbol{I}_{RS}\otimes\boldsymbol{J}\end{bmatrix}.$$

When benchmarking a table of monthly series classified by 12 regions and 18 industries using 10 annual benchmarks, the dimensions of matrices $\boldsymbol{\Omega H}_w'$ and $\boldsymbol{\Omega}_w=\boldsymbol{H}_w\boldsymbol{\Omega H}_w'$ are (25,920×2,280) and (2,280×2,280), respectively.

7 An Application: Benchmarking Canadian Retail Trade Series

In this section we apply the adjustment methods described so far to the Canadian retail trade series, released by Statistics Canada.[12] Monthly raw series (expressed in millions of Canadian dollars) are available according to two (single) breakdowns, i.e. by 18 trade groups (TG system) and by 12 provinces (PR system). Both systems are composed by series of different magnitudes (see Table 2 in Appendix A). Province PR300351 represents about 37% of the total Canadian retail sales, whilst the sum of provinces PR300601 and PR300611 does not reach even one percentage point. As far as trade group breakdown is concerned, nearly half of the total sales are achieved by the two items 010001 and 100001. ADF statistics, reported in the last column of Table 2, test the presence of unit root in the series (3 lags are considered in the ADF regression). The unit root hypothesis is not rejected at the 5% level in all cases; a common source of non-stationarity is likely to affect the whole system of retail sales.

Each component series and the Total Canada aggregate have been directly seasonally adjusted using X11-ARIMA. Seasonal adjustment procedures often generate two undesirable effects for a statistical office. Firstly, the annual totals of the SA series do not agree with those of the raw series (Quenneville *et al*., 2003). Second, the monthly contemporaneous sum from different systems does not comply with the monthly SA total series. Table 3 in Appendix A shows some descriptive statistics on both temporal and contemporaneous

12 We thank B. Quenneville, from Statistics Canada, who kindly made the SA series available to us.

discrepancies between the SA unbenchmarked monthly series and the relevant (temporal or contemporaneous) constraint.[13]

The discrepancies are negligible in all cases except for the monthly sum of PR component series, which shows large deviations from the Canada Total aggregate (see Figure 1 in Appendix B). The highest value is found in December 1997, which is more than 8 times larger than the mean absolute discrepancy. The sub-annual discrepancies show a clear seasonal pattern, so their behaviour cannot be classified as constant or erratic in time (Cholette, 1988).[14]

According to the nature of the data, we face a problem of one-way classification adjustment: the component series of each system must be adjusted so that they add up to the same Total Canada series. The latter can be considered either as a binding or unbinding constraint. The aim of this exercise is to evaluate the performance of different adjustments for PR and TG breakdowns in terms of discrepancies between benchmarked and unbenchmarked series. The benchmarking has been accomplished according to the proportional variant in order to conveniently deal with the marked differences in dimension of the component series. Different adjustments are devised to get benchmarked estimates in the two systems. The following four cases are considered (benchmarking of the TG system alone is not an interesting case given the small sub-annual and contemporaneous discrepancies[15]):

(1) PR system with no binding constraint (12 component series + Total Canada series are benchmarked);

(2) PR system with Canada total aggregate used as a binding constraint (only 12 component series are benchmarked) using both Denton's PFD variant and the benchmarking procedure working on growth rates described in Section 5;

(3) PR and TG systems with no binding constraint (12 + 18 component series are benchmarked, while the unbenchmarked SA Total Canada series remains unused);

(4) PR and TG systems still with unbinding constraint, but using the unbenchmarked SA Total Canada as a preliminary series.

[13] Statistics on the discrepancies with respect to the temporal constraints are derived as average of the discrepancies of the single variables.

[14] Dependencies from the past like those found in this case could be better taken into account using a 'data-based' benchmarking (Guerrero and Nieto, 1999, Di Fonzo and Marini, 2005).

[15] We present a variety of alternatives in order to appreciate how the methods work. However it should be stressed that, from a practical point of view, in this particular case it would probably reasonable (i) to benchmark TG system using Total Canada as either binding or unbinding constraint, and (ii) adjust PR component series using Total Canada series of step (i) as binding constraint. For all practical purposes, the estimates of PR component series obtained by this way are equivalent to the results of case (2).

The performance of the benchmarking procedure for each adjustment is evaluated by considering the proportional corrections c_{it}^{p}, defined as $c_{it}^{p} = \hat{y}_{it}^{*} / y_{it}$, $i = 1,\ldots,M$, and the additive corrections c_{it}^{a}, given by $c_{it}^{a} = \hat{y}_{it}^{*} - y_{it}$, $i = 1,\ldots,M$, where M is equal to 13, 12, 30 and 31 for cases (1), (2), (3) and (4), respectively. The additive corrections have been kept in consideration also for the monthly growth rates. Annual benchmarks, given by the sums of monthly raw series, are available for the period 1991-2000, while the unbenchmarked SA series cover the period 1991.01-2001.01 (the benchmarked estimate for the last month is then obtained by extrapolation[16]). In our notation we have $R = 12,\ S = 18,\ n = 121,\ N = 10$. In the following we present a set of tables summarizing the corrections made to levels and growth rates (%) of the unbenchmarked series. Although Denton's PFD variant preserves at best the levels, we have chosen to evaluate also rates of change because they can be considered as the "real" short-term information of a SA monthly series (see Section 5). In some cases we also plot the series of corrections made by different benchmarking procedures, in order to compare their patterns.

7.1 Benchmarking series by province

On case (1), provincial data are adjusted without a binding constraint according to the procedure described in Section 3.1. The benchmarked Total Canada series is forced to comply with the sum of the 12 benchmarked provincial series. The range of the proportional corrections to the levels (Table 4 in Appendix A) is 2.34%; the largest absolute correction occurred for January 1992 (-1.44%), due to the correction of opposite sign to December 1991, where the unbenchmarked Canada total aggregate was 181,535 millions of dollars higher than the sum of provincial unbenchmarked data. Other significant adjustments are found for provinces PR300241 and PR300351, whereas for the other provinces additive corrections to growth rates are much lower than 0.2%.

If the Canada total aggregate is in turn considered fully reliable, we fall into case (2), where the Canada total series is used as a binding constraint, which provincial data must fulfil. From Table 5 it emerges that provinces PR300351 and PR300241 show again the largest corrections (about 5.2% and 3.1% of range for month-to-month rates of change, respectively), but also PR300591 and PR300481 are visibly corrected (see Figure 2 in Appendix B).

Table 6 in Appendix A and Figure 3 in Appendix B present in turn the benchmarking results for the PR system while preserving the growth rates, as proposed in Section 5. It clearly appears that the peaks in the corrections are somewhat smoothed with respect to the results obtained using Denton's PFD variant, but now almost all the series are affected by corrections.

---

16 Notice that preliminary benchmarked estimates must still fulfil a contemporaneous aggregation constraint.

7.2 Simultaneous benchmarking of series by province and trade groups

Let us now move on to cases (3) and (4), that is to the simultaneous adjustment of provincial and trade groups systems of series. In case (3) the component series for both systems are adjusted without taking into account the Canada total aggregate either as an unbinding constraint or as a preliminary series (see Section 4.2 for details). The variables are treated as if they belong to a single system, consequently the proportional nature of the method adjusts the series in line with the total ranking of the 30 variables (see Table 7 in Appendix A). The growth rates of the two largest series, TG100001 and PR300351, show the largest corrections (ranging between [-1.39%, 1.87%] and [-1.38%, 0.96%], respectively). As far as TG system is concerned, significant corrections are found only for TG010001, whereas the remaining trade groups remain largely unchanged. In contrast, the provincial data show greater corrections with respect to the benchmarked values of case (1). However, the growth rates of the benchmarked Canada total aggregate are close to those of the preliminary series.

In case (4) we wish to evaluate whether introducing the Canada preliminary series can induce any improvement on the benchmarked estimates (Table 8). In fact, the range of corrections is slightly shrunk (from 1.23 to 1.06 for rates of change). Furthermore, the whole benchmarked system benefits (21 out of 30 variables show lower variability of the additive corrections to growth rates).

Finally, in Figure 4 in Appendix B we display the proportional corrections to the levels of the unbenchmarked Total Canada series made by the sums of the benchmarked component series in cases (1), (3) and (4), while in Figure 5 the additive corrections to month-to-month rates of change in cases (1) and (3)[17] are shown. From both graphs it clearly emerges that simultaneous benchmarking of both systems (either with or without a preliminary estimate of the contemporaneous constraint) reduces half of the correction induced by benchmarking system PR only.

8 Conclusions

In this paper we have proposed a solution to the problem of benchmarking a table of seasonally adjusted time series such that the benchmarked series fulfil both temporal and contemporaneous constraints. We derived a procedure – in line with the least squares adjustment technique of Stone *et al*. (1942) – which is grounded on a "movement preservation principle" (Denton, 1971), according to which the temporal profiles of the benchmarked series should be as similar as possible to those of the unbenchmarked counterparts. The proposed procedure has been used to restore the additivity of individually seasonally adjusted monthly Canadian retail trade series, classified by province and industry.

As a concluding remark we should stress that, in practical situations, the dimension of the problem (which is related to the number of the series in the systems and to the covered

17 The additive corrections in case (4) are practically indistinguishable from those of case (3).

time span) can be very large, making it difficult to handle the general benchmarking formula. In such cases, simplified expressions which exploit the constraint sparsity and the structure of the involved matrices can significantly reduce computer time and store requirements.

References

Aadland, D.M. 2000. "Distribution and interpolation using transformed data," *Journal of Applied Statistics* 27, pp. 141-56.

Barker, T.; F. van der Ploeg and M. Weale. 1984. "A balanced system of national accounts for the United Kingdom," *The Review of Income and Wealth* 30:4, pp. 461-85.

Bloem, A.M.; R.J. Dippelsman and N.Ø. Mæhle. 2001. *Quarterly National Accounts Manual. Concepts, Data Sources, and Compilation*. Washington DC: International Monetary Fund.

Bozik, J.E. and M.C. Otto. 1988. "Benchmarking: Evaluating methods that preserve month-to-month changes," Bureau of the Census, Statistical Research Division, CENSUS/SRD/RR-88/07.

Byron, R.P. 1978. "The estimation of large social accounts matrices," *Journal of the Royal Statistical Society A* 141:3, pp. 359-67.

Chen, Z.G. and E.B. Dagum. 1997. "A recursive method for predicting variables with temporal and contemporaneous constraints," American Statistical Association, Proceedings of the Business and Economic Statistics Section.

Cholette, P.A. 1984. "Adjusting sub-annual series to yearly benchmarks," *Survey Methodology* 10:1, pp. 35-49.

Cholette, P.A.. 1987. "Concepts, definitions and principles of benchmarking and interpolation of time series," Statistics Canada, Time Series Research and Analysis Division, Working Paper no. TSRA-87-014E.

Cholette, P.A. 1988. "Benchmarking systems of socio-economic time series," Statistics Canada, Time Series Research and Analysis Division, Working Paper no. TSRA-88-017E.

Cholette, P.A. and E.B. Dagum. 1994. "Benchmarking time series with autocorrelated survey errors," *International Statistical Review* 62:3, pp. 365-77.

Dagum, E.B.; P.A. Cholette and Z.G. Chen. 1998. "A unified view of signal extraction, benchmarking, interpolation and extrapolation of time series," *International Statistical Review* 66:3, pp. 245-269.

Deming, W.E. and F.F. Stephan. 1940. "On a least-squares adjustment of a sampled frequency table when the expected marginal totals are known," *Annals of Mathematical Statistics* 11 pp. 427-44.

Denton, F.T. 1971. "Adjustment of monthly or quarterly series to annual totals: An approach based on quadratic minimization," *Journal of the American Statistical Association* 66:333, pp. 99-102.

Di Fonzo, T. 2002. "Temporal disaggregation of a system of time series when the aggregate is known. Optimal vs. adjustment methods," Workshop on Quarterly National Accounts, Eurostat, Theme 2 Economy and finance, pp. 63-77.

Di Fonzo, T. and M. Marini. 2003. "Benchmarking systems of seasonally adjusted time series according to Denton's movement preservation principle," Università di Padova, Dipartimento di Scienze Statistiche, working paper 2003.9 (available at http://homes.stat.unipd.it/difonzo/pubblic_file/wp_2003_09.pdf).

Di Fonzo, T. and M. Marini. 2005. "Benchmarking a system of time series: Denton's movement preservation principle vs. a data based procedure," Proceedings of the Workshop in Frontiers in Benchmarking Techniques and their Application in Official Statistics, Luxembourg, Eurostat (to appear)..

Durbin, J. and B. Quenneville. 1997. "Benchmarking by state space models," International Statistical Review 65:1, pp. 23-48.

Eurostat. 1999. *Handbook of quarterly national accounts*. Luxembourg: European Commission.

Guerrero, V.M. and F.H. Nieto. 1999. "Temporal and contemporaneous disaggregation of multiple economic time series," *Test* 8:2, pp. 459-89.

Helfand, S.D.; N.J. Monsour and M.L. Trager. 1977. "Historical revision of current business survey estimates," American Statistical Association, Proceedings of the Business and Economic Statistics Section, pp. 246-50.

Hillmer, S.C. and A. Trabelsi. 1987. "Benchmarking of economic time series," *Journal of the American Statistical Association* 82:400, pp. 1064-71.

Laniel, N. and K. Fyfe. 1989. "Benchmarking of economic time series," American Statistical Association, Proceedings of the Business and Economic Statistics Section, pp. 462-66.

van der Ploeg, F. 1982. "Reliability and the adjustment of sequences of large economic accounting matrices," *Journal of the Royal Statistical Society A* 145:2, pp. 169-94.

van der Ploeg, F. 1984. "Generalized least squares methods for balancing large systems and tables of national accounts," *Review of Public Data Use* 12:1, pp. 17-33.

Quenneville, B. and E. Rancourt. 2005. "Simple methods to restore the additivity of a system of time series," Proceedings of the Workshop in Frontiers in Benchmarking Techniques and their Application in Official Statistics, Luxembourg, Eurostat (to appear).

Quenneville, B.; P. Cholette, G. Huot, K. Chiu and T. Di Fonzo. 2003. "Adjustment of seasonally adjusted series to annual totals," Proceedings of Statistics Canada Symposium 2003 (available at http://homes.stat.unipd.it/difonzo/pubblic_file/Statistics_Canada_Symposium_2003.pdf).

Salazar, E.L.; R.J. Smith and M. Weale. 1997. "Interpolation using a Dynamic Regression Model: Specification and Monte Carlo Properties," NIESR Discussion Paper no. 126.

Sefton, J. and M. Weale. 1995. *Reconciliation of national income and expenditure. Balanced estimates of national income for the United Kingdom, 1920-1990*. Cambridge: Cambridge University Press.

Smith, R.J.; M.R. Weale and S.E. Satchell. 1998. "Measurement error with accounting constraints: point and interval estimation for latent data with an application to U.K. gross domestic product," *Review of Economic Studies* 65, pp. 109-34.

Solomou, S. and M. Weale. 1993. "Balanced estimates of National Accounts when measurement errors are autocorrelated: the UK, 1920-38," *Journal of the Royal Statistical Society A* 156:1, pp. 89-105.

Stone, R.; D.G. Champernowne and J.E. Meade. 1942. "The precision of national income estimates," *Review of Economic Studies* 9, pp. 111-25.

Taillon, J. 1988. "Dessaisonalisation d'un tableau avec contraintes linéaires: Application aux flux financiers canadiens," *The Canadian Journal of Statistics* 16, pp. 97-107.

Weale, M. 1992. "Estimation of data measured with error and subject to linear restrictions," *Journal of Applied Econometrics* 7, pp. 167-74.

Appendix A

Table 1 **Types of benchmarking a system of two-way classified time series**

Type #	z	a	b	'equivalent' to type #
1	y	y	y	
2	y	y	p	
3	y	y	n	
	y	p	y	2
4	y	p	p	
5	y	p	n	
	y	n	y	3
	y	n	p	5
6	y	n	n	
*	p	y	y	
*	p	y	p	
*	p	y	n	
*	p	p	y	
7	p	p	p	
8	p	p	n	
*	p	n	y	
	p	n	p	8
9	p	n	n	
*	n	y	y	
*	n	y	p	
*	n	y	n	
*	n	p	y	
10	n	p	p	
11	n	p	n	
*	n	n	y	
	n	n	p	11
12	n	n	n	

y: a binding (yet benchmarked) series is available.

p: a preliminary (to be benchmarked) series is available.

n: neither a binding (yet benchmarked), nor a preliminary (to be benchmarked) series is available.

*: practically uninteresting, because from either a or b it is possible to recover z .

Table 2 **Descriptive statistics for Canadian retail trade series according to TG and PR breakdowns (SA unbenchmarked series, 1991.01-2001.01)**

	Mean	St. dev.	Min	Max	Range	ADF test*
Trade Group						
TG010001	4,162,300	337,533	3,560,975	4,880,131	1,319,156	-0.064
TG020001	335,966	39,654	254,507	386,634	132,127	-0.861
TG030001	1,002,352	93,201	792,671	1,168,250	375,579	-1.145
TG040001	138,831	7,698	120,603	154,953	34,350	-1.987
TG050001	134,597	7,886	119,708	156,544	36,836	-2.266
TG060001	348,012	27,598	288,205	407,955	119,750	-0.406
TG070001	450,122	97,062	300,829	649,648	348,819	0.966
TG080001	771,265	126,882	540,947	1,117,685	576,738	1.663
TG090001	190,415	23,219	146,660	257,167	110,507	1.112
TG100001	4,555,360	1,029,007	3,034,552	6,323,391	3,288,839	-0.078
TG110001	1,355,285	214,443	1,146,144	1,981,460	835,316	1.683
TG120001	1,049,627	156,652	833,242	1,350,548	517,306	0.332
TG131001	1,190,377	174,007	969,904	1,555,479	585,575	0.473
TG132001	865,874	153,611	674,546	1,192,825	518,279	1.458
TG140001	624,673	75,289	475,735	755,247	279,512	-0.879
TG150001	492,345	74,305	389,483	644,373	254,890	0.602
TG161001	527,567	55,679	460,871	658,551	197,680	0.715
TG162001	400,494	46,191	311,122	503,897	192,775	-0.520
Province						
PR300101	309,597	33,210	274,969	383,481	108,512	1.324
PR300111	79,635	12,761	60,462	106,020	45,558	0.974
PR300121	587,000	72,382	473,966	731,827	257,861	0.557
PR300131	460,852	64,271	370,680	595,039	224,359	1.271
PR300241	4,382,203	534,915	3,534,316	5,393,719	1,859,403	0.606
PR300351	6,929,302	1,064,662	5,522,751	9,150,612	3,627,861	1.483
PR300461	647,807	92,850	513,478	809,824	296,346	0.147
PR300471	561,345	84,943	429,425	693,946	264,521	-0.232
PR300481	2,038,662	357,028	1,560,450	2,783,023	1,222,573	1.315
PR300591	2,538,252	340,222	1,900,230	3,091,321	1,191,091	-0.924
PR300601	21,771	5,006	13,219	30,123	16,904	-1.211
PR300611	38,783	5,840	28,051	51,778	23,727	0.072
300001 – Total Canada	18,595,462	2,623,083	14,816,800	23,756,760	8,939,960	1.680

* The 5% critical value is -2.886.

Table 3 **Temporal and contemporaneous discrepancies for the unbenchmarked SA series according to TG and PR breakdown – Descriptive statistics**

	Contemporaneous discrepancies		Temporal discrepancies	
	Trade groups	Province	Trade groups	Province
Median	-1	5,172	-1	-1
Standard deviation	3	62,811	1	1
Min	-8	-181,535	-2	-2
Max	8	347,024	1	1

Table 4 **Proportional corrections (levels) and additive corrections (growth rates) made by benchmarking PR component series and Total Canada without binding constraint – Descriptive statistics**

	Levels					Growth rates (%)				
Variable*	Median	Min	Max	Range	St. dev.	Median	Min	Max	Range	St. dev.
PR300351 (1)	0.99989	0.99634	1.00478	0.00844	0.00102	-0.01	-0.43	0.54	0.97	0.14
PR300241 (2)	0.99997	0.99738	1.00320	0.00582	0.00066	-0.01	-0.24	0.33	0.57	0.09
PR300591 (3)	0.99997	0.99868	1.00179	0.00311	0.00037	0.00	-0.16	0.19	0.35	0.05
PR300481 (4)	0.99997	0.99900	1.00147	0.00247	0.00030	0.00	-0.14	0.15	0.29	0.04
PR300461 (5)	0.99999	0.99965	1.00046	0.00081	0.00010	0.00	-0.04	0.05	0.09	0.01
PR300121 (6)	0.99999	0.99971	1.00039	0.00068	0.00009	0.00	-0.04	0.05	0.09	0.01
PR300471 (7)	0.99999	0.99973	1.00041	0.00068	0.00008	0.00	-0.04	0.04	0.08	0.01
PR300131 (8)	1.00000	0.99976	1.00031	0.00055	0.00007	0.00	-0.03	0.04	0.07	0.01
PR300101 (9)	1.00000	0.99981	1.00021	0.00039	0.00005	0.00	-0.02	0.03	0.05	0.01
PR300111 (10)	1.00000	0.99996	1.00006	0.00010	0.00001	0.00	0.00	0.01	0.01	0.00
PR300611 (11)	1.00000	0.99998	1.00003	0.00005	0.00001	0.00	0.00	0.00	0.01	0.00
PR300601 (12)	1.00000	0.99999	1.00002	0.00003	0.00001	0.00	0.00	0.00	0.00	0.00
Canada	1.00021	0.98628	1.00968	0.02340	0.00278	0.03	-1.44	1.15	2.59	0.37

* Provinces are ordered by range of levels' correction; ranking by mean is indicated in parentheses.

Table 5 **Proportional corrections (levels) and additive corrections (growth rates) made by benchmarking PR component series with binding constraint – Descriptive statistics**

	Levels					Growth rates (%)				
Variable*	Median	Min	Max	Range	St. dev.	Median	Min	Max	Range	St. dev.
PR300351 (1)	0.99946	0.98128	1.02722	0.04594	0.00555	-0.06	-2.31	2.92	5.23	0.74
PR300241 (2)	0.99984	0.98655	1.01815	0.03160	0.00360	-0.04	-1.30	1.78	3.07	0.46
PR300591 (3)	0.99988	0.99323	1.01018	0.01695	0.00200	-0.02	-0.88	1.00	1.89	0.27
PR300481 (4)	0.99988	0.99487	1.00836	0.01350	0.00163	-0.02	-0.75	0.83	1.58	0.22
PR300461 (5)	0.99997	0.99820	1.00263	0.00443	0.00052	-0.01	-0.23	0.27	0.50	0.07
PR300121 (6)	0.99996	0.99853	1.00224	0.00370	0.00048	-0.01	-0.20	0.27	0.47	0.06
PR300471 (7)	0.99997	0.99862	1.00231	0.00369	0.00045	-0.01	-0.21	0.23	0.45	0.06
PR300131 (8)	0.99998	0.99878	1.00176	0.00298	0.00037	0.00	-0.16	0.20	0.36	0.05
PR300101 (9)	0.99999	0.99905	1.00118	0.00212	0.00025	0.00	-0.10	0.14	0.25	0.03
PR300111 (10)	1.00000	0.99978	1.00032	0.00054	0.00006	0.00	-0.03	0.03	0.06	0.01
PR300611 (11)	1.00000	0.99990	1.00016	0.00026	0.00003	0.00	-0.01	0.02	0.03	0.00
PR300601 (12)	1.00000	0.99997	1.00010	0.00012	0.00002	0.00	-0.01	0.01	0.02	0.00

* Provinces are ordered by range of levels' correction; ranking by mean is indicated in parentheses.

Table 6 **Proportional corrections (levels) and additive corrections (growth rates) made by benchmarking PR component series with binding constraint – Descriptive statistics**

	Levels					Growth rates (%)				
Variable*	Median	Min	Max	Range	St. dev.	Median	Min	Max	Range	St. dev.
PR300351 (1)	0.99962	0.98369	1.02294	0.03924	0.00474	-0.06	-1.97	2.51	4.48	0.63
PR300241 (2)	0.99979	0.98710	1.01775	0.03066	0.00355	-0.04	-1.30	1.75	3.05	0.46
PR300591 (3)	0.99980	0.99144	1.01297	0.02153	0.00257	-0.03	-1.09	1.28	2.37	0.34
PR300481 (4)	0.99980	0.99248	1.01186	0.01938	0.00234	-0.02	-1.02	1.18	2.20	0.31
PR300461 (5)	0.99983	0.99462	1.00849	0.01387	0.00167	-0.02	-0.70	0.83	1.53	0.22
PR300121 (6)	0.99983	0.99485	1.00827	0.01342	0.00164	-0.02	-0.68	0.83	1.51	0.21
PR300471 (7)	0.99982	0.99490	1.00830	0.01340	0.00163	-0.01	-0.70	0.82	1.53	0.21
PR300131 (8)	0.99984	0.99500	1.00796	0.01296	0.00158	-0.02	-0.65	0.79	1.44	0.21
PR300101 (9)	0.99985	0.99516	1.00764	0.01248	0.00151	-0.02	-0.62	0.76	1.38	0.20
PR300111 (10)	0.99985	0.99566	1.00713	0.01147	0.00140	-0.01	-0.57	0.72	1.29	0.18
PR300611 (11)	0.99986	0.99576	1.00704	0.01127	0.00138	-0.01	-0.56	0.69	1.26	0.18
PR300601 (12)	0.99985	0.99582	1.00699	0.01117	0.00137	-0.01	-0.56	0.71	1.27	0.18

* Provinces are ordered by range of levels' correction; ranking by mean is indicated in parentheses.

Table 7 **Proportional corrections (levels) and additive corrections (growth rates) made by benchmarking PR and TG component series without binding constraint – Descriptive statistics**

	Levels					Growth rates (%)				
Variable*	Median	Min	Max	Range	St. dev.	Median	Min	Max	Range	St. dev.
PR300351 (1)	0.99963	0.98767	1.01571	0.02804	0.00346	-0.04	-1.39	1.87	3.27	0.46
TG100001 (2)	1.00017	0.98506	1.00663	0.02157	0.00248	0.03	-1.38	0.96	2.34	0.31
PR300241 (3)	0.99992	0.99116	1.01051	0.01935	0.00224	-0.02	-0.78	1.14	1.92	0.29
TG010001 (4)	1.00018	0.99121	1.00788	0.01667	0.00211	0.02	-1.20	0.84	2.04	0.28
PR300591 (5)	0.99991	0.99554	1.00588	0.01034	0.00125	-0.01	-0.53	0.65	1.17	0.17
PR300481 (6)	0.99990	0.99662	1.00483	0.00821	0.00101	-0.01	-0.45	0.53	0.98	0.14
TG110001 (7)	1.00008	0.99729	1.00248	0.00519	0.00069	0.01	-0.40	0.26	0.66	0.09
TG131001 (8)	1.00005	0.99737	1.00235	0.00498	0.00060	0.01	-0.32	0.25	0.56	0.08
TG120001 (9)	1.00004	0.99766	1.00209	0.00444	0.00053	0.00	-0.28	0.23	0.51	0.07
TG030001 (10)	1.00003	0.99790	1.00183	0.00393	0.00050	0.01	-0.27	0.20	0.47	0.07
TG132001 (11)	1.00005	0.99822	1.00155	0.00333	0.00043	0.00	-0.23	0.18	0.41	0.06
PR300461 (13)	0.99997	0.99882	1.00152	0.00270	0.00032	0.00	-0.14	0.17	0.31	0.04
TG080001 (12)	1.00003	0.99835	1.00099	0.00264	0.00038	0.01	-0.22	0.15	0.37	0.05
TG140001 (14)	1.00002	0.99852	1.00098	0.00246	0.00031	0.00	-0.17	0.13	0.30	0.04
TG150001 (18)	1.00002	0.99880	1.00109	0.00228	0.00026	0.00	-0.12	0.09	0.22	0.03
PR300121 (15)	0.99997	0.99903	1.00129	0.00226	0.00030	0.00	-0.12	0.17	0.29	0.04
PR300471 (16)	0.99998	0.99909	1.00133	0.00225	0.00028	0.00	-0.13	0.15	0.27	0.04
TG161001 (17)	1.00002	0.99893	1.00092	0.00199	0.00027	0.00	-0.16	0.10	0.27	0.04
PR300131 (19)	0.99998	0.99920	1.00102	0.00182	0.00023	0.00	-0.09	0.13	0.22	0.03
TG070001 (20)	1.00002	0.99889	1.00066	0.00177	0.00022	0.00	-0.11	0.09	0.20	0.03
TG020001 (23)	1.00001	0.99930	1.00085	0.00155	0.00018	0.00	-0.08	0.07	0.15	0.02
TG162001 (21)	1.00002	0.99919	1.00073	0.00154	0.00020	0.00	-0.12	0.08	0.19	0.03
TG060001 (22)	1.00002	0.99927	1.00066	0.00139	0.00018	0.00	-0.10	0.07	0.17	0.02
PR300101 (24)	0.99999	0.99938	1.00068	0.00130	0.00016	0.00	-0.06	0.09	0.15	0.02
TG090001 (25)	1.00001	0.99949	1.00027	0.00078	0.00010	0.00	-0.06	0.04	0.10	0.01
TG040001 (26)	1.00001	0.99973	1.00041	0.00068	0.00008	0.00	-0.04	0.03	0.07	0.01
TG050001 (27)	1.00001	0.99974	1.00025	0.00051	0.00007	0.00	-0.05	0.03	0.07	0.01
PR300111 (28)	1.00000	0.99986	1.00018	0.00033	0.00004	0.00	-0.02	0.02	0.04	0.01
PR300611 (29)	1.00000	0.99994	1.00009	0.00016	0.00002	0.00	-0.01	0.01	0.02	0.00
PR300601 (30)	1.00000	0.99998	1.00005	0.00007	0.00001	0.00	0.00	0.01	0.01	0.00
Canada	1.00005	0.99297	1.00411	0.01114	0.00130	0.01	-0.67	0.56	1.23	0.17

* Provinces and trade groups are ordered by range of levels' correction; ranking by mean in the whole system is indicated in parentheses.

Table 8 **Proportional corrections (levels) and additive corrections (growth rates) made by simultaneously benchmarking Total Canada, PR and TG component series – Descriptive statistics**

	Levels					Growth rates (%)				
Variable*	Median	Min	Max	Range	St. dev.	Median	Min	Max	Range	St. dev.
PR300351 (1)	0.99961	0.98718	1.01666	0.02948	0.00362	-0.04	-1.47	1.95	3.42	0.48
PR300241 (3)	0.99991	0.99080	1.01114	0.02034	0.00234	-0.03	-0.82	1.19	2.01	0.30
TG100001 (2)	1.00016	0.98629	1.00612	0.01983	0.00229	0.03	-1.27	0.88	2.14	0.28
TG010001 (4)	1.00017	0.99194	1.00727	0.01534	0.00195	0.02	-1.11	0.77	1.88	0.26
PR300591 (5)	0.99990	0.99536	1.00623	0.01087	0.00130	-0.02	-0.56	0.67	1.23	0.17
PR300481 (6)	0.99990	0.99648	1.00512	0.00863	0.00106	-0.01	-0.47	0.55	1.03	0.14
TG110001 (7)	1.00007	0.99752	1.00229	0.00477	0.00064	0.01	-0.37	0.24	0.61	0.09
TG131001 (8)	1.00005	0.99759	1.00217	0.00458	0.00056	0.00	-0.30	0.23	0.52	0.07
TG120001 (9)	1.00004	0.99785	1.00193	0.00408	0.00049	0.00	-0.25	0.21	0.47	0.07
TG030001 (10)	1.00003	0.99808	1.00169	0.00361	0.00046	0.01	-0.25	0.18	0.43	0.06
TG132001 (11)	1.00005	0.99837	1.00143	0.00306	0.00039	0.00	-0.22	0.16	0.38	0.05
PR300461 (13)	0.99997	0.99877	1.00161	0.00284	0.00034	0.00	-0.15	0.18	0.32	0.05
TG080001 (12)	1.00003	0.99849	1.00091	0.00243	0.00035	0.01	-0.20	0.14	0.34	0.05
PR300121 (15)	0.99997	0.99899	1.00137	0.00238	0.00031	0.00	-0.13	0.18	0.30	0.04
PR300471 (16)	0.99998	0.99905	1.00141	0.00236	0.00029	0.00	-0.14	0.15	0.29	0.04
TG140001 (14)	1.00001	0.99864	1.00090	0.00226	0.00029	0.00	-0.16	0.12	0.28	0.04
TG150001 (18)	1.00002	0.99890	1.00100	0.00210	0.00024	0.00	-0.11	0.09	0.20	0.03
PR300131 (19)	0.99998	0.99916	1.00108	0.00191	0.00024	0.00	-0.10	0.13	0.23	0.03
TG161001 (17)	1.00002	0.99901	1.00085	0.00183	0.00025	0.00	-0.15	0.10	0.24	0.03
TG070001 (20)	1.00001	0.99898	1.00061	0.00162	0.00020	0.00	-0.10	0.08	0.19	0.03
TG162001 (21)	0.99998	0.99922	1.00066	0.00145	0.00018	0.00	-0.11	0.07	0.18	0.02
TG020001 (23)	1.00001	0.99936	1.00079	0.00143	0.00017	0.00	-0.08	0.06	0.14	0.02
PR300101 (24)	0.99999	0.99935	1.00072	0.00137	0.00017	0.00	-0.06	0.10	0.16	0.02
TG060001 (22)	1.00002	0.99933	1.00061	0.00128	0.00016	0.00	-0.09	0.06	0.15	0.02
TG090001 (25)	1.00001	0.99953	1.00025	0.00072	0.00009	0.00	-0.06	0.03	0.09	0.01
TG040001 (26)	1.00001	0.99975	1.00038	0.00063	0.00007	0.00	-0.04	0.03	0.06	0.01
PR300611 (29)	0.99965	0.99947	1.00002	0.00055	0.00013	0.00	-0.01	0.01	0.02	0.00
TG050001 (27)	1.00001	0.99976	1.00023	0.00047	0.00007	0.00	-0.04	0.02	0.07	0.01
PR300111 (28)	1.00000	0.99985	1.00019	0.00034	0.00004	0.00	-0.02	0.02	0.04	0.01
PR300601 (30)	1.00000	0.99998	1.00006	0.00008	0.00001	0.00	-0.01	0.01	0.01	0.00
Canada	1.00004	0.99355	1.00380	0.01025	0.00119	0.01	-0.61	0.52	1.13	0.16

* Provinces and trade groups are ordered by range of levels' correction; ranking by mean in the whole system is indicated in parentheses.

Appendix B

Figure 1 **Monthly discrepancies of PR system**

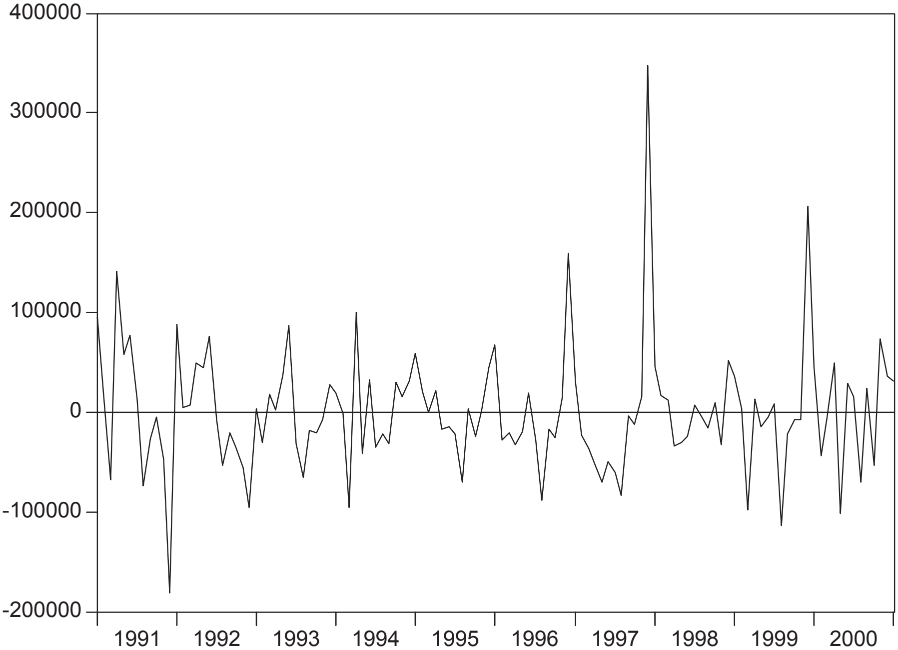


Figure 2 **Additive corrections to growth rates made by benchmarking PR component series with a binding constraint**

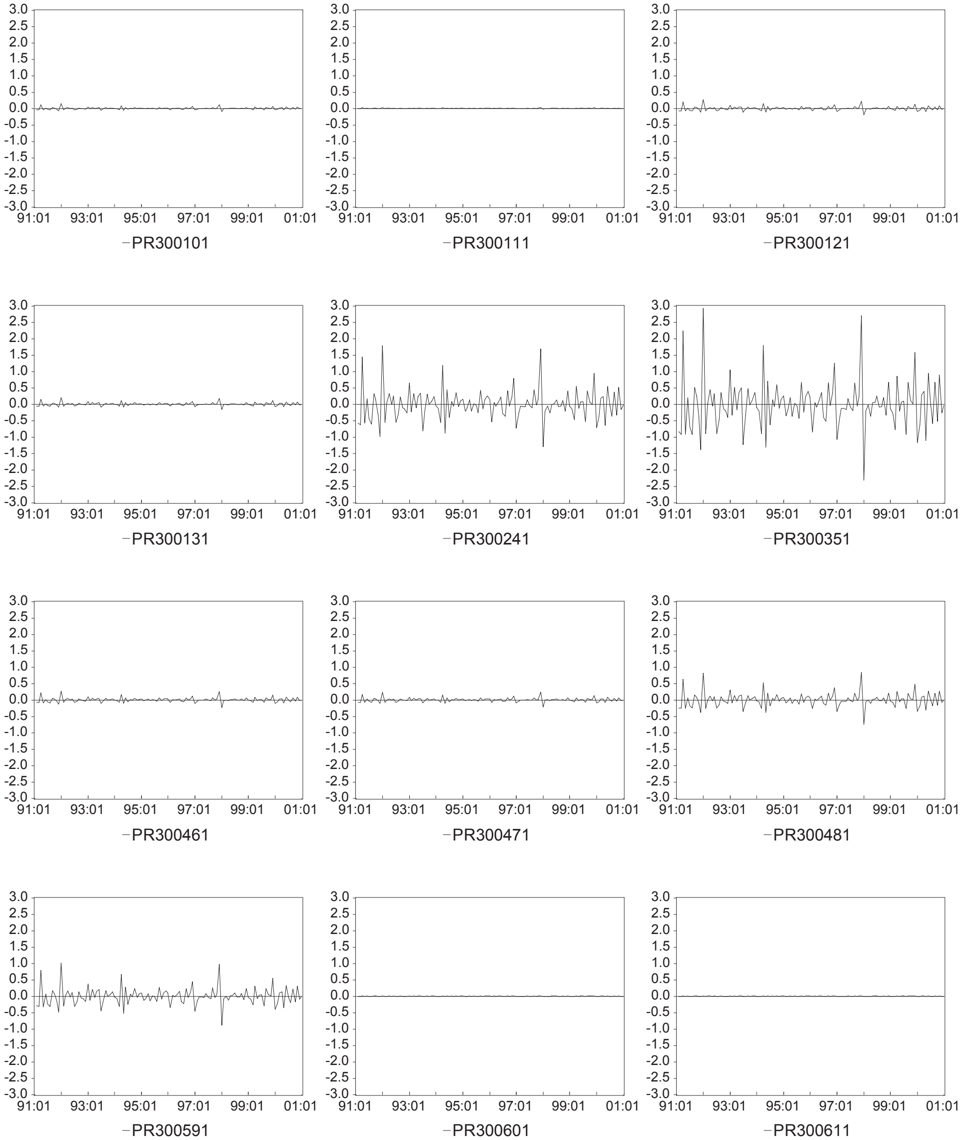


Figure 3 **Additive corrections to growth rates made by benchmarking PR component series while preserving the growth rates and with binding constraint**

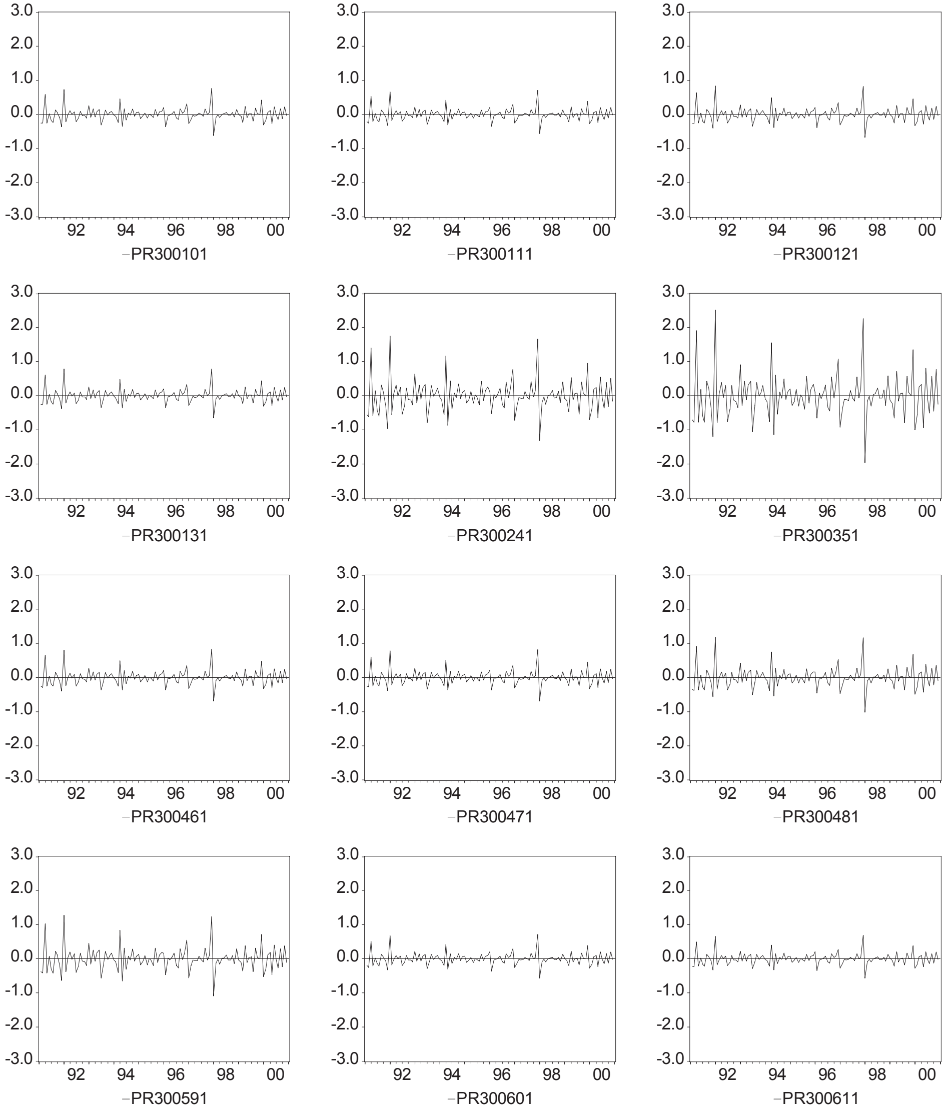
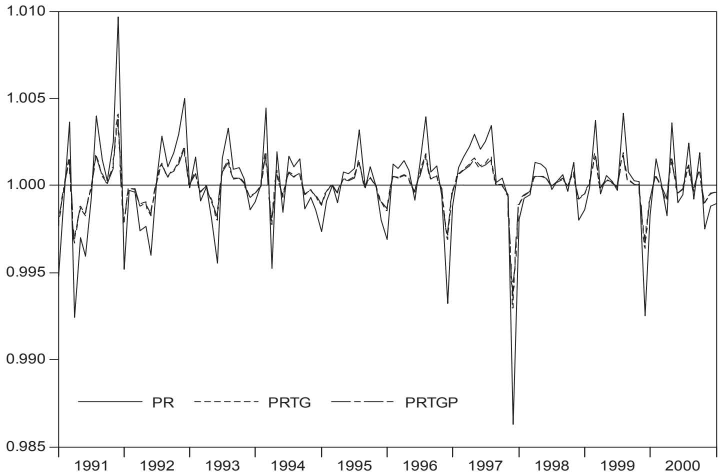
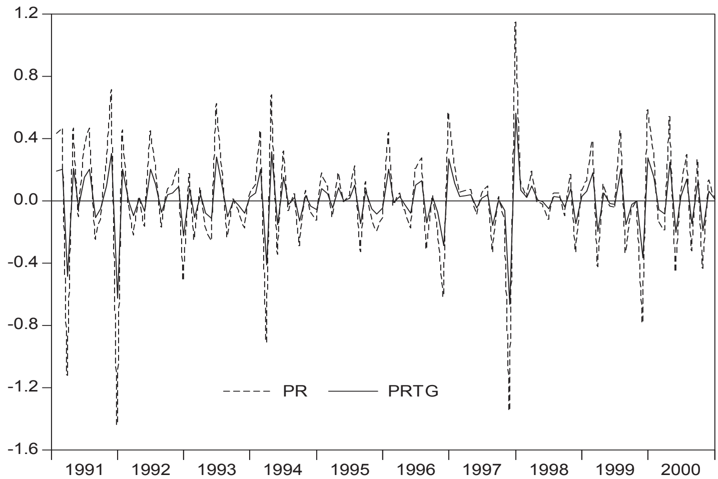


Figure 4 **Proportional corrections to the unbenchmarked SA Total Canada made by the sum of variously benchmarked SA series**



Notes: PR: case 1; PRTG: case 3; PRTGP: case 4

Figure 5 **Proportional corrections to the unbenchmarked SA Total Canada made by the sum of variously benchmarked SA series**



Notes: PR: case 1; PRTG: case 3.

From:
Journal of Business Cycle Measurement and Analysis

Access the journal at:
https://doi.org/10.1787/17293626

Please cite this article as:

di Fonzo, Tommaso and Marco Marini (2005), "Benchmarking Systems of Seasonally Adjusted Time Series", *Journal of Business Cycle Measurement and Analysis*, Vol. 2005/1.

DOI: https://doi.org/10.1787/jbcma-2005-5km7v1835wr5

OECD