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Measuring Capital
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Chapter 3

How asset values are determined

3.1. Concept

The central economic relationship that links the income and production perspectives to each other is the net present value condition: in a functioning market, the *stock value* of an asset is equal to the discounted stream of future benefits that the asset is expected to yield, an insight that goes at least back to Walras (1874) and Böhm-Bawerk (1891). Benefits are understood here as the income or the value of capital services generated by the asset.

In what follows, we shall consider a single asset, although this is clearly unrealistic: no firm, and much less a statistical agency will measure capital by looking at individual pieces of machinery or equipment. The typical case is to consider classes of assets, although an attempt is normally made to keep these classes of assets as homogenous as possible. For the moment, however, consider a single asset that is new, *i.e.* of age zero.

3.1.1. Income perspective

The value of this asset at the beginning of period t, P_0^t, to its owner corresponds to the discounted stream of future incomes generated by the asset. A subscript has been used to signal the age of the asset, in the present case zero because this is a new capital good. The flow of income for this asset will be labelled c_s^{t+s} where the superscript 't+s' indicates the period when the income arises and where the subscript 's' indicates again the age of the asset. A discount factor is needed as well to reflect the fact that people prefer immediate income to income in the future. The discount factor is labelled (1+r), where r is the nominal rate of return that the asset holder expects the asset to yield. Economic reasoning would suggest that this is the opportunity cost of the funds tied up in the asset: how much would an investor have earned (risk adjusted) if the funds had been invested elsewhere? At a minimum, the nominal rate of return should reflect the financing costs for the asset, for example the interest that the asset owner has to pay for a loan taken out to purchase the asset. Typically, however, the nominal rate of return will be higher than the interest rate paid on financing but there is no need to dwell on this distinction here. With the above remarks in mind, the fundamental equation relating the stock value of an asset to future income is:

$$p_0^t = c_0^t/(1+r) + c_1^{t+1}/(1+r)^2 + c_2^{t+2}/(1+r)^3 + \ldots + c_T^{t+T}/(1+r)^{T+1}. \quad (1)$$

The relationship (1) has been formulated with nominal benefits and a nominal discount rate. Alternatively, it could have been expressed with real benefits and a real discount rate. In this case, a general deflator such as the consumer price index would be used to express future income flows and the rate of return r^t would be adjusted for the rate of general inflation. Consistency is important here and it is incorrect to combine nominal future income with a real discount rate or vice versa.

Further, the net present value formula (1) assumes that income payments are received at the end of each year. National accounts conventions suggest that benefits should be measured as evenly spread throughout the accounting period. This complication will be considered in the implementation part of the *Manual*. For the present conceptual exposition we ignore the complication as it does not affect the main conclusions.

Some more explanation on the income flows f may be useful. For an owner-user of an asset, the income generated by the asset corresponds to the profits that the asset generates when used in production. In more precise accounting terms, it corresponds to the extra gross operating surplus that the owner can expect from the use of the asset in production. Thus, the income flow for an asset should be 'gross' in the sense that it is not corrected for, *i.e.* inclusive of, depreciation, the value loss of the capital good as it ages. The income flow for an asset is 'net' in the sense that the extra proceeds from sales that were possible because the capital good generated additional output, are corrected for average labour costs and intermediate inputs per unit of capital.

3.1.2. Cost perspective

In competitive markets, there are no expected residual profits above and beyond the costs of capital input. The implication is that gross operating surplus[1] – whatever is left over once labour and intermediate inputs have been paid – will be equal to the cost of capital input. Thus, gross operating surplus per asset – the income flow generated by it – can also be given a cost interpretation: more specifically, it corresponds to the unit user cost of the asset. The cost perspective also permits interpreting unit user costs as the price of capital services: a capital good of a particular type and of a particular age supplies one unit of age-and asset-specific capital services. The price for these services is c_s^{t+s} – a price that the owner-user 'pays to himself'.

The cost perspective can be developed directly by examining the costs that a firm would have to incur if, at the beginning of a period, it bought an asset, used it in production during that period and sold it at the end of the period. The following elements would be considered in computing these costs: *(i)* the purchase price of the asset at the beginning of the period – if it is a new asset, this would be p_0^t; *(ii)* the sales price of the asset at the end of the period, noting that the asset is now one year old: p_1^{t+1}; *(iii)* a discount rate r to reflect the fact that financial capital is bound in the asset while in usage during the period. Combining these elements, the costs for using the asset are $p_0^t(1+r)-p_1^{t+1}$. These are in fact the unit user costs, or the price of capital services for the asset (see Chapter 8 for an in-depth presentation), which had been labelled c_0^t. It is not difficult to show (see Section 19.1) that the net present value relationship (1) follows from the reasoning about the cost of using a capital good during one period: $c_0^t = p_0^t(1+r)-c_1^{t+1}$.

When the sequence $\{c^t\}$ is interpreted as a sequence of unit user costs or capital services prices, equation (1) can also be interpreted as a rule for cost allocation over time: the value of a new capital good has to be distributed over accounting periods because of its nature as an investment good. This allocation in time should be such that costs in future periods match capital services that are provided by the asset in each period and measures for quantities and prices of capital services fulfil exactly this role.

Another important link can be established now that c_0^t has been interpreted as the price for the capital services of a new asset in year t: when compared to the price of capital services of another asset of the same type but of different age, say one year, it is plausible to state that the ratio of capital services prices c_0^t / c_1^t should reflect the relative efficiency in production of the new compared to the one year old asset.

3.1.3. Market perspective

The net present value relation (1) can also be formulated for the stock value of an asset that is not new, *i.e.* for an asset with age greater than zero. For some used assets, there are markets, for others there are no markets. If a used asset market exists, and if an asset is offered for sale at a price that does not seem likely to generate a satisfactory rate of return, there will be no demand for that asset. If an asset is offered at a price that seems likely to generate a very high rate of return, there will be more demand than supply for the asset. In the first case, the price will be bid down, and in the second case the price will be bid up until the rate of return rises or falls to a "normal" level. Equation (1) can, therefore, also be given an interpretation of how asset prices are determined in a market economy.

Equation (1) is central for understanding the conceptual framework of this *Manual*. The net present value formula provides the link between stock measures, depreciation, and capital services: the value of the (net) stock of a particular age s enters via the price of the asset p_s^t; depreciation is part of the gross operating surplus term per unit of capital c_s^{t+s} that reflects income. This in turn equals unit user costs which constitute the price of capital services.

3.2. Relationship between capital service and asset prices for a single asset – a numerical example

This Section uses a simple numerical example to convey the main ideas behind a consistent set of capital measures. Box 3.1 spells out the numerical assumptions. The example starts out with Table 3.1 which shows how equation (1) can be used to calculate the price of an asset both when it is new and at every stage in its lifetime. Several assumptions are made to construct this example.

A 6-month old car may have lost none of its productive efficiency and yet it can only be sold at a 20% discount on the second-hand market. This distinguishes the age-efficiency from the age-price profile.

We take a cost perspective here (see Section 3.1.2 above) although nothing particular hinges on this, and we could equally have chosen a market or an income perspective for illustration. The first column in Table 3.2 shows the (future) service years of the asset. The perspective taken is from the beginning of year 1, looking ahead until the end of the asset's service life. The cost per unit of capital services that the asset is expected to provide each year is shown in the third column and corresponds to the sequence of c_0^t, c_1^{t+1}, c_2^{t+2} etc. in the net present value calculation (1). There are several ways to compute the evolution of these prices of capital services, given the data in Box 3.1 and only one option is presented here. Two factors impact on the change in the price of capital services: the rate at which the productive capacity of an asset declines as it becomes older, and the rate at which asset prices develop. The first effect is captured by the *age-efficiency profile*, shown in the second column of the table. Thus, during the first year of operation, the asset runs at 100% of its productive capacity, during the second period the figure is 88%, and so forth. The age-efficiency profile has been depicted as linear here and whether this is correct or not is an empirical issue.

The second effect that bears on the price of capital services is general changes in asset prices. Those were assumed to rise by 2% per period. The two effects can now be combined to yield the sequence of prices of capital services shown in the third column. By assumption it is $ 10 at the end of the first year (or equivalently, at the beginning of the second year). At the beginning of the third year, the price of capital services has fallen to $ 8.93, the product of a decline in efficiency to 88% and a 2% rise in asset inflation: $\$ 10*0.88*1.02 = \$ 8.93$. At the beginning of year 4, the capital service price is $\$ 10*0.75*1.02^2 = \$ 7.80$ and so on.

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Box 3.1. **Numerical example**

In the following chapters, a numerical example will be used that is based on the following assumptions about a fixed asset:

- Service life of 8 years
- Discount rate 5 %
- Price of capital services for a new asset, payable by the end of the first year is $ 10
- For simplicity, no general inflation
- Price of new asset is expected to rise by 2% per year
- Productive services of the asset decline by a constant amount over its service life (linear *age-efficiency* pattern)

Because the costs for capital services accrue in different years, their present value has to be obtained by discounting each year's rental by the discount factor (1+r), taken as 1.05 in this example. The fourth column of Table 3.1 shows the value of capital service prices, discounted to the beginning of year 1. This example assumes that payments are due at the end of each year and so the first year's cost of $ 10 is valued at $ 10/1.05 = $ 9.52 at the beginning of year 1; the payment of $ 8.93 expected at the end of the second year (or at the beginning of the third year) is worth only $ 8.93/1.05^2 = $ 8.10 at the beginning of year 1; the payment of 7.80 expected at the end of year 3 is worth only $ 7.80/1.05^3 = $ 6.74 etc. The total of these discounted capital services prices gives the value of the asset at the beginning of year 1, *i.e.* $ 40.12.

Table 3.1. **Relationship between capital service prices and asset value in year 1**

Year (t)	Age-efficiency	Price of capital service at beginning of period	Price of capital service discounted to beginning of year 1
1	100.0%		
2	87.5%	10.00	9.52
3	75.0%	8.93	8.10
4	62.5%	7.80	6.74
5	50.0%	6.63	5.46
6	37.5%	5.41	4.24
7	25.0%	4.14	3.09
8	12.5%	2.82	2.00
9	0.0%	1.44	0.97
10		0.00	0.00
Price of asset beginning of year 1			40.12

So far, we have considered the valuation of the asset at the beginning of the first year. Next, consider the same type of calculation one year later, *i.e.* at the beginning of year 2 and then two years later and so forth. This is captured in Table 3.3 below. The first four columns are identical to Table 3.1 but the fifth column shows the asset value at the beginning of the second year. For example, the capital service price of $ 8.93 prevailing in period 3 is the same as before but because time has moved on, it is now only discounted by one period: $ 8.93/ 1.05 = $ 8.50. The asset value at the beginning of the second year is then $ 32.12; the value at the beginning of the third year is $ 24.81 and so on. This sequence of asset values can be considered the price history of the asset, expressed in current prices of each period.

Table 3.2. **Relationship between capital service prices and asset value in all years**

			Price of capital service discounted to beginning of year							
Year (t)	Age-efficiency	Price of capital service at beginning of period	1	2	3	4	5	6	7	8
1	100.0%									
2	87.5%	10.00	9.52							
3	75.0%	8.93	8.10	8.50						
4	62.5%	7.80	6.74	7.08	7.43					
5	50.0%	6.63	5.46	5.73	6.02	6.32				
6	37.5%	5.41	4.24	4.45	4.68	4.91	5.15			
7	25.0%	4.14	3.09	3.24	3.41	3.58	3.76	3.94		
8	12.5%	2.82	2.00	2.10	2.21	2.32	2.43	2.55	2.68	
9	0.0%	1.44	0.97	1.02	1.07	1.13	1.18	1.24	1.30	1.37
10		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Price of asset beginning of year			40.12	32.12	24.81	18.24	12.52	7.74	3.98	1.37

With the price history at hand, an important link can be established – that between the age-efficiency profile and age-price profile. To show this link, a matrix is constructed below with the history of asset prices in the main diagonal (Table 3.3). Each line stands for a different year in the life of the asset and each column epitomises the asset's age.

Table 3.3. **Price history of asset**

	Age of asset								
Year (t)	0	1	2	3	4	5	6	7	8
1	**40.12**								
2	40.92	**32.12**							
3	41.74	32.77	**24.81**						
4	42.57	33.42	25.30	**18.24**					
5	43.43	34.09	25.81	18.61	**12.52**				
6	44.29	34.77	26.32	18.98	12.77	**7.74**			
7	45.18	35.47	26.85	19.36	13.03	7.89	**3.98**		
8	46.08	36.18	27.39	19.75	13.29	8.05	4.06	**1.37**	
9	47.01	36.90	27.94	20.14	13.56	8.21	4.14	1.39	**0.00**

It can now be seen that the diagonal entries, the price history of the asset, combine two effects:

- a (vertical) movement in time (from year 1 to 2 etc.) that reflects the general change of the price of the asset class in question. For example, the new asset at the beginning of year 1 has a price of $ 40.12; after one year, its value has dropped to $ 32.12. The first effect can be read by comparing vertically the price of a new asset in year 1 ($ 40.12) to the price of a new asset in year 2 ($ 40.92). The difference reflects the 2% change in new asset prices underlying the present example.
- a (horizontal) movement in the age of the asset (from being new – age zero – to being one year old etc.) that reflects the value change because the asset has become older. In the example at hand, the age-effect is given by the horizontal movement from $ 40.92 to $ 32.12 – that is by the price difference of a new and a one-year old asset at the beginning of year 2. In percentage terms, the relative price of a one year old asset compared to a new asset is $ 32.12/$ 40.92 = 78.5%, the relative price of a two year-old asset compared to a new asset is $ 24.81/$ 41.74 = 59.4% and so forth. These price comparisons of assets of different

age for a given year constitute the *age-price profiles* of assets and are directly linked to depreciation. In particular, the line for year 9 shows the entire age-price profile of the asset.

From the above discussion it should be apparent that the age-efficiency profile and the age-price profile of a class of assets come in pairs, and although they may be different, they are not independent of each other. This is important for empirical implementation where the starting point is either an age-price profile from which a consistent age-efficiency profile is derived or an age-efficiency profile from which a consistent age-price profile is derived. The two avenues are presented in detail in Part II of this *Manual*.

Table 3.4. **Linear age-efficiency and corresponding age-price profile**

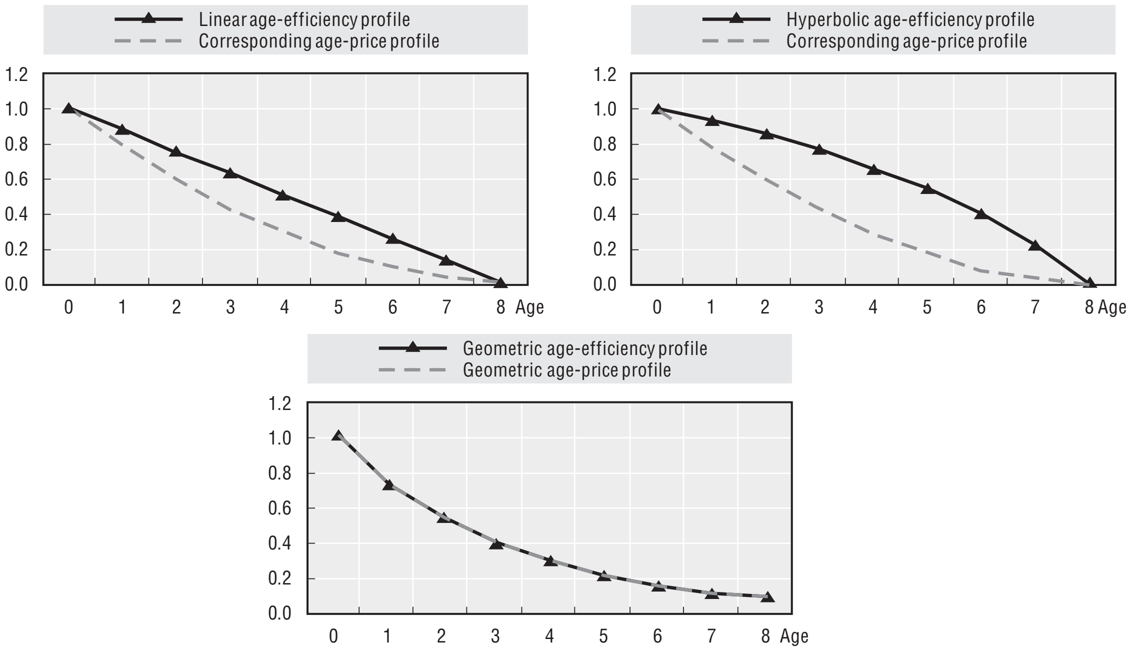
	Age								
	0	1	2	3	4	5	6	7	8
Age-efficiency profile	1.00	0.88	0.75	0.63	0.50	0.38	0.25	0.13	0.00
Age-price profile	1.00	0.79	0.59	0.43	0.29	0.17	0.09	0.03	0.00

The age-price and the age-efficiency profiles for the numerical example are shown in Table 3.4. The age-efficiency profile is taken directly from the first column of Table 3.1, and is based on the assumption of linearly efficiency decline. The age-price profile has been derived via Table 3.3, by comparing – for a given year – the price of capital goods of different ages with the price of a new capital good. One notes immediately that the two profiles are not identical. This can also be seen from the first graph in Figure 3.1: the linear age-efficiency profile gives rise to a convex-looking age-price profile. Other types of age-efficiency and age-price profiles are of course possible and indeed, the linearly declining efficiency profile may not be the most plausible approximation to the typical pattern of efficiency loss of an asset as it ages.

Two particular cases are worth mentioning here and are depicted graphically in Figure 3.1. The first case is a particular version of a hyperbolic age-efficiency profile where an asset's productive efficiency declines at a slow rate in the first years of its service life and at increasingly faster rates towards the end of the asset's service live. A hyperbolic age-efficiency profile gives rise to a convex age-price profile. The second special case arises when the age-efficiency or the age-price profile declines at a constant rate. It can be shown that in this case, the age-efficiency and the age-price profile are identical and both decline at the same rate. This constitutes significant practical advantages in implementing and computing capital measures and has been used in a vast majority of empirical studies of capital measures and depreciation.

The numerical example used here *assumed* a discount rate of 5% to put the calculations in place. In other words, the discount rate has been taken from outside, as an exogenous variable. As will be discussed later in this *Manual*, this is but one way of obtaining a discount rate or rate of return. In particular, a widely-used approach towards measuring the rate of return is computing it *endogenously* (see Section 8.3). It is worth flagging at this point already that an endogenous computation of the rate of return is difficult to reconcile with non-geometric age-efficiency and age-price profiles[2]. Thus, when rates of return are computed endogenously, it is best to combine them with geometric age-price and age-efficiency profiles.

Figure 3.1. **Age-efficiency and corresponding age-price profiles**



Notes

1. Taxes on production and mixed income are ignored for the moment.
2. There is an issue of simultaneity: if age-efficiency and age-price profiles are not geometric, and the starting point for computations is an age-efficiency profile, a rate of return is needed to derive the age-price profile. But to compute an endogenous rate of return, an age-price profile is needed. Conversely, if the starting point for implementation is an age-price profile, the age-efficiency profile is required to compute an endogenous rate of return.

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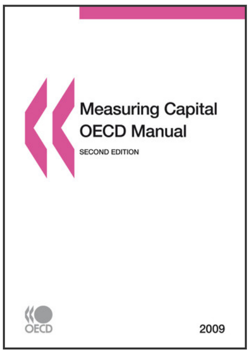

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