The Information Content of the Term Structure of Interest Rates: Theory and Practice

Frank Browne
Paolo Manasse

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WORKING PAPERS

No. 69 THE INFORMATION CONTENT OF THE TERM STRUCTURE OF INTEREST RATES: THEORY AND PRACTICE

by

Frank Browne and Paolo Manasse
Monetary and Fiscal Policy Division
Balance of Payments Division

September 1989
ECONOMICS AND STATISTICS DEPARTMENT

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The paper is devoted to an empirical examination of the information content in the term structure of nominal interest rates for future inflation. Tests of the ability of the term structure to forecast future changes in the inflation rate are carried out for six major OECD countries using monthly data. These tests demonstrate that the term structure does have considerable forecasting ability, particularly for rates taken from the short end of the maturity spectrum. However, with one exception, forecasting power tends to fade or disappear completely when the term structure in question is formed using yields on increasingly distant maturities as the long rate. This suggests that changes in the nominal term structure using such rates reflect mostly changes in the term structure of (ex post) real interest rates.

Cette étude présente une analyse empirique des informations apportées par la structure des taux d'intérêt sur l'évolution future de l'inflation. La relation entre la structure des échéances et l'évolution future du taux d'inflation y est testée pour six grands pays de l'OCDE, sur des données mensuelles. Les tests effectués révèlent que cette structure a effectivement un pouvoir prédictif considérable, notamment si l'on considère les taux les plus courts. En revanche, à une exception près, cette propriété s'estompe ou disparaît complètement lorsqu'on considère les taux associés à des échéances de plus en plus longues. On peut donc penser que l'évolution de la structure des échéances, pour les longs taux, reflète surtout les changements qui interviennent dans la structure des taux d'intérêt réels (ex post).
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I. INTRODUCTION

Financial market deregulation and innovation have considerably complicated the interpretation of money aggregates in the 1980s, making them less reliable indicators for monetary policy in some countries. As a result, the problem of detecting whether monetary policy is too tight or too loose at any moment in time has become much more difficult. A more pragmatic approach to monetary policy formulation has thus emerged in recent years, relying on more judgmental views and taking into account a wider array of potentially relevant financial and real indicators.

Among these indicators there is a growing interest in the term structure of interest rates as a measure of expected inflation (1). It is argued that a steepening of the yield curve reflects an expected acceleration of inflation which, by signalling an advanced warning of current inflationary pressures, may warrant a tightening of policy in order to obviate the necessity of much stronger and more disruptive counter-inflationary action later. For the term structure to be a useful leading indicator of inflation, however, two main conditions are required to be fulfilled: i) that the Fisher theory be valid for all asset maturities and ii) that expectations be rational. The former says that assets are perfect substitutes for commodities which translates into the more familiar condition that nominal interest rates incorporate inflation premia. If all assets are perfect substitutes for goods then assets are perfect substitutes for each other. Therefore the Fisher condition implies the expectations theory of the term structure of interest rates which says that assets at varying maturities are perfect substitutes or, stated more familiarly, that the current long rate of interest is a weighted average of expected future short rates. The corollary also holds since the failure of the expectations theory implies that the Fisher condition cannot be valid at all asset maturities. Two assets, for example, which are imperfect substitutes for each other cannot both at the same time be perfect substitutes for commodities. One of the reasons, therefore, why the slope of the term structure may be a poor predictor of future inflation may be due to less than perfect substitutability of assets at different maturities.

Time-varying risk premia, segmented markets and excess long rate sensitivity have all been propounded as causes of less than perfect substitutability between assets at different points in the maturity spectrum or for departures from the expectations theory. Given the expectations theory's central importance in the term structure literature, a selective review of the arguments and evidence for and against this theory is given in Section II of the paper. The main purpose of the paper, however, is to examine the ability of the term structure to forecast future inflation changes.

A model recently proposed by Mishkin (outlined in Section III of the paper) provides the basis for the empirical tests. These are carried out for six of the major OECD economies. The results are reported in Section IV and Section V contains some conclusions.
II. THE EXPECTATIONS THEORY OF THE TERM STRUCTURE

The usefulness of the term structure relationship as a leading indicator of future inflation relies heavily, though not critically, on the validity of the expectations theory of the term structure. This holds that the long-term interest rate is a weighted average of present and expected future short-term interest rates. If future short rates are expected to remain constant, then the long rate will equal the short rate. However, if future short rates are expected to increase at any time before the n periods to maturity of the long bond have elapsed, then the current long rate will exceed the current short rate and vice versa. The corollary of this is that the shape of the yield curve is a reflection of the market’s expectation of future short rates of interest. The expectations theory implies that securities of varying maturities are perfect ex ante or expected substitutes for one another.

An approximately equivalent argument of the expectations theory states that expected holding-period returns on bonds of all maturities are identical, or differ only by constant risk premia (2). Without any loss of generality, let “one period” be defined by the time to maturity of the short bond. The yield and holding return on this short bond held to maturity \( r_s \) are, by definition, the same. The "one-period" holding return on a long bond (i.e. a bond where maturity is longer than one period) is what one obtains by purchasing such a security, holding it for "one period" and then selling it at the prevailing price. The term premium is the difference between the expected holding return on the long bond and that on the short bond. Realised or ex post excess holding returns are, therefore, identically equal to the sum of the time varying term premium \( \Theta_t \) and expectational errors \( V_{t+1} \).

Expectations are assumed to be rational, and hence \( V_{t+1} \) is a serially uncorrelated error term reflecting "news" about the long rate. The term premium, \( \Theta_t \), therefore, reflects the extra return investors demand for holding the long rather than the short bond. The expectations theory of the term structure states that \( \Theta_t \) is a constant, i.e. \( \Theta_t = \theta \). Thus, the joint hypothesis of rational expectations and the expectations theory of the term structure states that observed excess returns cannot be forecast with information available at time \( t \). To refute this joint hypothesis, then, all that is required is to find some variables that can explain excess holding returns. Lagged values of excess holding returns seem an obvious candidate.

The choice of sample period in conducting such tests is important. Some researchers (Blanchard, 1984, Mankiw and Miron, 1986 and Belongia and Koedijk, 1988) have argued, in the United States context, that tests of the expectations theory of the term structure are likely to be sensitive to the changes in the monetary control procedures implemented by the Federal Reserve in October 1979. These changes allowed short-term interest rates to fluctuate much more than under the previous procedure. Given the importance of the United States in the world financial system, this increased variability is likely to have been transmitted to other OECD countries, also implying structural change for them after October 1979.
Tests of the expectations theory using excess holding period returns with monthly data were carried out for two periods: 1971M1 to 1979M9 and 1979M11 to 1989M4. The results (not reported here in detail but available from the authors) show that either one or more lagged values of excess holding period yields are highly significant for all countries and for both time periods. The joint hypothesis of the expectations theory and rational expectations is thus rejected for all countries in the sample (United States, Germany, France, the United Kingdom, Italy and Canada), for all long and short rates employed (see Section IV for details) and for both time periods.

Tests of the expectations theory reported in the literature are, as here, mostly unfavourable (3). The theory is rejected conditional on some maintained hypothesis about how expectations of future short rates are generated. In recent contributions this is always a rational expectations assumption. The inconclusiveness of the results of these tests is due to the fact that they are tests of a joint hypothesis: the expectations theory of the term structure and the rational expectations of future nominal short-term interest rates. The rejection of this joint hypothesis is interpreted by some as implying that time-varying risk premia exist (i.e. the expectations theory of the term structure is wrong) and by others that the long rate over- or under-reacts relative to a rational expectation of future short rates (i.e. rational expectations are rejected).

Empirical rejection of the expectations theory does not necessarily mean that there is no information in the term structure that is useful as an indicator for monetary policy. Variation in expected future short rates could still account for the bulk of the systematic variation in current long spot rates, or in current short forward rates. The problem is in extracting this information, given the noise created either by time-varying risk premia or by excess volatility of the long rate, or by both (see below).

The U.S. Federal Reserve relinquished the objective of interest rate smoothing between October 1979 and October 1982, adopting, at the end of this time period, a policy of partial smoothing. It is argued by Mankiw and Miron (1986) that this change in operating procedure will render interest rate behaviour more favourable to the expectations theory after October 1982. One such test (Hardouvelis, 1988), although rejecting the strict expectations theory, nevertheless finds that it has considerable forecasting power particularly after 1979. He also finds only modest forecasting ability before 1979, lending some corroboration to the Mankiw-Miron hypothesis. Another test using survey data on interest rate expectations (Froot, 1988) rejects the hypothesis that the forward rate is an unbiased predictor of the corresponding future spot rate. However, for one category of long-term bonds he fails to reject the expectations theory.

To what is the relatively poor performance of the expectations theory attributable? Three main factors have been advanced as explanations: time-varying risk premia; excess volatility of the long rate of interest; and some degree of market segmentation due to the preferred habitats of investors.

Explaining the failure of the expectations theory in terms of time-varying risk premia is somewhat vacuous in the absence of a further theory as to why risk premia themselves vary with time. Indeed, the most
striking rejections of the expectations theory tend to be for data drawn from the short end of the maturity spectrum such as three- and six-month Treasury bills [see, for example, Fama (1984), Jones and Roley (1983), Mankiw and Summers (1984) and Shiller, Campbell and Schoenholtz (1983)]. In order to explain the failure of the theory by time-varying premia in such cases requires identifying huge variations in risk within very short time periods, for which the objective conditions do not seem to exist.

Much of the effort of researchers has thus been devoted to a search for measures of risk that prove to be significantly related to the long-short spread (4). They have largely failed to come up with a risk measure that has significant explanatory power for variations in the term structure.

If it were possible to obtain such a measure of risk, and providing expectations are rational, then the slope of the yield curve could be purified of this element leaving the remainder to be interpreted in accordance with the tenets of the expectations theory of the term structure. In other words, an upward-sloping risk-adjusted yield curve would then reflect the markets' expectation of a future increase in short rates and vice versa.

An alternative hypothesis originally suggested by Shiller (1979) and also tested by Shiller, Campbell and Schoenholtz (1983), Campbell and Shiller (1987) and Mankiw and Summers (1984), is that it is myopic expectations, which provoke an overreaction in long rates to changes in contemporaneous short rates, that are responsible for the failure of the expectations theory.

However, the empirical tests of this excess sensitivity hypothesis decisively reject it. Long rates are found to underreact to current short rate changes. In other words, the long rate reacts disproportionately to expected future short spot rates. In sum, the excess sensitivity hypothesis cannot account for the observed departures from the predictions of the expectations theory.

The apparent inadequacies of the expectations theory to account fully for its observed movements have motivated some researchers to investigate the possibility that the term structure is compartmentalised to a greater or lesser extent. If so, then changes in the supply and demand for securities of a particular maturity will affect the shape of the yield curve. The view that borrowers and lenders may have preferred habitats at particular maturities which suit their specific needs, and from which they can only be coaxed by large interest differentials, has long been debated.

Empirical investigations of this theory have used structural supply–demand models and have employed the restrictions suggested by the theory of portfolio behaviour (see Brainard and Tobin, 1968 and Smith, 1975) to constrain the term structure equation implied by the data-admissible structural portfolio model. Friedman and Roley, both separately and jointly (Friedman, 1977, 1979; Roley, 1981 and Friedman and Roley 1979), have examined the determinants of the term structure using this approach. They claim that their implied expressions for the term structure of interest rates fits the data about as well as does the typical alternative approach of a single-equation reduced form. Nevertheless, their simulations for the effects of debt management policies indicate that such effects are quite small. Other single equation evidence also suggests that debt management effects are small
(see Modigliani and Sutch, 1966). Other authors, who have attempted more recently to test for market segmentations by the inclusion of proxy variables in single-equation reduced form term structure equations, have also found only limited support for this view (5).

In general terms, segmentation effects appear to be sufficiently small (and probably diminishing with time as a consequence of financial market deregulation) as not to compromise significantly the potential of the term structure to act as a leading indicator of future inflation changes.

III. THE TERM STRUCTURE AS A LEADING INDICATOR OF INFLATION

Mishkin (1988) has recently proposed a model for testing the forecasting ability of the term structure for future inflation. His model (for a detailed derivation see Appendix 1) is based on the Fisher equation for different maturities plus rational expectations. His forecasting equation is:

\[ \pi_m,t - \pi_n,t = \alpha_{m,n} + \beta_{m,n} [i_m,t - i_n,t] + \eta_t, \]

where

\[ \alpha_{m,n} = r_i^m - r_i^n, \]

\[ \eta_t = \varepsilon_m,t - \varepsilon_n,t - u_{m,t} + u_{n,t}, \]

\[ u_{m,t} = rr_{m,t} - rr_{i,m,t} \]

\[ u_{n,t} = rr_{n,t} - rr_{i,n,t}. \]

\( \pi_m,t \) and \( \pi_n,t \) are the realised inflation rates over \( m \) and \( n \) periods corresponding to the maturities of the long and short assets the yields of which are denoted by \( i_{m,t} \) and \( i_{n,t} \), respectively. The intercept in equation 1 is the difference in the average real rates over the corresponding maturities. \( \varepsilon_m,t \) and \( \varepsilon_n,t \) are inflation forecast errors over the \( m \)- and \( n \)-period horizons. If the real term structure is constant, (i.e. \( u_{m,t} = u_{n,t} = 0 \)), the somewhat complex error term boils down to \( \varepsilon_m,t - \varepsilon_n,t \). Given rational expectations the latter is independent of the contemporaneous nominal term structure (i.e. \( i_{m,t} - i_{n,t} \)), a necessary condition for consistent estimation. However, in the present context of overlapping observations this condition does not guarantee efficient parameter estimates (this issue will be taken up again in Section IV).

For heuristic purposes it is worthwhile noting that equation [1] can be rewritten as:

\[ r_{m,t}^{p} - r_{n,t}^{p} = -\alpha_{m,n} + (1 - \beta_{m,n}) [i_m,t - i_n,t] - \eta_t, \]

where \( r_{m,t}^{p} \) and \( r_{n,t}^{p} \) are the ex post real interest rates on \( m \) and \( n \) period bonds at time \( t \).
It is important to note that the interest and inflation rates entering equation [1] are expressed in an analogous manner on a continuously compounded basis at annual rates. To be more precise, let \( i_{j,t} \) be the annualised interest rate prevailing in period \( (\text{month})t \) [i.e. 8.65 per cent on 3-month treasury bills in April 1989 for the U.S. economy, for example]. The continuously compounded rate over the \( j \) periods (months) to maturity of the rate prevailing in period \( t \) converted to an annualised basis is:

\[
i_{j,t} = \left[ (i_{j,t}^{A}/100+1)^{1/12}-1 \right] \times 100, \quad j=m,n.
\]

Hence it represents the nominal \textit{ex post} gross return on investing 1 dollar in a fixed rate bond that matures at \( t+j \).

The inflation rate is compounded, not over the actual rate prevailing in period \( t \) (as with the interest rate), but over the actual inflation rates prevailing in all the future periods to maturity of the corresponding asset whose yield is being compared with the inflation rate, i.e.:

\[
\pi_{j,t} = \left[ (\pi_{t}^{A}/100+1)^{1/12}(\pi_{t+1}^{A}/100+1)^{1/12}... \pi_{t+j-1}^{A}/100+1)^{1/12}-1 \right] \times 100, \quad j=m,n
\]

where \( \pi_{t}^{A} \) is the annualised inflation rate in period \( t \).

If \( \beta_{m,n} \) in equation 1 is significantly different from zero then the slope of the term structure defined as \( i_{m}-i_{n} \) in period \( t \) has some forecasting ability for the actual inflation rate over the future time interval \((t+m)-(t+n))\), \( n < m \). The rejection of \( \beta_{m,n} \) equal to zero would be tantamount to a rejection of the null hypothesis that the term structure of real interest rates moves pari-passu with the term structure of nominal interest rates (see equation [2]). On the other hand, a rejection of the hypothesis that \( \beta_{m,n} \) equal one is a rejection of the hypothesis that all changes in the slope of the nominal term structure arise from variations in inflation and thus that the slope of the real term structure remain invariant with time. Such a result can also therefore be interpreted as saying that changes in the slope of the nominal term structure are informative with respect to changes in the slope of the term structure of real interest rates.

IV. RESULTS

A. Ordinary least squares

Tables 1.1, 1.2 and 1.3 report the estimated OLS forecasting equations for inflation for six leading OECD countries and for a variety of forecasting horizons. The forecasting horizons chosen are dictated by the data available on interest rates. A full description of the data is given in Appendix 3 to the paper.
### Table 1.1
TERM STRUCTURE FORECASTING EQUATIONS FOR INFLATION (OLS)

#### UNITED STATES

<table>
<thead>
<tr>
<th>Months ( (m,n) )</th>
<th>( \hat{\alpha}_{m,n} )</th>
<th>( \hat{\beta}_{m,n} )</th>
<th>( R^2 )</th>
<th>( F(%) )</th>
<th>( X^2_A(12) )</th>
<th>( X^2_B(1) )</th>
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<td>((6,3))</td>
<td>-0.65</td>
<td>1.52</td>
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<td>76</td>
<td>0.4</td>
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<tr>
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<td>(16.80)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>-0.55</td>
<td>0.84</td>
<td>0.42</td>
<td>78</td>
<td>97</td>
<td>1.9</td>
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<td>(8.84)</td>
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<td>((60,3))</td>
<td>43.1</td>
<td>0.17</td>
<td>0.02</td>
<td>2.8</td>
<td>91</td>
<td>14.6</td>
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<tr>
<td>(a)</td>
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<tr>
<td>(b)</td>
<td>30.9</td>
<td>-0.09</td>
<td>0.01</td>
<td>1.4</td>
<td>48</td>
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<td>(1.20)</td>
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<td>((60,6))</td>
<td>43.7</td>
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<td>0.00</td>
<td>1.1</td>
<td>91</td>
<td>9.6</td>
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<td>0.03</td>
<td>2.5</td>
<td>48</td>
<td>9.6</td>
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#### GERMANY

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<tr>
<th>Months ( (m,n) )</th>
<th>( \hat{\alpha}_{m,n} )</th>
<th>( \hat{\beta}_{m,n} )</th>
<th>( R^2 )</th>
<th>( F(%) )</th>
<th>( X^2_A(12) )</th>
<th>( X^2_B(1) )</th>
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<td>0.07</td>
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<td>49.8</td>
<td>10.6</td>
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<td>((132,3))</td>
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<td>(b)</td>
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<td>-</td>
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<td>-</td>
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<tr>
<td>((132,60))</td>
<td>-6.38</td>
<td>0.32</td>
<td>0.90</td>
<td>462.3</td>
<td>40.3</td>
<td>7.5</td>
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<td>(a)</td>
<td>(4.32)</td>
<td>(21.50)</td>
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<tr>
<td>(b)</td>
<td>-</td>
<td>-</td>
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</tbody>
</table>

Notes: \( X^2 \) is a Lagrange multiplier test for residual serial correlation (see Godfrey, 1978). The tabulated \( X^2 \) value for 12 degree of freedom at the 5 and 1 percent significance levels are 21.03 and 26.22, respectively.
\( X^2 \) is a test for heteroskedasticity. The tabulated values at 5 and 1 percent level for one degree of freedom are 3.84 and 6.63. The figures in parentheses are absolute t values.
Table 1.2
TERM STRUCTURE FORECASTING EQUATIONS FOR INFLATION (OLS)

FRANCE

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>$\hat{\alpha}_{m,n}$</th>
<th>$\hat{\beta}_{m,n}$</th>
<th>$\bar{R}^2$</th>
<th>F(1/)</th>
<th>$\chi^2_{A}(12)$</th>
<th>$\chi^2_{B}(1)$</th>
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<tr>
<td>(3,1)</td>
<td>(a) -0.51</td>
<td>1.06</td>
<td>0.53</td>
<td>146</td>
<td>123.8</td>
<td>23.6</td>
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<tr>
<td></td>
<td>(b) -1.27</td>
<td>1.41</td>
<td>0.80</td>
<td>438</td>
<td>91.7</td>
<td>6.0</td>
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<td></td>
<td>(3.26) (12.09)</td>
<td>(10.21) (20.94)</td>
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<tr>
<td>(6,1)</td>
<td>(a) -1.56</td>
<td>1.11</td>
<td>0.51</td>
<td>134</td>
<td>125.6</td>
<td>22.8</td>
</tr>
<tr>
<td></td>
<td>(b) -3.63</td>
<td>1.49</td>
<td>0.80</td>
<td>431</td>
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<td>(6,3)</td>
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<tr>
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<td>(10.61) (19.70)</td>
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UNITED KINGDOM

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>$\hat{\alpha}_{m,n}$</th>
<th>$\hat{\beta}_{m,n}$</th>
<th>$\bar{R}^2$</th>
<th>F(1/)</th>
<th>$\chi^2_{A}(12)$</th>
<th>$\chi^2_{B}(1)$</th>
</tr>
</thead>
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<tr>
<td>(60,3)</td>
<td>(a) 107.9</td>
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<td>0.03</td>
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</tr>
<tr>
<td></td>
<td>(b) 6.30</td>
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<td>0.25</td>
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</tr>
<tr>
<td></td>
<td>(13.2) (2.00)</td>
<td>(0.97) (4.26)</td>
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<tr>
<td>(120,3)</td>
<td>(a) 284.9</td>
<td>-0.42</td>
<td>0.13</td>
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<td>0.2</td>
</tr>
<tr>
<td></td>
<td>(b) -</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(11.62) (3.79)</td>
<td>(10.26) (4.37)</td>
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</tr>
<tr>
<td>(120,60)</td>
<td>(a) 171.0</td>
<td>-0.47</td>
<td>0.17</td>
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<td>85</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>(b) -</td>
<td>-</td>
<td>-</td>
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</table>

Notes (cont.): Time periods for estimation.

United States: (a) 72M1-79M9, (b) 79M11-84M4
Germany: (a) 74M1-79M9, (b) 79M11-84M4
France: (a) 78M1-88M10, (b) 79M11-88M10
United Kingdom (a) 72M1-79M9, (b) 79M11-84M3
Italy: (a) 76M11-88M10, (b) 79M11-88M10
Canada: (a) 72M1-79M9, (b) 79M11-85M4
### Table 1.3
TERM STRUCTURE FORECASTING EQUATIONS FOR INFLATION (OLS)

#### ITALY

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>( \hat{\alpha}_{m,n} )</th>
<th>( \hat{\beta}_{m,n} )</th>
<th>( R^2 )</th>
<th>( F(/) )</th>
<th>( \chi^2_A(12) )</th>
<th>( \chi^2_B(1) )</th>
</tr>
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<tr>
<td>(6,3)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(a)</td>
<td>-0.92 (3.06)</td>
<td>1.04 (13.46)</td>
<td>0.56</td>
<td>181.1</td>
<td>117.3</td>
<td>0.2</td>
</tr>
<tr>
<td>(b)</td>
<td>-1.82 (5.03)</td>
<td>1.26 (13.22)</td>
<td>0.62</td>
<td>174</td>
<td>92.3</td>
<td>0.2</td>
</tr>
<tr>
<td>(12,3)</td>
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<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>(a)</td>
<td>-3.10 (2.51)</td>
<td>1.09 (10.35)</td>
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<td>107</td>
<td>131.9</td>
<td>3.6</td>
</tr>
<tr>
<td>(b)</td>
<td>-5.42 (4.35)</td>
<td>1.21 (11.63)</td>
<td>0.57</td>
<td>135</td>
<td>87.3</td>
<td>0.7</td>
</tr>
<tr>
<td>(12,6)</td>
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</tr>
<tr>
<td>(a)</td>
<td>0.22 (0.24)</td>
<td>0.79 (6.66)</td>
<td>0.24</td>
<td>44</td>
<td>135.9</td>
<td>19.9</td>
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<tr>
<td>(b)</td>
<td>-3.68 (4.28)</td>
<td>1.19 (11.27)</td>
<td>0.56</td>
<td>127</td>
<td>87.0</td>
<td>0.7</td>
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</tbody>
</table>

#### CANADA

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>( \hat{\alpha}_{m,n} )</th>
<th>( \hat{\beta}_{m,n} )</th>
<th>( R^2 )</th>
<th>( F(/) )</th>
<th>( \chi^2_A(12) )</th>
<th>( \chi^2_B(1) )</th>
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<tbody>
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<td>(24,3)</td>
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</tr>
<tr>
<td>(a)</td>
<td>13.96 (11.35)</td>
<td>0.27 (3.12)</td>
<td>0.09</td>
<td>9.7</td>
<td>88.6</td>
<td>5.2</td>
</tr>
<tr>
<td>(b)</td>
<td>-2.69 (1.09)</td>
<td>0.68 (6.25)</td>
<td>0.30</td>
<td>39.1</td>
<td>84.2</td>
<td>1.4</td>
</tr>
<tr>
<td>(48,3)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>29.6 (13.7)</td>
<td>0.36 (5.73)</td>
<td>0.26</td>
<td>32.8</td>
<td>89.0</td>
<td>17.2</td>
</tr>
<tr>
<td>(b)</td>
<td>13.44 (2.23)</td>
<td>0.20 (2.02)</td>
<td>0.05</td>
<td>4.07</td>
<td>60.5</td>
<td>3.0</td>
</tr>
<tr>
<td>(48,24)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>14.15 (6.46)</td>
<td>0.50 (4.62)</td>
<td>0.18</td>
<td>21.4</td>
<td>85.6</td>
<td>13.2</td>
</tr>
<tr>
<td>(b)</td>
<td>12.60 (5.60)</td>
<td>-0.04 (0.56)</td>
<td>-0.01</td>
<td>0.3</td>
<td>56.1</td>
<td>3.2</td>
</tr>
</tbody>
</table>
For the United States, the estimated value of $\beta_{m,n}$ is positive and significantly different from zero for the 6- and 3-month treasury bill combination for both the pre- and post-October 1979 monetary policy regimes \((6)\). It is significantly greater than one for the pre-October 1979 interest rate targeting regime while it is not significantly different from one for the post-October 1979 non-borrowed reserves targeting regime. Using the 5-year (60 months) rate as the long rate with either the 3- or 6-month rate as the short rate yields either a zero or a theoretically perverse, but insignificant, value of $\beta_{m,n}$. In other words, the term structure determined by these pairs of rates failed to anticipate the rapid acceleration of inflation in the periods early 1972 to late 1974 and again in the late 1970s, nor the intervening rapid deceleration of inflation between the end of 1974 and the end of 1976. This result contrasts with that based on the 6- and 3-month treasury bill rates over the same period. The forecast is in the correct direction but is in excess of the value of one suggested by the null hypothesis. For the post-October 1979 period $\beta_{m,n}$ is not significantly different from one. Thus, for the United States, these results would suggest that the term structure for maturities at the short end of the maturity spectrum, specifically 3- and 6-month treasury bills, can forecast inflation over the corresponding future 3- to 6-month horizon. However, the slope of the term structure fails to forecast inflation when rates at the longer end of the maturity spectrum are employed as long rates. This result is not at all implausible. The more distant the future horizon the greater the scope for unpredictable shocks to influence the actual outcome for future inflation. For the slope of the yield curve formed by the 5-year government bond rate, as the long rate, to have been a good predictor of changes in the rate of inflation in the 1970s would have required bond holders to have been able to predict, not only the first and second oil crises, but also general government policy reaction to these events. For the 1980s, bondholders would have to have been able to predict the disinflationary Federal Reserve policy of the early 1980s. They would, furthermore, have had to be able to make predictions of these events approximately 5 years in advance of their occurrence.

This inability of the term structure to predict future changes in the inflation rate when bond yields from the longer end of the maturity spectrum are employed is a general feature of the results obtained here across countries. The one exception is Germany for which the 5- and 11-year government bond rates of interest relative to the 3-month Fibor Rate \((7)(8)\) as the short rate have considerable predictive ability for future inflation. Except for the pre-October 1979 period for the \(60,3\) combination the long-short differential attracts a $\beta_{m,n}$ differential which is significantly different from both zero and one.

Tests for France were performed using 1-, 3- and 6-month Fibor rates. In the absence of long time series, tests were confined to the period January 1978 to October 1988. $\beta_{m,n}$ is insignificantly different from one for all three \((m,n)\) Fibor rate combinations over the January 1978-October 1988 interval. The $\beta_{m,n}$ coefficient is greater than one when estimation is confined exclusively to the post-October 1979 regime.

There is only one "non-perverse" result for the United Kingdom. The long (5-year government security rate) -- short (3-month treasury bill rate) differential had some forecasting power for future inflation for the period November 1979 to March 1984. For the pre-October 1979 period, using either the 5-year or 10-year government security rate as the long rate yields significantly negative coefficients for $\beta_{m,n}$. 
No long rate of interest was available for Italy where the long-term government security market is not currently developed due to the progressive shortening of government debt. The 3-, 6- and 12-month treasury bill rates are only available from November 1976. Using this date as the start date for estimation and October 1988 as the end date yields results which are very similar to those reported above for France. For no combination of short and long rates is the $\beta_{n,n}$ parameter estimate significantly different from one for the earlier period. If attention is confined exclusively to the post-October 1979 period, $\beta_{n,n}$ is found to be significantly greater than one at the 5 per cent level for the (6,3) and (12,3) interest rate combinations and greater than one at the 10 per cent level for the (12,6) rate combination. This is another aspect in which the results are similar to those obtained for France.

The Canadian results suggest good forecasting power for the term structure particularly over the pre-October 1979 estimation period where the $\beta_{n,n}$ parameter estimate lies between zero and one for all three maturity combinations. For the post-October 1979 period, $\beta_{24,3}$ increases considerably while remaining significantly different from both zero and one. $\beta_{48,3}$ remains just significant for the second period while $\beta_{48,3}$ becomes insignificantly different from zero.

B. Further econometric considerations

A problem of "overlapping observations" can arise in models with forward-looking expectations whenever the sampling interval is finer than the forecast horizon. Forecast errors are not known until the forecast horizon is reached. Referring back to equation [1], we see that the forecast error term $\varepsilon_{m,t}$ are not realised until period $t+m$. Rational expectations can only rule out any correlation between the forecast error (realised at $t+m$) and the variables entering the conditioning information set (at $t$) which includes forecast errors realised at $t$ or earlier. They cannot rule out autocorrelation in forecast errors realised from $t+1$ to $t+m$, since they do not enter the conditioning information set. Hence the composite error term is likely to follow a MA process of order $(m-1)$. The consequence is the familiar efficiency loss of OLS due to serially-autocorrelated residuals (9). This shows up in virtually all the estimated equations in Table 1 as reflected in the very high values of the $X^2\_\Lambda$ tests for autocorrelation.

Another econometric problem arises from the fact that, unless the real rate is constant, nominal interest rates will not be independent of the error term (that includes real interest rates). To obtain consistent parameter estimates some form of instrumental variable estimation procedure is called for.

Because contemporaneous forecasting errors for different term structure forecasting equations are likely to be highly correlated, SURE estimation is likely to enhance the efficiency of parameter estimation.

The literature on estimation of models containing rational expectations has proposed three basic different approaches.

i) A full-information method (see for example Hansen and Sargent, 1980, 1982) consisting of the estimation of the complete
simultaneous system. The model solution is used to generate the expectational variables, giving rise to cross equation parameter constraints that assure consistency. The efficiency gains of this approach must be weighted against the difficulties in estimating a (possibly) large system, and against the risk the mis-specification in any equation will lead to inconsistent estimates of all the parameters.

ii) The unobservable variable approach does not impose a unique (convergent) solution to the rational expectation model, but treats expectational variables as unobservable state variables and applies Kalman filtering techniques. This approach is useful when testing for bubble-like phenomena, so that one does not want to rule out non-convergent solutions in the first place (Burmeister, E., Wall, K.D., 1982). This is not the case here.

iii) The most popular (and "easy") method is the limited information approach proposed by McCallum (1976). This consists of substituting actual future for expected values in the equations, and using instrumental variables to take care of the non-independence of those regressors not entering the conditioning information set and the "composite" error term, that now includes structural disturbances as well as forecast errors.

The McCallum technique however, is not easy to extent in order to handle the case of auto-correlated structural disturbances as occurs in the case in hand. Standard ways of correcting for autocorrelation, as the generalised two stage least squares proposed in Theil (1961), will make things "worse" as they would re-introduce inconsistency through the "filtering" of the instrumental variables and of the "composite" error term (see Cumby, Huizinga and Obstfeld, 1983). The two-step-two-stage least squares (2S2SLS) estimator employed here is designed to achieve efficient estimates by removing autocorrelation in residuals without loosing consistency in a rational expectation context. It is consistent, asymptotically normal and asymptotically efficient in the class of "generalised method of moments" estimators developed by Hansen (1982). All that is required is the existence of some instruments that are "predetermined" with respect to the error term, and that serial correlation in the model dies out in a finite time (hence the need to remove the AR component from the error term by quasi differencing).

The first step consists in obtaining a consistent estimate of the system by applying instrumental variables.

From this one recovers a consistent estimate of the variance-covariance matrix of residuals, and then applies GLS to the system (second step). Hence, this estimator will account for serial correlation in the error terms as well as cross-equation correlation in disturbances, while preserving consistency through instrumental variables. Furthermore, the technique accounts for heteroskedastic residuals (a problem in only a few of the equations as indicated by the $\chi^2_B$ statistic in Table 1) when obtaining an estimate of the variance-covariance matrix in the first step. It employs the weights for the sample moments suggested by Newey and West (1986) for the case in which this matrix is not positive definite.
C. Two-step two-stage least squares results

The 2S2SLS results (10) are reported in Tables 2.1, 2.2 and 2.3. For the United States all the $\beta_{m,n}$ parameter estimates lie between zero and one but, in common with the OLS estimates, only the $\beta_{o,j}$ coefficient is significantly different from zero. The slope of the nominal term structure at the short end of the maturity spectrum contains information about future inflation over the time interval from 3 to 6 months ahead. However, since $\beta_{e,3}$ is also significantly different from one the slope of the nominal term structure also contains information about the term structure of real interest rates. This contrasts with a result reported in Mishkin (1988) that "the term structure for maturities of six months or less contains almost no information about the path of future inflation" (p. 16, op. cit.). When the 5-year rate is used as the long rate, however, the results suggest that the term structure of nominal rates only contains information about the real term structure.

For Germany, the $\beta_{m,n}$ parameter estimates are not greatly altered from the corresponding OLS estimates with both $\beta_{132,3}$ and $\beta_{132,60}$ continuing to lie between zero and one again reflecting information in the term structure for both the real term structure and future inflation. As with the OLS estimates, the German 2S2SLS results are again the only ones which suggest forecasting power for the term structure for inflation over very long time horizons.

The estimated $\beta_{m,n}$ parameters are all lower and their corresponding estimated standard errors are all much higher for 2S2SLS compared to OLS for France. Inferences from these results are also qualitatively different in that the 2S2SLS $\beta_{m,n}$'s all lie between zero and one with both $\beta_{e,1}$ and $\beta_{e,3}$ significantly different from zero only at the 10 per cent level. A curious feature of these results is the large value of $\alpha_{3,1}$ (=47.1). This is hard to rationalise as the difference of the means of the 3- and 6-month real rates. For the United Kingdom the significantly negative $\beta_{m,n}$'s obtained with OLS estimation have disappeared to be replaced by insignificant $\beta_{m,n}$'s.

Except for the a priori implausible value for $\beta_{132,3}$ (=2.39) the results for Italy suggest considerable forecasting ability for inflation. $\beta_{e,3}$ is not significantly different from one while $\beta_{e,3}$ lies half-way between zero and one. The 2S2SLS results for Canada also indicate some forecasting power for future inflation which, not unreasonably, fades with lengthening maturity and forecast horizons.

The efficiency gains from SURE estimation can be appreciated by noting the huge cross-equation correlations between the errors in different equations.

D. General mechanisms behind these findings

Where the slope of the nominal yield curve fails to predict future inflation it, by definition, reflects variation in the slope of the (ex post) real yield curve. Changes in the real term structure affect the real economy. Therefore, the following general pattern could be tentatively inferred from the cross-country results reported: changes in the slope of the term structure, formed by using a long interest rate from the long end of the maturity spectrum, reflect exclusively changes in the real term structure (except for Germany) and may, therefore, forecast future output changes: changes in the slope of the term structure based on "long" and short rates
### Table 2.1
TERM STRUCTURE FORECASTING EQUATIONS FOR INFLATION
(2S2SLS ESTIMATES)

#### UNITED STATES

Time period for estimation: 72M1-79M4

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>$\hat{\alpha}_{m,n}$</th>
<th>$\hat{\beta}_{m,n}$</th>
<th>$\hat{\rho}_1$</th>
<th>$R^2$</th>
</tr>
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<tr>
<td>(6,3)</td>
<td>0.86</td>
<td>0.49</td>
<td>1.11</td>
<td>.979</td>
</tr>
<tr>
<td></td>
<td>(2.76)</td>
<td>(3.88)</td>
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</tr>
<tr>
<td>(60,3)</td>
<td>0.04</td>
<td>0.32</td>
<td>0.99</td>
<td>.990</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(1.07)</td>
<td>(135.74)</td>
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</tr>
<tr>
<td>(60,6)</td>
<td>0.52</td>
<td>0.35</td>
<td>0.98</td>
<td>.989</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(1.09)</td>
<td>(126.17)</td>
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</tr>
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</table>

Cross equation correlation matrix

\[
\begin{pmatrix}
1.00 \\
.58 \\
.55
\end{pmatrix}
\begin{pmatrix}
1.00 & .58 & .55 \\
.58 & 1.00 & 1.00 \\
.55 & 1.00 & 1.00
\end{pmatrix}
\]

$\hat{\varepsilon}_{1t} \sim \text{ARMA (1,5)}, \hat{\varepsilon}_{2t} \sim \text{ARMA (1,8)}, \hat{\varepsilon}_{3t} \sim \text{ARMA (1,8)}$

#### GERMANY

Time period for estimation: 74M1-79M4

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>$\hat{\alpha}_{m,n}$</th>
<th>$\hat{\beta}_{m,n}$</th>
<th>$\hat{\rho}_1$</th>
<th>$R^2$</th>
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</thead>
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<tr>
<td>(60,3)</td>
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<td>0.87</td>
<td>.948</td>
</tr>
<tr>
<td></td>
<td>(12.26)</td>
<td>(4.47)</td>
<td>(28.89)</td>
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</tr>
<tr>
<td>(132,3)</td>
<td>13.40</td>
<td>0.27</td>
<td>0.81</td>
<td>.958</td>
</tr>
<tr>
<td></td>
<td>(2.51)</td>
<td>(5.86)</td>
<td>(20.99)</td>
<td></td>
</tr>
<tr>
<td>(132,60)</td>
<td>-20.68</td>
<td>0.53</td>
<td>0.77</td>
<td>.958</td>
</tr>
<tr>
<td></td>
<td>(4.73)</td>
<td>(10.33)</td>
<td>(18.57)</td>
<td></td>
</tr>
</tbody>
</table>

Cross equation correlation matrix

\[
\begin{pmatrix}
1.00 \\
-0.90 \\
-0.87
\end{pmatrix}
\begin{pmatrix}
1.00 & -0.90 & -0.87 \\
-0.90 & 1.00 & 0.99 \\
-0.87 & 0.99 & 1.00
\end{pmatrix}
\]

$\hat{\varepsilon}_{1t} \sim \text{ARMA (1,5)}, \hat{\varepsilon}_{2t} \sim \text{ARMA (1,5)}, \hat{\varepsilon}_{3t} \sim \text{ARMA (1,5)}$
Table 2.2
TERM STRUCTURE FORECASTING EQUATIONS FOR INFLATION (2S2SLS ESTIMATES)

FRANCE
Time period for estimation: 79M1-88M10

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>$\hat{\alpha}_{m,n}$</th>
<th>$\hat{\beta}_{m,n}$</th>
<th>$\hat{\rho}_1$</th>
<th>$R^2$</th>
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<tbody>
<tr>
<td>(3,1)</td>
<td>47.10</td>
<td>0.57</td>
<td>1.00</td>
<td>.992</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(2.31)</td>
<td>(44.28)</td>
<td></td>
</tr>
<tr>
<td>(6,1)</td>
<td>-2.46</td>
<td>0.38</td>
<td>0.99</td>
<td>.998</td>
</tr>
<tr>
<td></td>
<td>(0.49)</td>
<td>(1.73)</td>
<td>(110.02)</td>
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</tr>
<tr>
<td>(6,3)</td>
<td>-0.32</td>
<td>0.53</td>
<td>0.97</td>
<td>.994</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(1.76)</td>
<td>(65.88)</td>
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Cross equation correlation matrix

\[
\begin{bmatrix}
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0.43 & 1.00 & 0.54 \\
0.57 & 0.54 & 1.00
\end{bmatrix}
\]

$\hat{\varepsilon}_{1t} \sim \text{ARMA (1,2)}, \hat{\varepsilon}_{2t} \sim \text{ARMA (1,5)}, \hat{\varepsilon}_{3t} \sim \text{ARMA (1,5)}$

UNITED KINGDOM
Time period for estimation: 72M1-79M3

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>$\hat{\alpha}_{m,n}$</th>
<th>$\hat{\beta}_{m,n}$</th>
<th>$\hat{\rho}_1$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(60,3)</td>
<td>-27.32</td>
<td>-0.01</td>
<td>0.99</td>
<td>.999</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.32)</td>
<td>(17.13)</td>
<td></td>
</tr>
<tr>
<td>(120,3)</td>
<td>229.6</td>
<td>0.01</td>
<td>1.04</td>
<td>.999</td>
</tr>
<tr>
<td></td>
<td>(6.66)</td>
<td>(0.28)</td>
<td>(48.10)</td>
<td></td>
</tr>
<tr>
<td>(120,6)</td>
<td>125.88</td>
<td>0.03</td>
<td>1.04</td>
<td>.999</td>
</tr>
<tr>
<td></td>
<td>(4.78)</td>
<td>(0.88)</td>
<td>(61.49)</td>
<td></td>
</tr>
</tbody>
</table>

Cross equation correlation matrix

\[
\begin{bmatrix}
1.00 & -0.27 & -0.39 \\
-0.27 & 1.00 & 0.98 \\
-0.39 & 0.98 & 1.00
\end{bmatrix}
\]

$\hat{\varepsilon} \sim \text{ARMA (1,5)}, \hat{\varepsilon}_t \sim \text{ARMA (1,5)}, \hat{\varepsilon}_t \sim \text{ARMA (1,5)}$
Table 2.3
TERM STRUCTURE FORECASTING EQUATIONS FOR INFLATION
(2S2SLS ESTIMATES)

ITALY

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>$\hat{\alpha}_{m,n}$</th>
<th>$\hat{\beta}_{m,n}$</th>
<th>$\hat{\rho}_1$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6,3)</td>
<td>-0.01</td>
<td>0.92</td>
<td>0.99</td>
<td>0.886</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(3.16)</td>
<td>(256.10)</td>
<td></td>
</tr>
<tr>
<td>(12,3)</td>
<td>0.26</td>
<td>2.39</td>
<td>1.03</td>
<td>0.960</td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td>(8.16)</td>
<td>(39.37)</td>
<td></td>
</tr>
<tr>
<td>(12,6)</td>
<td>0.02</td>
<td>0.50</td>
<td>1.01</td>
<td>0.973</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(4.71)</td>
<td>(55.01)</td>
<td></td>
</tr>
</tbody>
</table>

Cross equation correlation matrix

\[
\begin{pmatrix}
1.00 \\
0.80 & 1.00 \\
0.85 & 0.93 & 1.00
\end{pmatrix}
\]

$\hat{\varepsilon}_{1t} \sim$ ARMA (1,6), $\hat{\varepsilon}_{2t} \sim$ ARMA (1,6), $\hat{\varepsilon}_{3t} \sim$ ARMA (1,6)

CANADA

<table>
<thead>
<tr>
<th>Months (m,n)</th>
<th>$\hat{\alpha}_{m,n}$</th>
<th>$\hat{\beta}_{m,n}$</th>
<th>$\hat{\rho}_1$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(24,3)</td>
<td>14.19</td>
<td>0.25</td>
<td>1.04</td>
<td>.993</td>
</tr>
<tr>
<td></td>
<td>(6.49)</td>
<td>(2.58)</td>
<td>(64.21)</td>
<td></td>
</tr>
<tr>
<td>(48,3)</td>
<td>37.26</td>
<td>0.14</td>
<td>1.02</td>
<td>.997</td>
</tr>
<tr>
<td></td>
<td>(7.06)</td>
<td>(2.21)</td>
<td>(134.21)</td>
<td></td>
</tr>
<tr>
<td>(48,24)</td>
<td>25.79</td>
<td>0.08</td>
<td>1.01</td>
<td>.997</td>
</tr>
<tr>
<td></td>
<td>(2.43)</td>
<td>(1.11)</td>
<td>(166.60)</td>
<td></td>
</tr>
</tbody>
</table>

Cross equation correlation matrix

\[
\begin{pmatrix}
1.00 \\
0.81 & 1.00 \\
0.32 & 0.80 & 1.00
\end{pmatrix}
\]

$\hat{\varepsilon} \sim$ ARMA (1,5), $\hat{\varepsilon} \sim$ ARMA (1,5), $\hat{\varepsilon} \sim$ ARMA (1,5)
taken from the short end of the maturity spectrum are informative about both future inflation and the real term structure and, via the latter, future real output.

Some further caveats relating to our results need to be stated. Our forecasting equations make no attempt to explain the joint determination of nominal and real interest rates, as well of inflation rates.

Also, the fact that the longer maturity yields are to be thought of as "forward-looking" variables means, in general, that the slope of the yield curve will reflect unexpected as well as expected future disturbances (policies) in a different way. Imagine a "back-of-the-envelope" model (see Appendix 2 for a mathematical treatment) in which output is determined by aggregate demand, which, in turn, depends on the ex ante real long-term interest rate, in which the short-term nominal interest rate equilibrates the money market, in which inflation is determined by an augmented Phillips Curve and, finally, in which the expectations theory of the term structure holds. In such a model, the Fisher condition will hold exactly only in the stationary state. Thus we would expect an unanticipated monetary shock to cause the yield curve and inflation to move in the direction suggested by some of the empirical results, with the slope of the yield curve "underpredicting" inflation ($P_{m,n}<1$). See Figure 1 in Appendix 2.

However, the announcement of a future monetary expansion could well have the yield curve and inflation move in opposite directions (see Fig. 2 in Appendix 2); so one is left with the doubt that the actual estimates of the forecasting equation are not "structural", but simply reflect a particular configuration of shocks experienced over the estimation period.

Attempts by the monetary authorities to affect yields at different maturities by open-market operation might have introduced a considerable amount of "noise" into the data so that interest rates would reflect policy targets more than market expectations of inflation rates. The issue of credibility of policies and the openness of the economy should suggest no simple way of extrapolating from a single time series.

**V. SUMMARY**

The breakdown in the long-run relationship relating nominal money, on the one hand, to real output and prices, on the other, in the early 1980s has left monetary policy formulation in some OECD countries without a reliable anchor. Monetary aggregates are now being supplemented by an array of real and financial indicators in a more eclectic approach to policy formulation. One of the financial variables that attracts considerable attention in this respect is the term structure of interest rates, the shape of which has been advanced as an indicator of the markets' current expectation of future inflation. If these expectations are responding to actual current incipient inflationary pressures that have not yet become known through published price indices, then such an indicator would be extremely valuable. It would allow the monetary authorities to take pre-emptive action to prevent inflation emerging and becoming ingrained in peoples' expectations.
Testing of the forecasting power of the term structure for future inflation have been carried out here for six major OECD countries and for a variety of asset maturities. Careful attention had to be paid to econometric issues. To obtain consistent and relatively efficient parameter estimates in a context of rational expectations, overlapping observations, simultaneity and the likely contemporaneous cross-correlation in the forecast errors for different, but overlapping forecast horizons, called for the use of a special estimation procedure. Such a procedure has recently been proposed and was employed here. A general pattern emerges from these results. The term structure does have considerable forecasting ability but this fades as yields on assets of increasingly distant maturities are employed as "long" rates. The one exception is Germany for which the spread between the long (11-year maturity) and short (3-month maturity) has significant forecasting ability for inflation over an 11-year future horizon. For those countries that conform to the general pattern, the term structure at the longest end reflects variation in the real term structure and may thus contain useful information about the evolution of future output. For the United States, the results are somewhat at variance with those reported by Mishkin who concludes that "the term structure for maturities of six months or less contain almost no information about the path of future inflation" while he obtains considerable forecasting power using longer rates. However his "longer" rates relate to assets with 9 and 12 months to maturity. The one employed here, by contrast, is the yield on a 5-year (60 month) to maturity asset.

The current practice of using the spread between a very long rate (typically a 10-year government bond rate) and the 3-month treasury bill rate, as the short rate, to provide an indicator, albeit tentative, of the markets' expectation of future inflation may be suspect. The results obtained here would suggest that yields taken from the shorter end of the maturity spectrum (in the region of, say, three months to two years) are more reliable indicators of the markets' expectation. The use of such yields would also have the advantage of conforming more closely to the likely policy horizons of central banks.
APPENDIX 1: THE MISHKIN MODEL

Mishkin (1988) builds on Fama's (1975) study on variations in the level of nominal interest rates as forecaster of future inflation rate movements. According to the Fisher equation expected inflation over m periods is equal to the m-period nominal interest rate minus the m-period real interest rate, i.e.:

\[ E_t \pi_{m,t} = i_{m,t} - r_{r_{m,t}} \]  \hspace{1cm} (A1)

where \( E_t \pi_{m,t} \) is the expectation of inflation over the m period horizon at time \( t \), \( i_{m,t} \) is the m-period nominal interest rate of time \( t \) and \( r_{r_{m,t}} \) the m-period ex ante real interest rate at time \( t \). It is assumed that inflationary expectations are generated rationally, i.e.:

\[ \pi_{m,t} = E_t \pi_{m,t} + \epsilon_{m,t} \]  \hspace{1cm} (A2)

where \( \epsilon_{m,t} \) is the forecast error over the full m periods. Substituting A2 into A1 and rearranging yields:

\[ \pi_{m,t} = \alpha_m + \beta_m i_{m,t} + \eta_{m,t} \]  \hspace{1cm} (A3)

where:

\[ \alpha_m = - \overline{r_{r_{m,t}}} \]
\[ \beta_m = 1 \]
\[ \eta_{m,t} = \epsilon_{m,t} - u_{m,t} \]
\[ u_{m,t} = r_{r_{m,t}} - \overline{r_{r_{m}}} \]

To examine the information in the term structure of interest rates, as opposed to the level of the interest rate, about future changes in the inflation rate, Mishkin substracts equation A3 for the n-period inflation rate from equation A3 for the m-period inflation rate which gives the following:

\[ \pi_{m,t} - \pi_{n,t} = \alpha_{m,n} + \beta_{m,n} [i_{m,t} - i_{n,t}] + \eta \]  \hspace{1cm} (A4)

where:

\[ \alpha_{m,n} = \overline{r_{r_{n}}} - \overline{r_{r_{m}}} \]
\[ \beta_{m,n} = 1 \]
\[ \eta = \epsilon_{m,t} - \epsilon_{n,t} - u_{m,t} + u_{n,t} \]
\[ u_{n,t} = r_{r_{n,t}} - \overline{r_{r_{n}}} \]

and

\[ \epsilon_{n,t} \] the forecast error over the n-period horizon.

As the estimated value of \( \beta_{m,n} \) varies from zero to one the information content of the yield curve for future inflation increases while its information content for the term structure of real interest rates decreases.
APPENDIX 2: A SIMPLE DYNAMIC MACRO MODEL

Here we present a simple way to look at the relationship between the yield curve, inflation and output on the basis of a stylized dynamic model, where "smart" arbitrageurs interact with "dull" labour/goods markets.

The model is described by equations (A.1) to (A.6)

\[ y = -\sigma(R - \pi^e) + g \]  
\[ l = m - p = \phi y - \lambda r \]  
\[ \dot{\pi} = \psi y + \pi^e \]  
\[ e = R = R[R - r] \]  
\[ \pi^e = \Theta \equiv \dot{\pi} \]

where \( y, R, r, \pi^e, g, m, \psi, \lambda, \Theta \) represent output, the long and short interest rates, inflation, a fiscal shock, the price level, the money stock and its rate of growth. A bar indicates a steady-state value and a dot a time derivative. All variables, except \( r \) and \( R \), are expressed as logarithmic deviations from steady-state values.

Equation (A.1) represents aggregate demand. This is assumed to be a function of the expected real long-term interest rate, and a positive function of a fiscal shock. Equation (A.2) represents money market equilibrium, where money demand negatively depends on the short-term interest rate. Equation (A.3) can either be viewed as an "augmented" Phillips curve, or as a Lucas type aggregate supply schedule, with output deviations from "normal", \( \bar{y} = 0 \), positively related to price-surprises, \( \dot{\pi} - \pi^e \). Equation (A.4) is a linearized version of the arbitrage condition,

\[ r = R - R^e/R \]

around the steady state; equilibrium in the capital market requires the return on a short-term bond, \( r \), to be equal to the return on a consol (yielding 1$ per period), \( R \), plus expected capital gains, \( (-dR^e/dt)/R \). Provided the following transversality condition holds,

\[ \lim_{s \to +\infty} R(s)e^{-\tilde{R}(s-t)} = 0 \]
we can solve (3.4) forward obtaining the "expectations theory":

\[ R(t) = R \int_{t}^{\infty} r(s) e^{-\bar{\gamma}(s-t)} ds \]  \hspace{1cm} (A.6)

The current long-term interest rate must be proportional to the discounted stream of future expected short-term interest rates (we drop the e-superscript from now on).

In equation (A.5) we assume that while financial markets are characterized by perfect foresight, labour/goods market can only locate the "core" inflation rate, \( \pi = \Theta \), but not the medium-run adjustment path.

It is easily checked that the steady state of this economy is described by:

\[ \dot{\bar{\gamma}} = -\lambda \Theta (\lambda / \sigma) g \quad ; \quad \bar{r} = \Theta_+ (1 / \sigma) g = \bar{r} \]  \hspace{1cm} (A.7)
\[ \dot{\pi} = \pi = \Theta \quad ; \quad \dot{\gamma} = 0 \]  \hspace{1cm} (A.8)

The Fisher equation holds for interest rates in the long run, while nominal prices grow in line with money supply (homogeneity) and monetary and real shock do not affect the natural level of output (neutrality). Real balances are cut by an inflation tax, from (A.7).

First consider the case when we have an (unanticipated permanent) increase in the money growth (inflation expectations). The long-term interest rate jumps up to \( A \) in Fig. 1 in response to the expectation of higher future short rates, hence the yield curve slopes upward and capital losses \( (dR/dt>0) \) must be expected to maintain equilibrium in the capital market. In this case, therefore, the long rate must undershoot its higher new long-run value, so that the (expected) real long-term rate will be cut. Output will be raised consequently while inflation will overshoot, in the short run. On impact, there will be an excess demand for money, so that the short-term rate will increase, validating the expectations. Over time, real money balances will decrease putting an upward pressure on short rates, \( R \) will converge to its higher level, thus raising the real long-term rate and bringing output back to its "natural" level.

At the time of the shock, the term structure slope will steepen, although less than proportionately to the increase in inflation. However, it is easy to see that, in the case of a the announcement of a future monetary shock, a steepening of the yield curve portrays a short-run reduction in inflation, although inflation will still be higher in the long run (see Fig. 2). The reasons is that since the monetary shock does not materialise immediately, while the long-term rate reacts when the news hit the economy, here the real long-term rate increases and pushes the economy into recession.
Fig. 1

Graph showing

- $R$
- $E_t$
- $i' = 0$
- $SS'$
- $i = 0$
- $R' = 0$

Variables:

- $n, \theta$
- $\hat{e}(t)$
- $\hat{e} < 1$
- $R$
- $\frac{dR(t)}{d\theta} < 1$
- $y$
- $\frac{dy(t)}{d\theta} = \frac{dR(t)}{d\theta}$
- $r$
- $\frac{dr(t)}{d\theta} < 1$
- $p$
- $\frac{dp(t)}{d\theta} > 1$
- $R - r$
APPENDIX 3: DESCRIPTION OF DATA

Some of the internally available OECD interest rate data are not ideal for the purposes for which they are being employed. The severity of these data limitations vary from country to country. From the United States, at one extreme, where four reasonably good time series are available to Japan, at the other extreme, for which sufficiently good interest rate data were not available to do even a single test. A general problem which characterises the data for most countries is the following: the reported monthly interest rate are, for the most part, monthly averages of daily data. For conformity with price and output data, end-month figures may be preferable. However, as noted by Mishkin (1988), the appropriate dating for the CPI in a particular month is not clear (at least for the United States) since price quotations on the component items of the index are collected at different times during the month.

United States: The following four interest rate series are available for the United States, three of which are used in the tests performed: yields on 3- and 6-month treasury bills and on 5- and 10-year-to-maturity U.S. government bonds. The treasury bill yields are monthly averages computed from closing-bid quotations reported daily. Yields are calculated on a bank discount rate basis and, therefore, only approximate a true yield. The government bond yields are also monthly averages of daily data for 5- and 10-year notes and bonds on the secondary market.

Germany: Given data limitations tests were only possible by ignoring the private-public sector division. Thus the results reported for Germany may be affected by different perceptions of default risk on public and private sector bonds. 3-month Fibor (Frankfurt interbank offered rates) are available for the full-time period of interest. However, 6-month Fibor data are only available from September of 1985 and the time series for 3-month treasury bills ends in November 1981. Thus the only short maturity yield available is that for the 3-month Fibor which is a private sector economy yield. The yields on public sector bonds on the secondary market with (3-7) and (7-15) years maturity are the only long bond yields available. There is a private sector industrial bond yield available but the maturity to which it refers is not available.

The group yields for the type of government securities mentioned above are weighted by the outstanding amount of bonds included in the calculation. Yields are generally monthly averages of daily data. However, up to January 1986 the yields on government bonds outstanding were based on the yield on four bank week return dates in each month.

France: The yields used here relate to private sector assets. 1-, 3-, and 6-month PIBOR (Paris interbank offer rates) are used. These are again monthly average rates. The yield on private sector bonds on the secondary market is also considered. This has a term to maturity of at least seven years. It is not used in the empirical tests herein, however.

United Kingdom: Three interest rate series are employed in tests for the United Kingdom. The 91-day treasury bill rate is, until August 1977, the weighted average rate of discount on allotment for 91-day bills at the weekly tender on the last Friday of the month. Since then, the rates are monthly
averages of weekly data. The government security yields are gross redemption yields for selected maturities, derived from fitting observed yield maturity curves to a mathematical model of the government bond markets. Taxes are ignored. The figures are averages of Wednesdays until February 1980; from March 1980 until December 1981, figures are the average of all observations (3 per week); from January 1982 figures are averages of all working days.

**Canada:** The 3-month treasury bill rate is a weighted average of the yields on successful bids for 3-month treasury bills sold by tender the last Thursday of the month. As from April 1981, the data are monthly averages of the Thursday rates. The government bond yields on the secondary market employed are unweighted averages of yields for issues other than guaranteed issues. The quotes used are mid-points between bid and asked prices at the close of business on the last Wednesday of the month. As from April 1981, the data are monthly averages of Wednesday rates.

**Italy:** The 3-, 6- and 12-month treasury bill yields employed in the tests are end-of-month rates.

All interest rate data were obtained from the Directorate for Financial, Fiscal and Enterprise Affairs of the OECD while the source for data on inflation (based on the GNP/GDP deflator) and output is the Economics and Statistics Department of the OECD.
NOTES

1. Among other indicators that have been cited are commodity prices, the exchange rate, credit aggregates, cyclical indicators of real activity, as well as, of course, money aggregates.


3. There are numerous examples. For a recent selection of the literature see, for example, Jones and Roley (1983), Shiller, Campbell and Schoenholtz (1983), Mankiw (1986) and Hardouvelis (1988).

4. The risk measure employed by Mankiw (1986) is the absolute value of the percentage first forward difference in the long bond yield. Another measure frequently used is a moving standard deviation of short rates (see, for example, Modigliani and Shiller, 1973 and, more recently, Jones-Roley, 1983). Other measures of interest rate variability have been used by Mishkin (1982) and Bodie, Kane and MacDonald (1984). These measures tend to be ex post measures of volatility and to that extent are less than ideal.

5. Jones and Roley (1983) test for four such variables: treasury bill supplies, the unemployment rate, a risk variable and foreign holdings (specifically foreign central bank holdings of U.S. treasury securities), in a test of the joint rational expectations-expectations theory of the term structure hypothesis. The authors find that the last-mentioned variable has a significant effect at the 5 per cent level in a model that rejects the joint null hypothesis. Their rationale for this effect is as follows. Foreign central banks have a "preferred habitat" in three-month U.S. treasury bills. When investors observe high foreign holdings of treasury bills, they expect further purchase of this security in the next period and thus a lower short-term interest rate and an increase in the six-month holding-period yield. This implies that the risk of a capital loss is reduced, which lowers the required term premium. Shiller, Campbell and Schoenholtz (1983) also test to see if the relative volume of trade in securities at either end of the maturity spectrum succeeds in explaining the term premium. They find that the volume variable does help to explain excess holding returns and indeed it displaces the risk variable (measured as a moving standard deviation of the short interest rate) which was significant in an excess return equation when entered without the volume variable.

6. Note that using the 10-year government bond rate is not possible in the post-October 1979 regime since the calculation of the cumulative inflation rate over the following 10 years for April 1979 uses up all the remaining observations to April 1989.

7. The yield on the secondary market on public sector bonds are quoted for (3-7) and (7-15) year maturities. In both cases the mid-point of these ranges was chosen as the actual term to maturity.
8. Data availability constraints dictated that tests could only be carried out for Germany by combining yields on public and private sector securities.

9. Of course, if the real interest rate is not constant, $u_{n,t}$ could follow a higher-order MA process than $(m-1)$. However, allowing for higher order processes does not qualitatively alter the results.

10. The data transformations required to obtain cumulative inflation over the following 60-month (5-year) horizon combined with the 2S2SLS estimation procedure involves such a large loss of observations that estimation over the post-October 1979 period is not feasible except for France and Italy where only relatively very short maturity assets are employed.
REFERENCES


