The Contribution of Regions to Aggregate Growth in the OECD

Enrique Garcilazo
Joaquim Oliveira Martins

https://dx.doi.org/10.1787/5k3tt0zzp932-en
This series is designed to make available to a wider readership selected studies on regional development issues prepared for use within the OECD. Authorship is usually collective, but principal authors are named. The papers are generally available only in their original language English or French with a summary in the other if available.

The opinions expressed in these papers are the sole responsibility of the author(s) and do not necessarily reflect those of the OECD or the governments of its member countries.

Comment on the series is welcome, and should be sent to either gov.contact@oecd.org or the Public Governance and Territorial Development Directorate, 2, rue André Pascal, 75775 PARIS CEDEX 16, France.

OECD Regional Development Working Papers are published on http://www.oecd.org/gov/regional/workingpapers

Applications for permission to reproduce or translate all or part of this material should be made to: OECD Publishing, rights@oecd.org or by fax 33 1 45 24 99 30.

© OECD 2013
THE CONTRIBUTION OF REGIONS TO AGGREGATE GROWTH IN THE OECD

by

Enrique Garcilazo and Joaquim Oliveira Martins

OECD, Regional Development Policy Division

Abstract

This paper investigates the contribution of regions to aggregate growth in the OECD. We find a great degree of heterogeneity in the performance of OECD TL3 regions and among the OECD regional typology (urban, intermediate and rural). While the distribution in GDP and GDP per capita growth rates follows an approximately normal distribution, the regional contributions to aggregate growth follow a power law, with a coefficient around 1.2 (in absolute terms). This implies that Few-Large (FL) regions contribute disproportionately to aggregate growth whereas Many-Small (MS) individual regions contribute only marginally. Nevertheless, because the number of these smaller regions is very large and the decay of their contribution to growth is slow (generating a fat tail distribution), their cumulated contribution is actually around 2/3 of aggregate growth. For the period 1995-2007, only 2.4% of OECD TL3 regions contribute to 27% of OECD GDP growth, but the remaining 97.6% corresponds to 73%. We also found that the distribution of growth rates by size follows a non-monotonic pattern, with the largest concentration of above average regional growth rates being concentrated for middle-sized regions. This heterogeneity suggests that the possibilities for growth seem to exist in many different types of regions.

1. Garcilazo: Enrique.Garcilazo@oecd.org; Oliveira: joaquim.oliveira@oecd.org OECD, Regional Development Policy Division, 2 Rue André Pascal, 75775 Paris Cedex 16. The authors would like to thank Giulia Ajmone-Marsan, Monica Brezzi, Philip McCann, Andrés Rodríguez-Pose, William Tompson and Paolo Veneri for helpful comments and discussions. The views expressed are those of the authors and do not reflect those of the OECD or its Member countries.
1. Introduction

Heterogeneity in regional performance is a stylized fact among OECD countries (OECD, 2009), both among similar type of regions (e.g. urban, intermediate and rural) and across regions within the same country. The fact that heterogeneity persists over time suggests that cumulative factors at the regional level yield significant differences in productivity and consequently income levels among regions (Acemoglu and Dell, 2009). This contrasts with the view that income differences will eventually even out as corrective market forces will induce firms to move to areas where labour is cheaper, or labour to move to areas where wages are higher. A wide body of literature has examined the main factors driving the performance of regions (for a recent literature review see Roberts and Setterfield, 2010) focusing mainly on endogenous factors yielding differences in regional performance and hence the observed heterogeneity.

Increased polarisation of regions is consistent with previous studies documenting the existence of two opposing global trends, namely globalisation and localisation (McCann, 2008). At the international scale, Philip McCann argues, economic growth is being dominated by networks of particular major urban centres, while at the local scale production increases as distance-transport costs fall to very low levels in the traditional core periphery setting within the New Economic Geography (Krugman and Venables, 1995).

The debate about the regional structure of economic activities in one country and its overall performance remains unsettled. Is it better to concentrate economic activities in order to benefit fully from agglomeration economies or to have more evenly distributed contributions to aggregate growth? There are opposite views concerning this link. World Bank (2009), on the one hand, recommends focusing on few largest cities, viewed as main engines of national growth, as critical pillars for development, while OECD (2009, 2011) defended the view that growth can be found in many different types of regions and these should also contribute and participate in the development process. The implicit assumption being that, even if their potential can be different, there are always some untapped sources of growth in every region (see also OECD, 2012). Perhaps the most important one is related to the convergence potential existing in many lagging regions. In this context, a recent third view promoted by the Inter-American Development Bank2 (IADB) and the McKinsey Global Institute (Dobbs et al. 2011) places more emphasis on the role of intermediate regions. Along these lines, should economic development policies be ‘space-blind’ and let market forces alone determine the location of economic activities, or should policies contain ‘place-based’ elements because regional growth factors can only be fully mobilised in this way? Addressing these questions requires a full research programme, as argued in Rodríguez-Pose (2010).

This paper only aims at providing a first step in this direction by analysing how different types of regions contribute to aggregate growth. Accordingly, the paper focuses on the regional contributions to aggregate growth rather than regional growth rates. Contributions to growth are the product of two components: a size component, measured by the GDP of regions, and a growth component, measured by their subsequent growth rate. By construction, the sum of all regional contributions to growth is equal to the observed aggregate growth rate between two periods.

The main results can be summarised as follows. While the distribution in GDP and GDP per capita growth rates follows an approximately normal distribution, the regional contributions to aggregate growth follow a power law, with a coefficient of around 1.2 (in absolute terms). This implies that a few large (FL) regions contribute disproportionately to aggregate growth whereas many small regions (MS) contribute only marginally. Nevertheless, because the number of these smaller regions is very large, their cumulated

---

contribution to growth is actually dominant. For the period 1995-2007, only 2.4% of OECD TL3 regions contribute to 27% of OECD GDP growth, but the remaining 97.6% corresponds to 73%. Moreover, despite the important contribution of large regional hubs to national growth, we find that over time, during our period of analysis from 1995-2007; their contribution is declining vis-à-vis second tier regions. Our analysis also confirms scale-free properties in our growth contributions given that the magnitudes of the growth contributions by FL and MS are roughly maintained for more aggregated or for country-level regional data.

The structure of the paper is as follows. The next section examines stylised facts concerning regional heterogeneity of regional growth rates and their persistence over time across a large set of large (TL2-level) and smaller (TL3-level) OECD regions, among types of regions (urban, intermediate and rural) and also among regions within countries. The third section of the paper analyses regions’ contributions to aggregate OECD growth over the period 1995-2007. We quantify the aggregate effects of the main growth-hubs (FL) compared with all other regions (MS) and find an approximate (1/3, 2/3) rule. A final section concludes.

2. A large dispersion of regional growth rates

During the period 1995-2009, OECD countries appeared to have entered into a process of overall convergence in GDP per capita due to a catching up process from the bottom of the distribution. The catching up process was notably driven by the faster rate of growth -- than the OECD average -- of former Eastern European countries. As a consequence, the Gini coefficient of GDP per capita across OECD countries has steadily decreased since late 1990s (Figure 1).

![Figure 1. Inequality among OECD countries, 1995-2009](image)

This process of inter-country convergence seems to have taken place in parallel with increased regional disparities. Indeed, the within-country Gini coefficient of GDP per capita has increased substantially (Figure 2). Over the period 1995-2007, the spread between extreme values of GDP, GDP per capita and productivity growth across OECD TL2 and TL3 regions is larger -- by a factor of two and three, respectively -- than the spread across countries (OECD, 2009).

3. For a description of the OECD Classification of Regions see Annex 1.
At a first sight, this simultaneous process of convergence of countries and divergence of regions could be due to the forces of agglomeration and concentration within countries. In other words, agglomeration forces could be generating an increase of average country-level productivity, at the ‘cost’ of an increased regional disparity. But the real world is a bit more complex than this simple and intuitive picture.

To see this point, Figure 3 displays the relation between the level of regional GDP per capita in 1995 and its subsequent growth rate over the period 1995-2007. We also identified the OECD regions at the TL3 level that are categorised as predominantly urban and predominantly rural (see Annex 1). Four types of growth patterns can be identified in the figure: (I) rich regions (above average GDP per capita) growing above OECD average; (II) rich regions growing below average; (III) less-developed regions growing below average; and, (IV) less-developed regions growing above OECD average.

Had the agglomeration forces only predominated, the relation between the level of GDP per-capita and subsequent growth rates would have been positive and quadrants (I) and (III) would be the most populated. With mainly regional catching-up mechanisms at work, regions in quadrants (II) and (IV) would display the highest density of data points. The figure however does not yield such regular patterns among both urban and rural regions populating all four quadrants. It can also be seen that many urban regions grew faster than rural ones, but also many rural regions out-performed urban regions in terms of GDP per capita growth rates over the period considered. The shape of the data cloud could therefore result from a tension between convergence and agglomeration forces.

The corresponding picture for the TL3 intermediate regions (Figure 4) actually shows that convergence forces tend to dominate at that scale, with a more pronounced negative relation between GDP per capita levels and growth rates. Figure 4 also shows an increased dispersion of regional per-capita
growth rates for lagging regions. This implies that some regions far away from the production frontier are catching-up quite rapidly while other may be losing sizeable growth opportunities.

Figure 3. Initial level and annual average growth rates of GDP per capita among predominantly urban and rural OECD TL3 regions, 1995-2007

![Graph showing growth rates of GDP per capita among urban and rural OECD TL3 regions](image)

NB: The vertical and horizontal lines correspond, respectively, to the OECD urban and rural average growth rates and the average income level. Regions from the United States, Mexico, Switzerland, Canada, Australia, New Zealand and Iceland are missing due to lack of GDP data at TL3.

Source: Author’s calculations based on OECD Regional Database (2010)

Figure 4. Initial level and annual average growth rates of GDP per capita among intermediate OECD TL3 regions, 1995-2007

![Graph showing growth rates of GDP per capita among intermediate OECD TL3 regions](image)

Source: Author’s calculations based on OECD Regional Database (2010)

4. At the national level, Jones and Romer (2011) also find that there is a higher degree of variation in GDP per capita growth rates in countries far away from the income frontier as opposed to countries close to the frontier.
To sum-up, regional growth does not seem to emerge in particular types of regions, with the corollary that opportunities for growth may exist in all regions. Even when controlling for national factors and adjusting for the national growth rates, the large heterogeneity of these regional growth rates persists (results available upon request).

We also estimated the kernel densities of annual average growth rates in GDP and GDP per capita over the period 1995-2007 for 816 OECD TL3 regions where data are available (Figures 5). The distributions of GDP and GDP per capita are centred around a mean of 3% for GDP and 2% for GDP per capita, respectively, but there is departure from normality assumptions. The distributions display asymmetric long tales, with some regions with very high GDP growth rates, while some others display very negative rates of GDP per capita growth. A possible explanation is that some regions have very high GDP growth because of massive inflow of workers, which tends to reduce their growth in GDP per capita terms. Conversely, the decline of some regions ceteris paribus tends to be more pronounced on a GDP per capita basis.

Figure 5. Distributions in GDP and GPD per capita growth among OECD TL3 regions, 1995-2007

![Kernel density estimate for GDP growth rates](image1)

![Kernel density estimate for GDP per capita growth rates](image2)

Note: GDP data for Turkey are only available for 1995-2001, and for the United States for 1997-2007. TL3 data are not available for Australia, Canada, the United States and Mexico.

Source: Calculations based on OECD Regional Database (2010)

3. Linking regional to aggregate performance: a distributed growth model

This regional growth heterogeneity can be mapped into aggregate growth by calculating the regional contributions to growth ($C_i$), as follows:

$$C_{i,t} = w_{i,t-1} \cdot \frac{\Delta GDP_{i,t}}{GDP_{i,t-1}}$$

where

$$w_{i,t-1} = \frac{GDP_{i,t-1}}{\sum_i GDP_{i,t-1}}$$

(1)

Where $GDP_{i,t}$ is the GDP in region $i$ and year $t$. By construction, the sum of all regional $C_i$ over all regions in a country or in a group of countries is equal to the growth rate of the whole economy. Large regions with high growth rates will have the largest impact to aggregate growth, while small regions with low rates will have the lowest impact. In the middle, the impact of a large region displaying very low growth rates could be equivalent to the one of a small region displaying very fast growth rates. The net effect will depend on balance between the two elements.
Using kernel density estimates, the regional contributions to aggregate OECD growth at the TL3 level for the period 1995-2007 display a much more skewed distribution than the regional growth rates above (Figure 6). Apart from few negative contributions, its shape follows approximately a power law (see Annex 2).

**Figure 6. Contributions to OECD GDP growth, TL3 regions 1995-2007**

![Kernel density estimate](attachment:image.png)

NB: The contributions to growth are normalised to (=aggregate growth rate=1). GDP data for Turkey are only available for 1995-2001 and TL3 data are not available for Australia, Canada, Mexico, the United States and New Zealand.

*Source:* Authors' calculations based on OECD Regional Database.

The positive regional contributions to GDP growth are characterised by two fat tails. The fat tail close to the y-axis corresponds to the disproportionate contribution of very few regions (11 OECD regions have a contribution ranging from 1% to 4% of OECD aggregate growth), whereas the long tail near the x-axis corresponds to a very large number of regions, each contributing only marginally. Overall, 87% of the regions (around 700) have growth contributions below 0.2%. Hereafter, we will designate the first tail with the *Few-Large* contributors by the FL-tail, and the second tail with *Many-Small* contributors by the MS-tail.

**Relative contributions to growth of the FL and MS-tails**

If one ranks each region by the size of its contribution to growth and plots the relation between the size of the contribution and its rank, we obtain again a close fit, although not perfect, of a power-law distribution (Figure 7).

Among TL3 regions for which data are available, the largest contributors to aggregate growth are Tokyo (4.1%) followed by Gyeonggi-do in Korea (2.5%), Seoul and Madrid (1.9%). Just the top-20 TL3 contributors to aggregate growth represented only 2.4% of the regions (the FL-tail) and yet contributed to almost one third (*i.e.* 27%) of overall OECD GDP growth during the period 1995-2007. None of the

---

5. Note at the TL3 level, the GDP data are missing from Australia, Canada, Mexico, the United States and New Zealand.
remaining 97.6% of regions of the MS-tail individually contributed more than 0.7% of GDP, but their combined contribution amounts to almost three-quarters of aggregate growth. This relative relationship shows that we are in presence of a distributed growth model, where the role of the large agglomerations is unquestionable, but the cumulated power of many smaller regions cannot be ignored. A simple fit of the power law of the regional contributions to aggregate growth displays a coefficient of around 1.2 (Figure 7, in absolute terms), which is a more skewed distribution than the usual Zipf’s law for cities (Gabaix, 1999a).

Figure 7. Distribution of TL3 regions’ contributions to OECD GDP growth, 1995-2007

The contributions to aggregate growth appear to be mainly dominated by the size-effect (e.g. regional GDP), with very large urban regions dominating the contributions to OECD growth. In general, bigger regions tend to have larger contributions. However, there are some exceptions, for example, Berlin. Therefore the size-effect is a necessary but not a sufficient condition for being in FL-group. Moreover, despite the dominance of the size effect, the heterogeneity of regional growth may play an important role too.

To see this point, Figure 8 displays regional growth rates relative to the size effect captured by share in OECD GDP (in logs). It can be seen that that the distribution of growth rates does not appear totally independent of GDP size as a pure random model of regional growth would suggest.6 When analysing both

---

6. In a pure random model where the growth rates would be independent of the size of regions, the relation would take the form of flat band centred around the (un-weighted) average growth rate of regions and its
the average and the dispersion of growth rates by size class it can be seen that both distribution parameters increase, reach a peak, and then decrease with size (Figure 9), generating a non-monotonic profile. Therefore, the specific dynamics of regional growth rates for middle-sized regions\(^7\) can generate substantial differences in their contribution to aggregate growth. One possible explanation could be that some of these regions can cumulate both the advantages of convergence and the benefits of agglomeration, thus maximising their growth potential. This empirical fact seems to be consistent with Cuberes (2010) who found that the average rank of the fastest-growing cities has been increasing over time.

\textbf{Figure 8. Regional growth rates vs. initial GDP shares by OECD TL3 regions, 1995-2007}

![Regional growth rates vs. initial GDP shares by OECD TL3 regions, 1995-2007](image)

Note: Data are provided at the level of TL3 except for all OECD countries and for TL2 for United States, Canada, Austria and Mexico given data for these latter four countries is missing at TL3. Growth shares are in %.
Source: Authors’ calculations using the OECD Regional database

This particular shape (i.e. a power law) of regional contributions may carry important policy implications. Ensuring that the few regions with the strongest contribution to aggregate growth continue to be competitive is essential. At the same time, opportunities for growth at the aggregate level are also possible when a large number of the remaining regions improve their performance, notably by exploiting their potential for productivity catching-up (see OECD, 2009, 2012). OECD (2011) argues that ‘place-based polices’ defined by the modern regional policy paradigm are best suited for this task.

width (or the dispersion of regional growth rates) would be also roughly independent of size. See Gabaix and Ioannides (2004) for a discussion on the link between a Gibrat random growth process and a Zipf law for cities.

7. The peak in the distribution for average and dispersion of growth is reached for the size class of regions representing between 0.03-0.04% of OECD GDP. Typically, middle-sized TL3 regions of 100,000-200,000 inhabitants, such as Derby (UK), Salamanca (ESP) or Upsala (SWE) would fit in this category.
Figure 9. Mean and dispersion of growth rates by size class, OECD TL3 regions, 1995-2007

Note: Data are provided at the level of TL3 except for all OECD countries and for TL2 for United States, Canada, Austria and Mexico given data for these latter four countries is missing at TL3. Growth shares are in %. The figures first rank TL3 regions from high to low in GDP shares and partitions the distribution into ten class groups. The mean is the average growth rate in each class group and the dispersion is the variance of growth rates in each class group.
Source: Authors' calculations using the OECD Regional database

Region’s groupings and contributions to growth

Power laws can be tested visually by plotting the logarithm of regional contributions to growth and their respective log rank in the sequence, and examining for a linear relationship (see Annex 2). This exercise also allows us the identify regions with similar contributions to aggregate growth and quantify their overall effects on aggregate growth.

Figure 10 plots the logarithm of TL3 regions’ growth contributions and their respective log rank in the sequence. Our visual test does not entirely find a linear relationship; rather we find a quasi-linear relationship where the upper ranked regions follow an imperfect line and the lower ranked regions a
concave relation. This relation appears to follow a combination of two distributions: a stretched exponential and a lognormal distribution.8

While the first 2% of the regions (right part of Figure 10) corresponds to big urban agglomerations (including Tokyo, Seoul, Madrid, Paris, London, Rome, Stockholm, Attiki, Milano, Barcelona, Miasto Warszawa and München), the cohort of about 51% of regions that follows them – contributing almost two-thirds of aggregate growth – includes a wide variety of predominantly second-tier urban and intermediate regions. In fact the majority of regions from this group (51%) are intermediate regions according to the OECD classification (see Annex 1), highlighting the importance of intermediate regions for aggregate growth. The group that follows comprises of 31% of regions, contributing 9% of aggregate growth, and finally the remaining 15% of regions contribute close to nothing.

Figure 10. Log rank vs. log contributions to growth, OECD TL3 regions, 1995-2007

Note: There are no GDP data for TL3 regions in Australia, Canada, Mexico, the United States and Switzerland.
Source: Authors’ calculations using the OECD Regional database.

Figure 11 zooms into the rank-growth contribution for each of the four groups displayed in Figure 10. Whereas the power law relation holds in groups 1 and 2, it breaks down for groups 3 and 4 where the decay of the contributions is almost linear. This means that the power law holds for roughly half of the regions corresponding to nearly 90% of the contributions. This combination of power-law behaviour together with a randomic component emerges in many other fields of economic phenomena, such as the size and growth of enterprises or income distribution.9

8. More testing would be needed for formally verify these apparent distributional forms. Note that a stretched exponential distribution takes the functional form of $c(r) = Ae^{-\left(\frac{r}{\tau}\right)^d}$.  
This suggests that power laws can have scale free properties, as they tend to replicate at smaller scales, but not completely. Similarly, on a country-by-country basis, most of the distributions of the regional contributions to growth follow a quasi-power law.10

Figure 12 summarises the magnitudes of the contributions of the four groups of regions in terms of population and growth. The big-hub regions (group 1) correspond to 13% of the OECD population and contribute to 26% of aggregate growth. In some sense, the population in this group has more than proportional contribution to growth. The second group consisting mostly of second-tier urban and intermediate regions accounts for 66% of the population and 64% of growth, or roughly a one to one correspondence between population and contribution to growth. Group 3 contributes to 9% of aggregate growth and accounts for 16% of the population, while the remaining 5% of the population mainly in predominantly rural regions represents 1% of growth.

**Figure 11. Contributions to growth across region groupings, OECD TL3 regions, 1995-2007**

Note: The four regional groupings correspond to the segments identified in Figure 10. Source: Authors’ calculations using the OECD Regional database.

---

10. To save space, these results are not provided here, but are available upon request.
Figure 12. Contributions to growth by percentage of population, OECD TL3 regions, 1995-2007

![Chart showing contributions to OECD aggregated growth by percentage of population.](chart)

Note: There are no GDP data for TL3 regions in Australia, Canada, Mexico, the United States and Switzerland.
Source: Authors' calculations using the OECD Regional database.

4. Conclusions and further research

In this paper we have investigated the contribution of OECD regions to aggregate growth. We find a great degree of heterogeneity in the performance of OECD TL3 regions and among types (urban intermediate and rural) of regions. While the distribution in GDP and GDP per capita growth rates follows an approximately normal distribution, the regional contributions to aggregate growth follow a power law, with a coefficient of around 1.2 (in absolute terms). This implies that a few large regions contribute disproportionately to aggregate growth whereas most individual regions contribute only marginally. However, the size-effect is a necessary but not sufficient condition for large regions to have a significant contribution to aggregate growth. Nevertheless, because there is a large number of this latter group of regions, their cumulated contribution to growth is dominant. For the period 1995-2007, only 2.4% of OECD TL3 regions contribute to 27% of OECD GDP growth, but the remaining 97.6% corresponds to 73%, roughly a (1/3, 2/3) empirical rule.

Within this distributed growth model, the distribution of growth rates by size follows a non-monotonic pattern with the largest concentration of above average regional growth rates being concentrated for middle-sized regions. One possible explanation could be that some of these regions can cumulate both the advantages of convergence and the benefits of agglomeration, thus maximising their growth potential. This point will be left for further research.

This empirical analysis has important policy implications. First, as the patterns of growth at the regional level follow an approximate power law, the concept of an ‘average region’ has little meaning and may be not an adequate target for policy. Second, while policy makers should ensure the few regions with the strongest contribution to aggregate growth continue to be competitive and maintain their levels of income, improving the performance of periphery and even lagging regions should not be neglected because their cumulated contribution is dominant. Indeed, although the potential is certainly not equal, possibilities for growth seem to exist in many different types of regions.
BIBLIOGRAPHY


ANNEX 1: OECD REGIONAL GRIDS AND TYPOLOGY

Regional grids

In any analytical study conducted at sub-national level, defining the territorial unit is of prime importance, as the word region can mean very different things both within and among countries. In order to have a measure that is comparable, the OECD has developed a regional typology for classifying regions within each member country.

The classification is based on two territorial levels. The higher level (Territorial Level 2 – TL2) consists of 335 large regions, while the lower level (Territorial Level 3 – TL3) is composed of 1,679 small regions. All the regions are defined within national borders and in most cases correspond to administrative regions. Each TL3 region is contained within a TL2 region.

This classification – which, for European countries, is largely consistent with the Eurostat classification – helps to compare regions at the same territorial level. Indeed these two levels, which are officially established and relatively stable in all member countries, are used as a framework for implementing regional policies in most countries.

OECD regional typology

The OECD typology classifies TL3 regions as predominantly urban, predominantly rural and intermediate. This typology, based on the percentage of regional population living in rural or urban communities, allows for meaningful comparisons among regions of the same type and level. The OECD regional typology is based on three criteria. The first identifies rural communities (kommun in Sweden) according to population density. A community is defined as rural if its population density is below 150 inhabitants per square kilometre (500 inhabitants for Japan to account for the fact that its national population exceeds 300 inhabitants per square kilometre). The second criterion classifies regions according to the percentage of population living in rural communities. Thus, a TL3 region is classified as:

- predominantly rural (rural), if more than 50% of its population lives in rural communities.
- predominantly urban (urban), if less than 15% of the population lives in rural communities.
- intermediate, if the share of population living in rural communities is between 15% and 50%.

The third criterion is based on the size of the urban centres. Accordingly:

- A region that would be classified as rural on the basis of the general rule is classified as intermediate if it has an urban centre of more than 200,000 inhabitants (500,000 for Japan) representing no less than 25% of the regional population.
- A region that would be classified as intermediate on the basis of the general rule is classified as predominantly urban if it has a urban centre of more than 500,000 inhabitants (1,000,000 for Japan) representing no less than 25% of the regional population.
ANNEX 2: POWER LAWS

A power law is a special type of mathematical relationship between two quantities. If one quantity is the frequency of an event, and the other is the size of the event, then the relationship has a power-law distribution when the frequency of the event decreases at a greater rate than the size increases (Figure A.2.1). Technically, a power law is any polynomial relationship that exhibits the property of scale invariance. The most common power laws relate two variables in the following functional form:

\[ f(x) = a x^k + o(x^k) \]  \hspace{1em} (A1)

- where \( a \) and \( k \) and constants
- \( o(x^k) \) is an asymptotically small function of \( x^k \)
- \( k \) is called the scaling exponent where the word scaling denotes the fact that a power law satisfies:

\[ f(cx) = a c^k x^k \sim f(x) \]  \hspace{1em} (A2)

Taking logarithms of (A1) reveals a linear relation with a slope \( k \). Re-scaling the argument produces a linear shift of the function up or down buy leaves both basis form and the slope \( k \) unchanged.

Common procure to estimate power laws include using Zipf Regressions (Gabaix 1999a, Gabaix 1999b, Balakrishnan et al., 2008) and Hill Estimators (Hill, 1975). At first glance a visual test can detect power laws by sorting the regions with the largest contribution to growth into a sequence and then assigning each region a rank within the sequence and plotting a log-log relation between their rank in the sequence and their contribution to growth. A straight line with negative slope should empirically confirm the power law behaviour. Formally, according to Prieto and Sarabia (2010) power laws are defined with the following cumulative distribution function:

\[ F(x) = \Pr(X \leq x) = \left( \frac{x}{\sigma} \right)^{-\alpha}, \quad x \geq \sigma \geq 0 \]  \hspace{1em} (A3)

And \( F(x)=0 \) if \( x \geq \sigma \), where \( \sigma \geq 0 \) is a shape parameter and \( \sigma \) is a scale parameter. The \( \alpha \) parameter will be called the Pareto coefficient. By taking logarithms of both sides of equation A3, we obtain a linear expression in log \( x \):

\[ \log(1 - F(x)) = \log(\sigma^\alpha) - \alpha \log x \]  \hspace{1em} (A4)

Therefore a power law with Pareto coefficient \( \alpha \) can be seen as a straight line with negative slope \( \sigma \) on a log-log plot. In terms of the rank we have:

\[ \log(rank) = C - \alpha \log(size_{(i)}) \]  \hspace{1em} (A5)