The Broader Benefits of Transportation Infrastructure

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Abstract

Assessments of the economic benefits of transportation infrastructure investments are critical to good policy decisions. At present, most such assessments are based of two types of studies: micro-scale studies in the form of cost-benefit analysis (CBA) and macro-scale studies in the form of national or regional econometric analysis. While the former type takes a partial equilibrium perspective and may therefore miss broader economic benefits, the latter type is too widely focused to provide much guidance concerning specific infrastructure projects or programs. Intermediate (meso-scale) analytical frameworks, which are both specific with respect to the infrastructure improvement in question and comprehensive in terms of the range of economic impacts they represent, are needed. This paper contributes to the development of meso-scale analysis via the specification of a computable general equilibrium (CGE) model that can assess the broad economic impact of improvements in transportation infrastructure networks. The model builds on recent CGE formulations that seek to capture the productivity penalty on firms and the utility penalty on households imposed by congestion (Meyers and Proost, 1997; Conrad, 1997) and others that model congestion via the device of explicit household time budgets (Parry and Bento, 2001, 2002). The centerpiece of our approach is a representation of the process through which markets for non-transport commodities and labor create derived demands for freight, shopping and commuting trips. Congestion, which arises due to a mismatch between the derived demand for trips and infrastructure capacity, is modeled as increased travel time along individual network links. Increased travel time impinges on the time budgets of households and reduces the ability of transportation service firms to provide trips using given levels of inputs. These effects translate into changes in productivity, labor supply, prices and income. A complete algebraic specification of the model is provided, along with details of implementation and a discussion of data resources needed for model calibration and application in policy analysis.

1. INTRODUCTION

Most contemporary assessments of the economic effects of transportation infrastructure investments fall into two major categories, one at the micro-scale and the other at the macro-scale. Micro-scale assessments follow the procedures of cost-benefit analysis (CBA). They use information on the likely outcomes of a proposed project – its effect on travel times, traffic flows, emissions, accidents, etc. – to estimate a pecuniary value of its lifetime benefit. That benefit estimate is then contrasted with lifetime...
project costs to determine whether it is economically productive. Such ex ante analyses are often required as a justification for devoting public funds to a proposed project. (For a review see Mackie and Nellthorp, 2001.)

Macro-scale studies include econometric analyses that relate the aggregate investment in (or stock of) transportation infrastructure to economy-wide measures of economic performance. For the most part, they specify production or cost functions in which public infrastructure is regarded as an input to production by private firms in a region or nation. The estimated production and cost functions provide evidence of the contribution that infrastructure investment makes towards augmenting the productivity of private firms and, in some cases, make it possible to calculate a rate of return on aggregate infrastructure investment. (For a review see Lakshmanan and Anderson, 2002.)

The two approaches are complementary. Micro-scale analyses have the advantage of being able to measure the impacts of adding or improving a specific infrastructure element, but the scope of their economic assessment is limited to effects on users of the element in question or closely related elements and to firms and individuals in its immediate locale. The macro-scale analyses capture a broader range of economic impacts, but they treat infrastructure investment as a homogenous good (measured in dollars or network miles.) and are therefore of little use for assessing the worth of specific investments. Further, the macro-scale approach sheds little, if any, light on the mechanisms that drive the observed economic impacts.

To provide a more complete picture of the economic impacts of infrastructure, an intermediate level of analysis is needed. For convenience, we refer to this level as “meso-scale,” although models in this category might be applied at a variety of geographical scales. We define three requisites for models in this class.

1. Unlike macro-scale analyses they should incorporate information about specific additions or improvements to transportation infrastructure networks (although not necessarily at the level of detail found in micro-scale analyses.)
They should trace the economic processes that are triggered by infrastructure improvements. (As we
will explain below, these may take the form of static general equilibrium effects or dynamic
developmental effects.)

Finally, in order to assess the relative magnitude of different economic mechanisms and to inform
policy, they should be amenable to empirical implementation using data that are either available or
obtainable at reasonable cost.

As a contribution toward the development of meso-scale analyses, we introduce a computable
general equilibrium (CGE) model that incorporates a number of novel mechanisms for tracing the effects
of additions to the capacity of a transport network through the broader economy. Infrastructure
investments are modeled as reducing travel times over links in a network. The key novelty is to
incorporate travel time explicitly in the utility and profit maximization problems of households and firms.
For households, travel time for commuting and consumption activities enters a time budget that also
includes time devoted to work and leisure. For firms providing transportation services, travel time affects
the number of trips that can be provided by a given stock of vehicles, which in turn affects the prices of
intermediate and final goods.

The remainder of the paper is organized as follows. Sections 2 and 3 discuss the broad economic
impacts of transportation infrastructure and the state of the art in assessing those impacts. Section 4
reviews the relatively brief literature on assessing economic impacts of infrastructure investment with a
CGE framework. Section 5 constitutes the meat of the paper, presenting an overview of our model, a
complete algebraic specification, details of implementation and a discussion of data needs. Section 6
provides a discussion and summary.

2. CONTEXT: THE BROADER ECONOMIC IMPACTS OF INFRASTRUCTURE INVESTMENT
The role of transportation infrastructure in the economy is multifaceted and plays out over a long period of time. It is unlikely that any modeling framework can capture all possible mechanisms. For our purpose, it is useful to make a distinction between two classes of economic impacts, which we call static general equilibrium impacts and dynamic developmental impacts. Static general equilibrium impacts comprise a broad range of effects coursing through the economy consequent on the time and monetary savings induced by the infrastructure improvements. Such temporal and monetary savings alter, in turn, the marginal costs of transport producers, individuals’ mobility and the demand for goods and services in the context of lowered congestion. As these changes ripple through the market mechanisms, endogenous changes occur in employment, output, and incomes. Dynamic developmental impacts ensue from the mechanisms set in motion when transport infrastructure improvements activate a variety of interacting processes that yield over time many sectoral, spatial, and regional effects which augment productivity. They produce transformations in the structure and pattern of the economy – such as changes in the spatial pattern of production; creation of new industries and inter-industry linkages; changes in the lifestyles and preferences of households; and the evolution of institutions and markets. While static general equilibrium impacts arise from the actions of a well-defined set of economic agents through the medium of markets, dynamic developmental impacts involve complex interactions of economic, social, cultural and institutional factors and are more idiosyncratic in nature. We therefore attempt to capture only the former category of impacts in the CGE model.

General equilibrium effects occur within a system of market relationships that is stable and relatively well understood. Most economic activities require some movement of goods and people. Production requires the movement of intermediate inputs to the production site, the movement of workers back and forth between their homes and places of employment (commuting) and the movement of finished goods to market. Consumption activities also require movement as in the case of household trips for shopping and recreation. To the extent that improvements to transportation infrastructure reduce the cost of movement of goods and people, they affect the levels of economic activity in all parts of the economy.
A number of general equilibrium mechanisms are described in detail below. But for the purpose of illustration, consider the effect of infrastructure on employment. Most transportation analyses start with the explicit or implicit assumption that the number of people who commute to work over a given network is fixed. In an economy like the US, however, labor supply is by no means perfectly inelastic because significant segments of the population, such as mothers with children and individuals past the normal retirement age, face decisions as to whether to enter or remain in the labor force. Labor supply is normally associated with the wage, but since commuting represents a significant cost of labor force participation, infrastructure improvements could entice more people to work. Of course, in a general equilibrium framework this would represent a shift in the labor supply function, which in turn would affect the equilibrium wage and employment level.

A peculiar aspect of transportation infrastructure investments is that the cost reductions they generate are often realized in time savings rather than monetary savings. Returning to the commuting example, a road expansion that relieves congestion might have a minor effect on a commuter’s out of pocket cost (e.g. lower fuel costs due to efficiency improvements that stem from changes in the driving cycle) but a major effect on commuting time. Time is a scarce resource for any potential worker, so less time spent commuting means more time is available for work, leisure, consumption activities, childcare, etc. Thus, in assessing the general equilibrium impacts of transportation infrastructure, the household time budget is as important as the household expenditure budget. Depending on the magnitude of the wage relative to the marginal utility of leisure, the impact of the decrease in commuters’ time costs on the labor supply may be larger than that of their pecuniary savings.

A general equilibrium perspective on transportation infrastructure recognizes that reductions in the pecuniary and time costs of transportation can lead to increases in the levels of various economic activities and thereby to increased derived demand for transportation services. Thus, induced traffic flows are a natural outcome of market mechanisms. To many transportation analyses, such flows are seen as negating benefits from transportation infrastructure. A project whose congestion reduction effect disappears due to increased traffic within a few years of its implementation is seen as a failure. This point
of view may be appropriate from an environmental perspective, where the goal of the project is to reduce emissions via improved traffic flow, but there are conceptual difficulties from a broad economic perspective. Induced trips are derived from increases in economic activities (labor supply, production, consumption, recreation) that lead to increased welfare, so as long as there are more trips there is presumably a benefit. This has an important implication: from a broader economic perspective, the benefits of a transportation infrastructure project cannot be assessed solely in terms of resultant travel time savings. This is especially true over the medium to long run, when the additional economic activity made possible by the expansion of infrastructure capital stock increases the derived demand for transportation to the point where it once again approaches the transportation network’s capacity.

The fact that we do not try to capture dynamic developmental impacts in the CGE model is not meant to detract from their importance. Impacts of this type are most pronounced in low-income countries, where infrastructure improvements often represent significant and non-marginal enhancements of infrastructure capacity, which (along with the transport services they make possible) can facilitate interregional trade and integration. As infrastructure and service improvements lower money and time costs and increase accessibility to various market actors—input suppliers, workers and customers—market expansion, increased interregional integration and sustaining growth occurs over time. The underlying mechanisms include gains from trade, technology shifts, and gains from agglomeration supported by transport. A well-studied example of such developmental transformation is the experience of the U. S. Midwest consequent on a 400% expansion of the rail network between 1848 and 1860 – essentially linking the Midwest to Northeastern U. S. and the world economy. There is considerable evidence that the development of railroads accelerated the settlement, agricultural expansion, and growth and diversification of manufacturing, and initiated dynamic sequences that integrated the New England and Mid-Atlantic regions with the Midwest (Fogel, 1964, Fishlow 1965, Lakshmanan and Anderson 2007). A more recent example of such developmental effects of major road investments is discernable in Sri Lanka (Gunasekara, Anderson and Lakshmanan 2007 forthcoming). The broader literature on transport and economic development suggests that transport infrastructure facilitates the transformation of
low-income economies from subsistence to commercial agriculture, the development of basic, transport-intensive industries and the growth of cites (Haynes and Button, 2001).

It would be a mistake to think that developmental impacts occur only at an early stage of economic development. Even in a mature economy, transportation infrastructure improvements might promote structural changes such as increased decentralization or agglomeration of economic activity; changes in the way business enterprises conduct operations such as inventory management, logistics and other practices; enhanced opportunities for face-to-face interaction; and a range of new recreational opportunities (Anderson and Lakshmanan, 2007.) These impacts, which affect the long-term evolution of the economy, are difficult to measure and even more difficult to predict. Nevertheless they are important, and a better understanding of developmental effects should lead to better decision-making on transportation infrastructure.

3. CONVENTIONAL METHODS OF IMPACT ASSESSMENT

As we have stated earlier, current methods of impact assessment include the micro-scale CBA and macro-scale econometric studies. CBA is nearly universal as a means of assessing the desirability of specific projects. Conceptually, economic benefits are assessed as the consumer surplus, defined in relation to the demand curve for the infrastructure facility in question. The effect of the infrastructure improvement is represented as a rightward shift in the infrastructure supply curve, which results in a fall in the price of using the facility—as usually defined in units of time as opposed to money—for any given level of demand. The associated economic benefit thus has two components: one based on the cost savings enjoyed by the number of travelers who used the facility prior to the improvement, and a second representing the benefits to new travelers who now choose to use the facility because of its lower price.
Since the benefit is calculated in terms of time savings, it is necessary to apply a value of time to recast the total benefit in monetary terms so that it can be compared against the project’s cost. Benefits may also be adjusted for the value of environmental externalities and traffic accidents. Since benefits accrue annually over the lifetime of the facility and most costs are incurred at the beginning of its lifetime, present values of the flows of benefits and costs are calculated to make them comparable.

In practice, the result of CBA can be highly sensitive to the assumed value of time and discount rates. If these values are accurate, however, the beauty of CBA lies in the theoretical argument that consumer surplus, which is a measure of travelers willingness-to-pay, captures the full range of economic benefits. For example, other measurable benefits, such as property appreciation near the improved facility, are chiefly outcomes of reduced travel time so including them in benefit calculation constitutes double-counting (Forkenbrock and Foster, 1990).

Even proponents of CBA concede that there are broader economic impacts that are not captured, but argue that the magnitude of these impacts for any particular project is probably small (Mackie and Nellthorp, 2001). But such impacts summed across a number projects may be substantial, which suggests that CBA is more appropriate for assessing individual projects than for assessing a program of infrastructure spending. As an indication that certain broader impacts are excluded from CBA results, notice that economic benefits are measured almost exclusively in terms of time savings. As we noted earlier, general equilibrium benefits can accrue even in the absence of time savings.

To the extent that an infrastructure spending program significantly influences relative prices, its effects are likely to be felt in markets that are removed from those under the narrow consideration of micro-level CBA. (e.g., consider the impacts on West-coast commodity markets of a significant infrastructure investment at the Port of Long Beach.) In such cases, analysis which (i) ignores changes in prices by treating the latter as strictly exogenous and (ii) considers only those impacts which are spatially or temporally proximate—or confined to transportation or related sectors—may well fail to fully account

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1 The theoretical justification rests on the assumption of perfect competition. Venables and Gasiorek (1999) develop a theoretical framework for assessing impacts under the assumption of monopolistic competition.
for the benefits of the investment in question. In traditional CBA the issue boils down to the conditions under which the value of time is a theoretically valid measure of for the monetary impacts of these myriad inter-market adjustments, and the extent to which these conditions are likely to be satisfied in practice.

Macro-scale assessments of the economic impact of productivity analysis generally take the form of production and cost functions in which transportation infrastructure is included as an argument on the right-hand-side. (For a review see Lakshmanan and Anderson, 2002.) Despite their rigorous grounding in economic theory, there is a “black-box” quality about them because public capital does not function like private capital in the production technology. For example, no firm has exclusive use of a highway, and for any firm one might consider, there are large segments of the highway network that it does not use at all. Still, a firm might benefit from a highway that it does not use directly via the indirect means of reduced input costs. Clearly the mechanisms by which private productivity is enhanced by transportation infrastructure are varied and complex. Thus, a positive output elasticity tells us that some economic benefit is occurring, but sheds little light on the underlying mechanisms (Anderson and Lakshmanan, 2007). In particular, it is often very difficult to discern how much of the observed impact is due to developmental as opposed to general equilibrium influences.

A further limitation of macro-level studies is that they treat transportation infrastructure as a homogeneous good that can be measured in dollar terms. Such a measurement has some validity in the case of private capital, because it is not unreasonable to assume that the value of a capital good reflects its competitively determined marginal revenue product. In the case of transportation infrastructure, which is allocated via mechanisms that are likely to emphasize distributional goals or political expediency over economic efficiency, such an assumption is questionable. It is highly likely that investments of some types and in some locations are more productive than others.

In short, the results of macro-studies point to an important relationship between public capital and private productivity, but provide little in the way of either explanation or policy guidance.
4. A REVIEW OF GENERAL EQUILIBRIUM ANALYSES OF CONGESTION

We focus our attention on two sets of simulation studies, which develop models of the interplay between infrastructure and congestion at the level of the aggregate economy. The first, by Mayeres and Proost (1997), Conrad, 1997 and Conrad and Heng (2000), define an explicit index of congestion \( Z \), modeled a function of the level of utilization of aggregate transportation infrastructure or capacity, where the latter is expressed in terms of either aggregate transport activity or the size of the vehicle capital stock. Congestion incurs a productivity penalty on firms and a utility penalty on consumers. The former manifests itself through the reduced speed with which firms are able to ship their goods to market, while the latter does do via the diminished quality of transport services consumed by households. The second set of studies (Parry and Bento, 2001; 2002) model congestion through the device of an explicit household time budget. Increases in travel times with the expansion of transport activity cause a reduction in labor supply and the consumption of both leisure and services of transport producers.

Mayeres and Proost (1997) construct a stylized applied general equilibrium model which captures the essence of the congestion problem without simulating the process by which infrastructure spending affects the value of time. They consider a simple economy made up of a utility-maximizing representative household and two representative firms, summarized algebraically as follows:

\[
\max_{C, \Lambda, q_F, R} U(C, \Lambda; Z^{-z}q_p) \tag{MP1}
\]

subject to:

\[
C + q_p + R \leq Z^{-z} f_1(h_1, q_p) \tag{MP2}
\]

\[
q_F = f_2(h_2) \tag{MP3}
\]

\[
h_1 + h_2 + \Lambda \leq H \tag{MP4}
\]

\[
Z = 1/[1 - (q_p + q_F)/(CAP + R)]^{1.5} \tag{MP5}
\]
In eq. (MP1) the household derives utility \((U)\) from consumption of a final good \((C)\), passenger transport \((q_P)\) and leisure \((\Lambda)\). Eq. (MP2) says that firm 1 combines inputs of labor \((h_1)\) and freight transportation \((q_F)\) according to the production function \(f_1\) to produce the final good, which may be consumed directly, allocated to passenger transport services, or used to create new transport infrastructure \((R)\). Firm 2 produces freight transportation services from labor \((h_2)\) according to the production function \(f_2\) in (MP3), and the household’s labor endowment \((H)\) constrains labor-leisure choice in (MP4): Eq. (MP5) specifies how the imbalance between aggregate transport activity and infrastructure capacity \((CAP)\) gives rise to congestion according to a capacity restraint function based on Evans (1992). In turn, \(Z\) adversely influences both the productivity of the final goods producer and the quality of passenger transport enjoyed by the household, according to the elasticities \(\xi\) and \(\varpi\), respectively. Infrastructure investment alleviates congestion by expanding transit capacity, though at the cost of reduced consumption.

Conrad (1997) and Conrad and Heng (2002) apply these ideas in the context of a large-scale recursive-dynamic CGE model (GEM-E3). They elaborate the mechanisms underlying eq. (MP5) by developing an explicit model of the influence of aggregate infrastructure on the utilization of vehicle capital stocks. Their economy is made up of a representative utility-maximizing household and \(j = \{1, ..., J, Tr\}\) firms, where firm \(Tr\) is a producer of transportation services. Firms’ capital stocks are partitioned into intersectorally mobile “jelly” capital \((k_j)\) and transportation capital \((k_{tj})\), which represents vehicles and is a fixed factor. In the simplest version of their model the aggregate quantity of transportation infrastructure \((KI)\) is constant, and its divergence from the socially optimal level \((KI^*)\) is responsible for congestion which reduces the productivity of \(k_t\):

\[
\max_{c_j} U(C_1,...,C_J; Z^{-\xi} C_P) \quad \text{(CH1)}
\]

subject to

\[
\sum_v X_{v,j} + C_j \leq f(X_{1,j},...,X_{I,j}; h_j, k_j, k_{tj}^*) \quad \text{(CH2)}
\]

\[
k_{tj}^* = k_{tj} Z^{-\alpha} \quad \text{(CH3)}
\]
\[ k_{tj} = k^{0}_{tj} \exp(-a / KI) \]  

(CH4)

\[ Z = \exp \left( -\sum_j a\omega_j \left( \frac{1}{KI} - \frac{1}{KI^*} \right) \right), \sum_j \omega_j = 1 \]  

(CH5)

\[ H = \sum_j h_j, \]  

(CH6)

\[ K = \sum_j k_j. \]  

(CH7)

\( KI \) fixed

(CH8)

\[ KI^* \approx \kappa \sum_j \pi^{KT}_{j} \frac{kt_j}{P^{KI}} \]  

(CH9)

In (CH1) the household derives utility from consumption of \( C_j \) units of each good, with congestion diminishing the quality of consumed transportation services. The \( j^{th} \) firm produces a unique good which is both consumed and used as an intermediate input \( (X_{ij}) \) to the \( i \) other firms (CH2). Production is described by a nested CES function, \( f_j \), which combines intermediate inputs with labor \( (h_j) \), jelly capital \( (k_j) \) and effective units of transportation capital \( (kt^j) \). The latter consists of a benchmark quantity of fixed capital \( (k^{0}_{tj}) \) whose productivity is exponentially augmented by infrastructure in (CH4) and attenuated by congestion in (CH3). These influences are modulated by the coefficient \( a \) and the elasticity \( \varpi \), respectively, and the factor \( \exp(-a / KI) < 1 \) can be interpreted as a capacity utilization measure. Equilibrium between the demands for labor, capital and infrastructure and the endowments of these factors \( (H, K \text{ and } KI) \) is given by eqs. (CH6)-(CH8), and eq. (CH5) defines congestion in terms of the weighted average utilization rate of transportation capital relative to the optimal utilization level, with industry weights \( \omega_j \). Conrad (1997) derives the condition for the optimum (CH9) under the assumption that there exists an exogenous government-cum-social planner whose objective is to minimize the economy’s total expenditure on transportation. The resulting supply function for \( KI^* \) is denominated over the quantities of firms’ transportation capital stocks, their shadow prices \( (\pi^{KT}_{j}) \), the marginal social cost of infrastructure provision \( (P^{KI}) \), and the elasticity of transport capital with respect to infrastructure \( (\kappa) \).
This approach has the advantage of being straightforward to numerically parameterize.\textsuperscript{2} However, its main limitation is that it does not explicitly relate congestion to investment in infrastructure and the value of time (e.g., the Lagrange multiplier on eq. (MP4)), whose role in CBA is to indicate when the marginal benefits of alleviating the former exceed the marginal costs of the latter. The relevant mechanism is captured by the second set of studies, which model the production of travel as requiring inputs of time, which explicitly enter into a household time budget constraint.

Parry and Bento (2001) emphasize the impact of substitution among differentially congested modes of travel on time expenditures. Theirs is a stylized model of commuting—production is modeled in the simplest possible way and freight transport is not considered. The economy is made up of a utility-maximizing household, a final goods producer and three transport firms (indicated by the subscript $m$), each of which corresponds to a particular mode: congested roads ($R$), public transit ($P$) and non-congested roads ($F$).

\[
\max_{c, x, q, c, q} U(c, \lambda) \quad \text{(PB1)}
\]

subject to:

\[
c + \sum_m x_m \leq \min(H, Q) \quad \text{(PB2)}
\]
\[
Q = f(q_R, q_F, q_P) \quad \text{(PB3)}
\]
\[
q_m = \begin{cases} 
\min(X_m / \nu_m, T_m / \tau_m) & m = P, F \\
\min(X_m / \nu_m, D(T_m, d_i - d_s q_m) / \tau_m) & m = R
\end{cases} \quad \text{(PB4)}
\]
\[
H + \lambda + \sum_m T_m \leq \bar{T} \quad \text{(PB5)}
\]

The household derives utility from consumption of the final good and leisure, (PB1). Eq. (PB2) says that the output of the final goods firm is produced from labor and aggregate transportation services ($Q$) according to a fixed-coefficients technology, and can either be consumed or allocated to intermediate

\textsuperscript{2} The main empirically-derived inputs employed by Conrad-Heng are benchmark estimates of the transportation and infrastructure capital stocks, the aggregate cost of congestion, and the elasticity of congestion with respect infrastructure spending, which, along with assumed industry weights, $\omega_j$, permits the $a$ parameter in the congestion function to be calibrated.
uses by the transport firms \((X_m)\). Transport services are defined in (PB3) as a composite of the trips on the different modes \((q_m)\), with \(f\) used to indicate a constant elasticity of substitution (CES) aggregator function. In turn, the production of trips in eq. (PB4) necessitates use of the intermediate commodities and travel time \((T_m)\). For public transit and uncongested roads, trip generation is modeled using a Leontief transformation function, whose coefficients \((v_m\) and \(\tau_m)\) indicate the per-trip expenditures of money and time. The implication is that for these modes the level of congestion is exogenous, with constant marginal time expenditures \(\tau_m\). By contrast, on congested roads the level of congestion is endogenous. The modeling device used to represent this is the CES aggregator function \(D\), which defines the degree of substitutability between travel time and “available road capacity”, given by the linear function \(d_1 - d_2 q_R\) (where \(d_1\) and \(d_2\) are constants). Finally, the time budget constraint (PB5) requires that the sum of labor supply, leisure and total commuting time exhaust the household’s time endowment \((T)\).

The simple logic of the model is that production creates a derived demand for transport. As trips via congested modes (in this case roads, \(R\)) rise, so does congestion, which in turn reduces available capacity and time spent traveling by those modes, inducing substitution of trips to less congested alternatives. The critical parameters governing this process are the elasticities of substitution among transit modes in \(f\) and between travel time and unused mode capacity in \(D\), and the coefficients of the road availability function.

Parry and Bento’s (2002) extension enumerates trips on congested freeways \((RF)\) and alternate back roads \((RB)\) as additional modes of travel, includes negative externalities such as accidents and air pollution (which we indicate using the function \(E\)), and represents congestion in terms of travel time using a more traditional approach.

\[
\max_{C, Q, \Lambda, q_{RF}, q_{RB}, q_P, q_F} U(C, Q, \Lambda) - E(q_{RF}, q_{RB}, q_P, q_F) \tag{PB1'}
\]

subject to (PB3), (PB5) and:

\[
C + \sum_m X_m \leq H \tag{PB2'}
\]
\[
X_m = \nu_m q_m \quad \text{(PB6)}
\]
\[
T_m = \tau_m q_m \quad \text{(PB7)}
\]
\[
\tau_m = \tau_n \left[ 1 + 0.15 \left( \frac{q_m}{\text{CAP}_m} \right) \right], \ m = RF, RB \quad \text{(PB9)}
\]

Aggregate transportation services are now included as an additional argument in the household’s utility function, along with non-congestion externalities (PB1’). As before, the sole factor of production is labor, whose supply is traded off against leisure and travel according to the time budget constraint (PB5). Here, however, (PB2’) assumes that each unit of labor produces one unit of the final good, which can be consumed or used to pay for trips, whose transformation into aggregate transport services follows (PB3). As in (PB4), trips incur fixed marginal pecuniary costs (PB6), but marginal expenditures of time (PB7) which increase with congestion. Eq. (PB10) defines the latter relationship using the classic Bureau of Public Roads (BPR) capacity restraint formula.

In both Parry-Bento models, the Lagrange multiplier on eq. (PB5) represents the “true” marginal utility of time, which takes into account the general equilibrium interactions among the labor supply, the consumption of the final good and leisure, and the supply-demand balance for trips by different modes. Nevertheless, the value of time which emerges from this analysis still does not completely account for the channels through which congestion’s effects are felt. In particular, the simple representation of production fails to capture the way in which travel delays impact firms or may themselves be exacerbated by the shipment of finished goods to retail markets or households’ retail purchasing behavior.

Likewise, the specification of substitution possibilities in transportation is simplistic. The Parry-Bento models are “maquettes” which resolve only a few, very aggregate modes of travel and can afford to rely on synthetic benchmark distributions of trips.\(^3\) In section 5.2.5 we caution that moving to the use of real-world data to numerically calibrate the aggregator functions for transportation services may be quite

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\(^3\) Parry and Bento (2001) distribute trips equally among modes, while in Parry and Bento (2001) trips are allocated 33 percent to each of peak-period freeway and public transit and 17 percent each to back roads and off-peak freeway travel.
involved. The remainder of the paper addresses these issues and examines their implications for constructing large-scale transport-focused CGE models.

5. A HYBRID MESO-MACRO APPROACH

Our proposed approach is a hybrid one which seeks to capture meso-level details of infrastructure, congestion and transport within the traditional framework of a macro-level CGE model. We consider a static economy with $N$ representative profit-maximizing firms, each of which produces a single, distinct commodity. Firms and commodities come in two varieties, $I$ non-transport producers and their associated goods and services, which we index using the subscript $i = \{1, ..., I\}$, and $M$ transport or logistics firms and their associated services, which we index with the subscript $m = \{1, ..., M\}$. To distinguish between firms and the goods which they produce, we introduce the subscript $j = \{1, ..., I\}$ to enumerate non-transport producers. Furthermore, we define the set of transport producers in such a way that each firm corresponds to a single disaggregate mode of transit, e.g., rail as well as truck freight shippers, air passenger and freight travel, own-supplied passenger road travel using private vehicles, purchased local/interurban passenger transit by road and rail, etc. Non-transport firms supply goods and services to satisfy the intermediate demands of other firms as well as the final demands of households. Transportation firms provide freight transport services to the non-transport firms and passenger transport services to households. Households in the economy are modeled as a representative utility-maximizing agent who owns the factors of production (hours of labor, $H$, and capital, $K$) and rents them out to the firms in exchange for factor payments which finance the consumption of commodities.

The centerpiece of our approach is a representation of the process though which the operation of markets for non-transport commodities and labor creates derived demands for transportation. In particular, we assume that:
(a) Each unit of non-transport commodity requires freight trips to be shipped to sources of intermediate or final demand.

(b) The representative agent’s final purchases of these goods and services require retail shopping trips in order for them to be converted into utility, and

(c) The agent’s rental of labor to firms requires commuting trips.

The three kinds of mobility are distinguished using the superscripts $TF$, $TC$ and $TH$, respectively.

We assume that households face two budget constraints, a pecuniary constraint that commodity purchases not exceed factor income, and a temporal constraint that the duration of travel for shopping and commuting, hours of work and leisure not exceed an aggregate endowment of time. The latter sets up a tension between travel time expenditures for the purposes of consumption and income generation. For households to increase their consumption they need more income, which in the short run can only be obtained by renting additional hours of labor to firms, with the possible side-effect of more and/or longer journeys to work. However, their ability to earn is constrained by the fact that consuming more non-transport goods requires additional retail trips, and concomitant expenditures of time.

Firms do not have explicit time budgets, nevertheless we assume that time constraints influence production in an implicit fashion. We treat non-transport firms as “mills”, whose products are manufactured using labor, capital and intermediate inputs from other non-transport firms. In order for these products to be consumed they must be shipped to markets, which creates a demand for trips supplied by the transportation firms. A key feature of the model is that the latter firms do not produce trips directly. Rather, their outputs take the form of generalized transportation services such as vehicles operated on the road, trains on tracks, planes in the air, etc.—whose capacity is determined by the firms’ stocks of transportation capital (i.e., vehicles). This allows us to model the mechanism by which congestion imposes a productivity penalty on firms: speed, which along any given segment of the transportation network is equivalent to the inverse of the travel time, is necessary to transform these services into trips. Thus, given a certain capacity to produce transport services, increases in travel time translate into fewer trips. Other things equal, the main consequence is a fall in the productivity of inputs
to transportation and in rise in the average cost of trips, a decline in movements of passengers and non-transport goods, and a reduction in production and consumption.

These devices allow us to model the impacts of congestion in a novel way. Congestion is the increase in travel time arising out of the imbalance between the aggregate derived demand for mobility and the capacity of the stock of transportation infrastructure to support the desired flux of trips. Our way of representing travel admits three channels through which congestion may exert a drag on activity, corresponding to (a)-(c), above:

- An increase in the duration of households’ retail trips per unit of consumption, which attenuates consumption through the time budget constraint. Depending on the relevant elasticities, the time spent on work or leisure may rise or fall as well, but the standard result is a decline in utility.
- A reduction in average productivity of transport-producers. Because congestion increases the duration of freight trips of a given distance, it reduces the number of trips which transport producers can supply with a given fleet of vehicles. The consequent dissipation of operating time—and thus revenue—drives a wedge between the marginal cost of each transportation firm’s output and the unit value of its trips consumed by firms and households, much like a tax.
- Dissipation of time in commuting, acting through the time budget constraint to reduce the economy’s aggregate labor supply. This effect is similar to a tax on labor.

To effectively inform traditional microeconomic cost benefit analysis, any attempt to capture these influences using top-down economy-wide models must address a number of issues. First, considering the impacts of congestion in the aggregate will yield limited insights, as typically only a fraction of the links in an economy’s transport network will be congested. These are the ones which are candidates for infrastructure projects. Following from this observation, a second consideration is that trips should be thought of as differentiated commodities, whose equilibrium allocation among network links will be a function of transport producers’ marginal costs, firms and households’ demands for goods and passenger mobility, and the distribution of travel times/congestion. In general, infrastructure investments that are sufficiently large will give rise to simultaneous non-marginal changes in all of these variables, the
character of which will depend on both magnitude of the investments and their location on the network. Finally, in order to properly capture the dissipative effect of congestion, a distinction needs to be made between the production of transport services and the consumption of the trips which they make possible. Logistics and passenger transport firms will allocate their outputs to the network segments which yield the highest marginal revenue, while households and non-transport firms will allocate their demand for trips to links with the lowest marginal cost. The key challenge is therefore to develop a computationally tractable way of modeling the equilibrium between the supply-side transformation of transport services to trips and the demand-side aggregation of trips into passenger and freight movements so as to resolve the substitution of trips from congested links to uncongested alternatives.

The simplest way of doing this is to keep the spatial details of the network structure to a minimum. Our strategy is to assume the existence of a generic transport network with \( l = \{1, \ldots, L\} \) links, amongst which trips generated by the \( m \) transport producers are allocated in a competitive fashion. This choice allows us to specify a top-down model of intermediate complexity which is able to capture the macroeconomic feedbacks which affect—and are affected by—Wardropian equilibria, while serving as a bridge to more disaggregate network equilibrium models (e.g., Ferris et al, 1999). We make the key simplifying assumption that trips across different mode-link alternatives are imperfect substitutes, with differing marginal costs to transport consumers and differing marginal revenues to transport producers. Thus, when non-transport firms ship their product to market, or households supply labor or consume a particular commodity, each of these actors simultaneously chooses travel distances/routes and modes by allocating trips over \( l \) and \( m \) so as to minimize total transportation expenditure. Symmetrically, each logistics or passenger transport firm simultaneously chooses travel distances/routes and payloads by allocating the transportation services it produces to trips by \( l \) and \( j \) in the case of freight, \( l \) and \( i \) in the case of retail shopping, and just \( l \) in the case of commuting, so as to maximize revenue.

We operationalize these ideas by specifying transportation demands as constant elasticity of substitution (CES) functions of trips by mode. Thus, freight transport demand by the \( j^{\text{th}} \) firm, \( Q_{j}^{f} \), is
modeled as a CES aggregate of the trips, $q_{j,l,m}^{TF}$, made by transport mode $m$ on network link $l$ to ship $j$’s product to intermediate and final consumers. Similarly, we model the aggregate household demand for transportation to consume the $i^{th}$ commodity, $Q_i^{TC}$, as a CES function of the retail trips, $q_{i,l,m}^{TC}$, across all combinations of transit modes and links, and the aggregate demand for transportation to supply labor, $Q_{TH}$, as a CES function of the various mode-and link-specific commuting trips, $q_{l,m}^{TH}$. We use the same device on the supply side, specifying the trips undertaken by each transport producer as a constant elasticity of transformation (CET) expansion of that firm’s output. Thus, firm $m$’s trips $q_{j,l,m}^{TF}$, $q_{i,l,m}^{TC}$ and $q_{l,m}^{TH}$ are modeled as a CET function of the aggregate supply of transportation services by mode, $Y_m$. Parry and Bento (2001) note that the ability to substitute between transport modes mitigates the cost of reducing congestion. Our assumption that households and firms substitute among both transport modes and network segments means that the elasticities in the aforementioned CES and CET functions will likely be a key influence on the marginal benefit of investments to increase the capacity of congested links.

The attractive feature of this formulation is that it automatically generates different levels of congestion for each transport mode and network link. Congestion is a function of the total flux of trips generated by each mode across a given link,

$$G_{l,m} = \sum_i (q_{i,l,m}^{TC} + q_{j,l,m}^{TF}) + q_{l,m}^{TH},$$

(1)

and the design capacity of the particular segment, $CAP_{l,m}$, such that as $G_{l,m}$ exceeds $CAP_{l,m}$, travel time on that link, $\tau_{l,m}$, increases rapidly. A convenient representation of this phenomenon is the BPR formula (PB9):

$$\tau_{l,m} = \tau_{l,m}^0 \left(1 + 0.15 \left( \frac{G_{l,m}}{CAP_{l,m}} \right)^4 \right),$$

(2)

in which $\tau_{l,m}^0$ is the mode- and link-specific free-flow travel time.
The dependence of capacity on infrastructure investment is an exogenous input to the model that must be developed from the existing characteristics of the transit network, the links which are candidates for improvement, and the projected changes in traffic flows resulting from the proposed project. Eqs. (1) and (2) establish the crucial connection between household trips to consume goods or commute back and forth to work, and the trips undertaken by logistics firms to deliver commodities. Growth in any one type of transportation adds to the total flux of trips across the transport network, raising trip times, and inducing economic actors to re-allocate trips to less congested mode-link alternatives, as well as cut back on overall travel. As mentioned above, the consequence of this is a decline in both the quantity of labor supplied to producers and the goods and services consumed by households. In this way infrastructure capacity acts as a fundamental brake on the expansion of economic activity.

5.1. Algebraic Summary of the CGE model

We begin with a description of the households in our simulated economy. We assume the existence of a representative agent whose utility is represented by the nested CES function shown in Figure 1A. At the top level of the nesting hierarchy, the agent obtains utility \( U \) from non-transport consumer commodities \( \hat{C}_i \) and leisure \( \Lambda \), with elasticity of substitution \( \sigma \) and technical coefficients \( \alpha_i \) and \( \alpha_\Lambda \):

\[
U = \left( \sum_i \alpha_i \left( \hat{C}_i \right)^{(\sigma - 1) / \sigma} + \alpha_\Lambda \hat{\Lambda} \right)^{\sigma / (\sigma - 1)}.
\]

The second level of the utility hierarchy describes how the demands for commodities create derived demands for personal transportation. We specify each unit of delivered commodity as a CES composite of transportation services, \( Q_{i TC} \) (i.e., trips for the purpose of retail purchases), and the relevant commodity \( \hat{C}_i \) whose consumption necessitates transport expenditures, with elasticity of substitution \( \sigma^C_i < 1 \) and technical coefficients \( \beta^C_i \) and \( \beta^C_i \):
Each retail commodity is itself a composite of the goods and services actually being purchased by consumers and freight transportation services, which we discuss in more detail below. Note that eq. (4) implicitly captures Lakshmanan and Hua’s (1983) distinction between discretionary and non-discretionary transportation: while travel is a necessary input to consumption ($\sigma_i^C < 1$) the actual quantity of travel undertaken by households is discretionary. Consequently, our formulation captures the ability of consumers to substitute passenger travel ($Q_{TC}^i$) for the freight transportation component of $\hat{C}_i$, e.g., by opting to travel to retail outlets to purchase goods versus having them delivered directly to the consumer’s place of residence. At the lowest level of the hierarchy the transportation services necessary to consume good $i$ are a CES composite of the shopping trips ($q_{i,lmn}^{TC}$) which occur on each link of the transport network and mode of transit:

$$
\hat{C}_i = \left( \beta_i^{TC} \left( Q_{TC}^i \right)^{\sigma_i^{TC} - 1} + \beta_i^C \left( \hat{C}_i \right)^{\sigma_i^C - 1} \right)^{\sigma_i^{TC} \left( \sigma_i^{TC} - 1 \right)}.
$$

Here $\sigma_i^{TC}$ is the substitution elasticity and $\gamma_{i,m}^{TC}$ are technical coefficients which indicate how the retail trips associated with each commodity are distributed across mode-link alternatives in the benchmark data used to calibrate the model.

Households’ rental of their factor endowments is modeled in a similar fashion. We assume that transport services are not necessary to supply capital to firms. However, to supply $H$ units of labor the agent must utilize transportation services, $Q^{TH}$ (i.e., trips for the purpose of commuting). Accordingly, the aggregate supply of labor ($\hat{H}$) is modeled as a CES composite, with elasticity $\sigma^H$ and coefficients $\beta^{TH}$ and $\beta^H$:

$$
\hat{H} = \left( \beta^{TH} \left( Q^{TH} \right)^{\sigma^H - 1} + \beta^H \left( \hat{H} \right)^{\sigma^H - 1} \right)^{\sigma^H \left( \sigma^H - 1 \right)}.
$$
As with retail trips, we model $Q_{TH}$ as a CES composite of the commuting trips $(q_{i,m}^{TH})$ by each network link and transit mode, with elasticity $\sigma_{TH}$ and mode-link coefficients $\gamma_{i,m}^{TH}$:

$$Q_{TH}^{T} = \left( \sum_{i} \sum_{m} \gamma_{i,m}^{TH} \left( q_{i,m}^{TH} \right)^{\sigma_{TH} / (\sigma_{TH} - 1)} \right)^{\sigma_{TH} / (\sigma_{TH} - 1)}.$$

(7)

The representative agent’s budget constraint mandates that the value of consumption at retail goods prices ($\tilde{P}_i$) exhaust the income from factor rentals:

$$\sum_{i} \tilde{P}_i \tilde{C}_i \leq \theta H + rK,$$

(8)

where $\theta$ denotes the marginal utility of time—i.e., the wage net of the marginal cost of commuting, and $r$ is the capital rental rate. The agent’s time constraint mandates that the total expenditure of time on trips for retail consumption and commuting (summed over all network links and modes), plus labor and leisure time, exhaust the agent’s endowment of time, given by $\overline{T}$:

$$\sum_{i} \sum_{m} \tau_{i,m} \left( \sum_{i} q_{i,i,m}^{TC} + q_{i,i,m}^{TL} \right) + H + \Phi \leq \overline{T}.$$

(9)

The key variable in this expression is $\tau_{i,m}$, the average trip time on each network segment, which by (1) and (2) reflects the tension between the total flux of trips on that segment and its capacity.

The organization of production is shown in panels B and C Figure 1. In panel B, the output of each of the $j$ non-transport firms ($Y_j$) is modeled using a CES production function denominated over inputs of intermediate commodities ($\tilde{X}_{i,j}$), labor ($\tilde{h}_j$) and capital ($k_j$):

$$Y_j = \left( \sum_{i} \delta_{i,j}^{NT} \tilde{X}_{i,j}^{(\sigma_{i,j}^{NT}-1)/\sigma_{i,j}^{NT}} + \delta_{H,j}^{NT} \tilde{h}_j^{(\sigma_{i,j}^{NT}-1)/\sigma_{i,j}^{NT}} + \delta_{K,j}^{NT} k_j^{(\sigma_{i,j}^{NT}-1)/\sigma_{i,j}^{NT}} \right)^{\sigma_{i,j}^{NT} / (\sigma_{i,j}^{NT} - 1)},$$

(10)

with substitution elasticity $\sigma_{j}^{NT}$ and distribution parameters $\delta^{NT}$. Similar to the households, the supply of commodities creates a derived demand for transportation services. We assume that the delivery of $\chi_{i}^{TP}$ units of commodity $i$ to intermediate and final users requires a unit of freight transportation services.
As a result, the supply of non-transport commodities are a Leontief composite of produced commodities and transportation:

$$\bar{Y}_i = \min \left( X_i^{TP}, Q_i^{FP}, Y_i \right).$$  \hspace{1cm} (11)

In turn, freight transport is a CES composite of delivery trips \((q_{j,l,m}^{TP})\) by network link and transit mode:

$$Q_j^{TP} = \left( \sum_l \sum_m Y_{j,l,m}^{TP} \left( q_{j,l,m}^{TP} \right)^{\sigma_j^{TP} - 1} \right)^{1/\sigma_j^{TP}},$$  \hspace{1cm} (12)

with elasticity \(\sigma_j^{TP}\) and mode-link coefficients \(\gamma_{j,l,m}^{TP}\).

The production function for the \(m\) transportation services is illustrated in panel C, and is essentially the same as that for non-transport commodities:

$$Y_m = \left( \sum_{l,m} Z_{l,m}^{TP} \left( X_{l,m}^{TP} \right)^{\sigma_m^{TP} - 1} \right)^{1/\sigma_m^{TP}},$$  \hspace{1cm} (13)

with substitution elasticity \(\sigma_m^{TP}\) and distribution parameters \(\delta_m^{TP}\). However, to translate between transport producers’ outputs and the trips necessary to deliver passengers and freight along each link, we use the following CET formulation with transformation elasticity \(\psi_m^{TP}\) and distribution parameters \(\mu_m^{TP}\):

$$Y_m = \left( \sum_{l,m} Z_{l,m}^{TP} \left( X_{l,m}^{TP} \right)^{\psi_m^{TP} - 1} \right)^{1/\psi_m^{TP}}.$$  \hspace{1cm} (14)

The variable \(Z_{l,m}\) is particularly important. It is a link-specific productivity penalty which is a function of each link’s average travel time in (2), and is intended to capture the adverse impact of congestion on the ability of transport firms to translate service outputs into movements of goods and passengers. Thus, the more congested a given link, the more units of \(Y_m\) necessary to generate an additional trip on that link, reducing the average and marginal productivity of the inputs to (13).

The model is closed by specifying market clearance conditions for the supply of composite non-transport commodities:
\[ \hat{Y}_i = \sum_j \hat{X}_{i,j} + \hat{C}_i, \]  

and the representative agent’s primary factor endowments:

\[ H = \sum_j \tilde{h}_j + \sum_m \tilde{h}_m, \]  

\[ K = \sum_j k_j + \sum_m k_m. \]

In an appendix to the paper we provide a detailed mathematical elaboration of how the foregoing elements may be used to construct an operational CGE model. The assumption of Walrasian competitive equilibrium allows us to express the production and aggregation relations in eqs. (3)-(7) and (10)-(14) in terms of their dual cost functions and associated conditional input demand functions. The latter can then be substituted into the supply-demand balance constraints (6), (8), (9) and (15)-(17) to yield a system of nonlinear equations (\(\Xi\)) in commodity and factor prices, the activity levels of firms and the income level of the representative agent. The result is the Walrasian excess demand correspondence of the economy.

The advantage of our approach is two-fold. First, on the consumer side, the price variable consisting of the Lagrange multiplier on eq. (9) is the endogenous value of time, which takes full account of not only the channels through which congestion’s impacts manifest themselves, but also the general equilibrium interactions among these effects. Second, in both eq. (9) and the cost functions for transport producers (i.e., the dual of eq. (14)) congestion plays the role of a vector of differentiated, link-specific taxes on trips. This is very useful, as it enables us to simulate congestion as an endogenous, nonlinear tax whose level is determined by trip volumes according to eqs. (1)-(2).

5.2. Data and Calibration

Numerical solution of the model requires that values be specified for the parameters of the excess demand correspondence, \(\Xi\). This procedure, known as calibration, is likely to be especially challenging given that it requires the integration of economic and transportation data which are often incommensurate.
Calibrating the purely macroeconomic, non-transport related components of the CGE model is a fairly simple task, involving the selection of values for the elasticities $\sigma^U$, $\sigma^{NT}_j$ and $\sigma^T_m$ based on empirical estimates, and the computation of values for the coefficients $\alpha_i$, $\alpha_{\lambda}$, $\delta^{NT}$ and $\delta^T$ using a national- or regional-level social accounting matrix (SAM). (For details see, e.g., Sue Wing, 2004.) We anticipate that it will be somewhat more difficult to find econometric estimates for, or calibrate using related empirical studies, values for the substitution elasticities $\sigma^C_i$, or to infer values for the coefficients $\chi^{TF}_i$, $\beta^{TC}_i$ and $\beta^C_i$ from data on freight transport margins. And since published estimates for the elasticities $\sigma^{TC}_i$, $\sigma^{TP}_j$, $\sigma^{TC}_i$, $\psi^{TF}_i$ do not exist, developing the data and econometric procedures to estimate these parameters will likely involve a large amount of effort. A rough-and-ready way to proceed is to set up the model using the values in the range assumed by Parry and Bento (2001, 2002), and perform sensitivity analysis.

Calibrating the coefficients $\gamma^{TF}_{j,l,m}$, $\gamma^{TC}_{l,j,m}$, $\gamma^{TH}_{l,m}$, $\mu^{TP}_{j,l,m}$, $\mu^{TC}_{l,j,m}$ and $\mu^{TH}_{l,m}$ in the trip aggregation and transformation functions is likely to be a significant undertaking. The key difficulty lies in defining the topology of the transport network and associated traffic flows at a geographic scale for which there are published inter-industry economic accounts. Although survey data on commuting and freight traffic flows are readily available at the level of metropolitan statistical areas (MSAs), input-output data are rarely tabulated at such a fine spatial scale. At the opposite extreme, while it is straightforward to construct an aggregate CGE model using a SAM constructed from the transportation satellite account make and use tables (Fang et al, 2000), for modes of surface travel such as commuting or retail shopping which are important contributors to congestion, it is not obvious how to represent major congested network links at such a highly aggregate scale.

Thus, apart from the normative question of what is the most appropriate geographic scale at which our model should be specified, data constraints dictate the practical necessity of structuring a reduced-form representation of the transportation network so that it is both sufficiently simple to calibrate the $\gamma$ and $\mu$ parameters and able to be matched to a regional SAM. At the current stage of this research,
the most promising source of economic data would seem to be county-level SAMs developed by IMPLAN which are coterminous with MSAs that straddle major transportation corridors in the eastern U.S.

6. DISCUSSION AND SUMMARY

Provision of transportation infrastructure is one of the most visible, vital and costly ways in which the public sector contributes to the private economy. Yet decisions about the levels and allocations of transportation infrastructure investments must currently be made with incomplete information about their economic impacts. Analytical tools are limited to micro-scale analyses, which may not capture the full range of economic benefits induced by a project or program, and macro-scale analyses, which are too broadly defined to provide guidance on the relative benefits of specific projects and programs. The situation calls for analytical tools defined as a “meso-level” that can provide impacts assessments that are both comprehensive and capable of representing specific expansions of infrastructure capacity, following the three criteria we define in the introduction. This paper contributes to the development of such tools by specifying a CGE model that is specifically designed to assess the broader economic impacts of transportation infrastructure investments.

The model specified above draws on the limited CGE literature on transportation infrastructure – especially the household time budgets of Parry and Bento (2001, 2002). It goes beyond existing models both in terms of its technical specification and its overall scope. It specifies a set of derived demands for transportation services that arise from production, consumption and labor supply activities. It represents transportation infrastructure as a set of capacitated mode-link combinations on which flows are assigned and congestion is modeled as increases in travel time. By embedding travel time explicitly in the determination of household utilities and the prices and quantities of commodities
produced in the economy, it incorporates in the model the phenomenon of congestion that is based on a comprehensive, endogenous definition of the value of time.

Existing CGE models designed for the analysis of transportation infrastructure can, for the most part, be classified as maquettes – that is, simplified or scaled down models designed to make rough estimates of relative magnitudes or as steps toward the creation of more comprehensive models. By contrast, the model specified above is intended as a practical tool for policy analysis. There are, however, significant hurdles to overcome before it can be made operational, including defining an appropriate geographical scale for its application, an appropriate level of detail for the set of mode-link combinations and appropriate data and parameters for calibration.

The question that naturally arises is whether it is worth the effort. To some extent this comes down to the empirical question of whether the broader economic benefits captured in the CGE model are of significant magnitude relative to the more direct effects captured in CBA. But looking beyond the “bottom line” of aggregate benefits, the CGE model generates a range of information that cannot be obtained from existing models such as whether the benefits of a capacity expansion accrue mostly to firms or households, whether household benefits are mostly in consumption activities or commuting and whether some industries benefit more than others. Such information may be useful in assessing whether specific objectives that policy makers attach to a project are likely to be met. Also, the CGE specification is especially well-suited to assessing the impact of infrastructure programs because the implementation of two or more capacity expansions can be modeled simultaneously. This will be useful in identifying complementarities among projects by seeing, for example, whether the benefits of projects A and B implemented simultaneously exceed the sum of the benefits of A and B implemented independently.

Ultimately, the value of a model such as the one we have specified lies in its laying bare a plausible set of economy-wide interactions that are triggered by an improvement in transportation infrastructure. In other words, it is an attempt to move beyond “black box” and “bottom line” approaches to policy models to an approach that explains rather than just captures economic impacts. Naturally, laying the underlying mechanisms bare opens the door to criticism based on underlying assumptions –
especially as regards market imperfections. Furthermore, we recognize that there are a range of dynamic, developmental impacts that the model does not include. Still, we believe that specifying and calibrating our model is a useful step toward a better understanding of the economy-wide consequences of transportation infrastructure in the economy.

7. APPENDIX: IMPLEMENTATIONAL DETAILS

Walrasian general equilibrium prevails when the price of commodities equals their marginal cost of production with firms earning zero profits, there is zero excess demand for commodities and factors, and consumer’ income equals their expenditure. These conditions form the basis for CGE models in a complementarity format, which specify the economy as a vector of zero profit, market clearance, income balance, and auxiliary equations. Each equation is paired with an associated dual variable with respect to which it exhibits complementary slackness (see, e.g., Rutherford, 1995; Sue Wing, 2004):

1. **Zero profit conditions for firms and households.** These specify the equilibrium between commodity prices and firms’ unit cost functions, and between the marginal utility of income and the aggregate expenditure function. They complementary to the activity levels of firms and the utility level of the representative agent.

2. **Market clearance conditions for commodities and factors.** These specify the equilibrium between the aggregate demands for commodities and factors—which are functions of prices and activity levels, and their aggregate supplies—typically indicated by firms’ activity levels and households’ factor endowments. They are complementary to commodity and factor prices.

3. **Income balance conditions.** These specify the equilibrium between the value of households’ expenditures and the value of their income, and are complementary to the income levels of the households.
4. **Auxiliary equations.** These typically represent some sort of constraint on the economy that is a function of both an auxiliary variable and other endogenous variables, which requires them to be solved for along with the remaining variables. They are complementary to the auxiliary variable. Henceforth we use the shorthand symbol “⊥” to represent these complementary relationships.

7.1. **Zero Profit Conditions and Associated Demand Functions**

As before, we begin with the households in the economy. Recasting the representative agent’s utility maximization problem as a dual expenditure minimization permits us to solve for the unit expenditure function, $\varepsilon$, dual to (3):

$$
\varepsilon = \left( \sum_i \alpha_i^{\sigma_u} \hat{P}_i^{1-\sigma_u} + \alpha^{\sigma_v} \theta^{1-\sigma_v} \right)^{1/(1-\sigma_v)} \quad \perp U \tag{18}
$$

where $\hat{P}_i$ is the price of the $i$th consumption good-transport services aggregate, and $\theta$ is the value of time. This expression can be thought of as a zero-profit condition for the “production” of a utility good, to which aggregate utility is the complementary activity variable. By Shepard’s Lemma, the derivatives of the zero-profit condition with respect to the prices of the inputs yields the conditional input demands. Accordingly, final demands for commodities and leisure are given by:

$$
\hat{C}_i = \alpha_i^{\sigma_u} \hat{P}_i^{1-\sigma_u} \varepsilon^{\sigma_u} U \tag{19}
$$

$$
\Phi = \alpha^{\sigma_v} \theta^{1-\sigma_v} \varepsilon^{\sigma_v} U \tag{20}
$$

Cost minimization in the aggregation of transport services and physical goods in eq. (4) gives rise to the following zero profit condition, which is the unit cost function for $\hat{C}_i$:

$$
\hat{P}_i = \left( \beta^{\sigma_u} \left( \hat{P}_i^{\sigma_u} \right)^{1-\sigma_u} \left( \beta^{\sigma_v} \hat{P}_i^{1-\sigma_v} \right)^{1/(1-\sigma_v)} \right) \quad \perp \hat{C}_i \tag{21}
$$

where $P_i^{\sigma_u}$ and $\hat{P}_i$ are the consumer prices of retail mobility and final commodity sales associated with good $i$. The conditional demands for these inputs are given by:
\[ Q_{TC}^i = (\beta_i^\gamma)^{\sigma_i^\gamma} (P_{iTC})^{-\sigma_i^C} \tilde{P}_i^{\sigma_i} \tilde{C}_i, \]  
(22)

\[ \tilde{C}_i = (\beta_i^\gamma)^{\sigma_i^\gamma} (\tilde{P}_i)^{\sigma_i} \tilde{P}_i^{\sigma_i} \tilde{C}_i. \]  
(23)

Similarly, the unit cost function arising from cost-minimizing aggregation of labor hours and commuting to produce supplied labor in (6) is:

\[ \tilde{w} = \left( (\beta_{TH}^H)^{\sigma_{TH}} (P_{TH})^{-\sigma_{TH}} + (\beta_{TH}^H)^{\sigma_{TH}} \theta_{TH}^{\sigma_{TH}} \right)^{1/(1-\sigma_{TH})}, \]  
\[ \cong \tilde{H} \]  
(24)

where \( \tilde{w} \) is the wage and \( P_{TH} \) is the marginal cost of commuting trips. Then, the conditional demands for trips and aggregate labor are given by:

\[ Q_{TH}^i = (\beta_{TH}^H)^{\sigma_{TH}} (P_{TH})^{-\sigma_{TH}} \tilde{w}_{TH}^{\sigma_{TH}} \tilde{H}, \]  
(25)

\[ H = (\beta_{TH}^H)^{\sigma_{TH}} \theta_{TH}^{\sigma_{TH}} \tilde{w}_{TH}^{\sigma_{TH}} \tilde{H}. \]  
(26)

The zero profit conditions corresponding to the cost-minimizing allocation of trips by mode and link in eqs. (5) and (7) are

\[ P_{iTC} = \left( \sum \sum \gamma_{i,l,m}^{TC} \sigma_{i,l,m}^{TC} \left( P_{iTC}^{TC} \right)^{1-\sigma_{i,l,m}^{TC}} \right)^{1/(1-\sigma_{i,l,m}^{TC})}, \]  
\[ \cong Q_{TC}^i \]  
(27)

\[ P_{TH} = \left( \sum \sum \gamma_{i,l,m}^{TH} \sigma_{i,l,m}^{TH} \left( P_{TH}^{TH} \right)^{1-\sigma_{i,l,m}^{TH}} \right)^{1/(1-\sigma_{i,l,m}^{TH})}, \]  
\[ \cong Q_{TH}^i \]  
(28)

where \( P_{iTC}^{TC} \) and \( P_{TH}^{TH} \) are the marginal costs of trips on a given mode-link alternative incurred by the representative agent in order to consume good \( i \) and journey to work, respectively. The associated conditional demands for trips by link, mode and commodity are:

\[ q_{i,l,m}^{TC} = \gamma_{i,l,m}^{TC} \sigma_{i,l,m}^{TC} \left( P_{iTC}^{TC} \right)^{1-\sigma_{i,l,m}^{TC}} Q_{TC}^i, \]  
(29)

\[ q_{i,l,m}^{TH} = \gamma_{i,l,m}^{TH} \sigma_{i,l,m}^{TH} \left( P_{TH}^{TH} \right)^{1-\sigma_{i,l,m}^{TH}} Q_{TH}^i. \]  
(30)
Turning now to the firms in the economy, cost minimization by producers of non-transportation goods and services in eq. (10) results in the following zero-profit condition:

\[
P_j = \left( \sum_i \left( \delta_{i,j}^{NT} \sigma_i^{NT} \bar{P}_i^{j-\sigma_i^{NT}} + \delta_{H,j}^{NT} \sigma_i^{NT} \bar{w}_i^{j-\sigma_i^{NT}} + \delta_{K,j}^{NT} \sigma_i^{NT} r^{j-\sigma_i^{NT}} \right) \right)^{1/(1-\sigma_i^{NT})}, \quad \perp Y_j \tag{31}
\]

where \( P_j \) is the producer price of each non-transport commodity and \( r \) is the capital rental rate. The zero-profit condition for logistics firms in eq. (13) takes a somewhat different form, owing to the CET specification of production. In particular, transportation services are not traded, and so do not have an explicit price within the model. Producers therefore equate the marginal revenue from revenue-maximizing allocation of trips in (14) to the marginal cost from cost-minimizing transport service production in (13):

\[
\sum_i \left( \sum_j \left( \mu_{i,j,m}^{TP} \right)^{\sigma_i^{TP}} \left( Z_{i,m} P_{i,j,m}^{TP} \right)^{1-\sigma_i^{TP}} + \sum_j \left( \mu_{i,j,m}^{TC} \right)^{\sigma_i^{TC}} \left( Z_{i,m} P_{i,j,m}^{TC} \right)^{1-\sigma_i^{TC}} + \left( \mu_{i,j,m}^{TH} \right)^{\sigma_i^{TH}} \left( Z_{i,m} P_{i,j,m}^{TH} \right)^{1-\sigma_i^{TH}} \right) \right)^{1/(1-\sigma_i^{TP})} = \left( \sum_i \left( \delta_{i,m}^{TP} \sigma_i^{TP} \bar{P}_i^{j-\sigma_i^{TP}} + \left( \delta_{H,m}^{TP} \right)^{\sigma_i^{TP}} \bar{w}_i^{j-\sigma_i^{TP}} + \left( \delta_{K,m}^{TP} \right)^{\sigma_i^{TP}} r^{j-\sigma_i^{TP}} \right) \right)^{1/(1-\sigma_i^{TP})} \quad \perp Y_m \tag{32}
\]

The left-hand side of the foregoing expression clearly demonstrates that the impact of congestion is identical to a tax on trips that is differentiated by link. This result turns out to be very useful, because it enables the level of congestion, \( Z_{i,m} \), to be modeled as an endogenous, nonlinear tax. We elaborate on this point below.

The associated conditional demands for inputs of intermediate commodities, labor and capital are found by applying Shepard’s lemma to the right-hand sides of (31) and (32):

\[
\tilde{X}_{i,j} = \left( \delta_{i,j}^{NT} \sigma_i^{NT} \left( \bar{P}_i \right)^{j-\sigma_i^{NT}} \left( P_j \right)^{\sigma_i^{NT}} \right) Y_j, \tag{33}
\]

\[
\tilde{X}_{i,m} = \left( \delta_{i,m}^{TP} \sigma_i^{TP} \left( \bar{P}_i \right)^{j-\sigma_i^{TP}} \right) \left( \sum_i \left( \delta_{i,m}^{TP} \sigma_i^{TP} \bar{P}_i^{j-\sigma_i^{TP}} + \left( \delta_{H,m}^{TP} \right)^{\sigma_i^{TP}} \bar{w}_i^{j-\sigma_i^{TP}} + \left( \delta_{K,m}^{TP} \right)^{\sigma_i^{TP}} r^{j-\sigma_i^{TP}} \right) \right)^{1/(1-\sigma_i^{TP})} Y_m, \tag{34}
\]

\[
\tilde{h}_j = \left( \delta_{H,j}^{NT} \sigma_i^{NT} \bar{w}_i^{j-\sigma_i^{NT}} \left( P_j \right)^{\sigma_i^{NT}} \right) Y_j, \tag{35}
\]

32
\[
\tilde{h}_m = \left( \delta_{H,m}^T \right)^{\alpha_m^T} \tilde{w}^{1-\alpha_m^T} \left( \sum_i \left( \delta_{i,m}^T \right)^{\alpha_i^T} \tilde{P}_i^{1-\alpha_i^T} + \left( \delta_{H,m}^T \right)^{\alpha_m^T} \tilde{w}^{1-\alpha_m^T} + \left( \delta_{K,m}^T \right)^{\alpha_m^T} \tilde{r}^{1-\alpha_m^T} \right) \right)^{\alpha_m^T \left( 1 - \alpha_m^T \right)} Y_m, \tag{36}\]

\[
k_j = \left( \delta_{K,j}^T \right)^{\alpha_j^T} \tilde{r}^{1-\alpha_j^T} \left( p_j \right)^{\alpha_j^T} Y_j, \tag{37}\]

\[
k_m = \left( \delta_{K,m}^T \right)^{\alpha_m^T} \tilde{r}^{1-\alpha_m^T} \left( \sum_i \left( \delta_{i,m}^T \right)^{\alpha_i^T} \tilde{P}_i^{1-\alpha_i^T} + \left( \delta_{H,m}^T \right)^{\alpha_m^T} \tilde{w}^{1-\alpha_m^T} + \left( \delta_{K,m}^T \right)^{\alpha_m^T} \tilde{r}^{1-\alpha_m^T} \right) \right)^{\alpha_m^T \left( 1 - \alpha_m^T \right)} Y_m. \tag{38}\]

As well, the associated conditional supplies for trips are found by invoking Shepard’s lemma on the left-hand side of (32):

\[
q_{j,l,m}^{TF} = Z_{j,m}^{\psi_l^T} \left( \mu_{j,m}^{TF} \right)^{\psi_l^T} \left( p_{j,m}^{TF} \right)^{-\psi_l^T} \left( \sum_i \left( \delta_{i,m}^T \right)^{\alpha_i^T} \tilde{P}_i^{1-\alpha_i^T} + \left( \delta_{H,m}^T \right)^{\alpha_m^T} \tilde{w}^{1-\alpha_m^T} + \left( \delta_{K,m}^T \right)^{\alpha_m^T} \tilde{r}^{1-\alpha_m^T} \right) \right)^{\alpha_m^T \left( 1 - \alpha_m^T \right)} Y_m. \tag{39}\]

\[
q_{j,l,m}^{TC} = Z_{j,m}^{\psi_l^T} \left( \mu_{j,m}^{TC} \right)^{\psi_l^T} \left( p_{j,m}^{TC} \right)^{-\psi_l^T} \left( \sum_i \left( \delta_{i,m}^T \right)^{\alpha_i^T} \tilde{P}_i^{1-\alpha_i^T} + \left( \delta_{H,m}^T \right)^{\alpha_m^T} \tilde{w}^{1-\alpha_m^T} + \left( \delta_{K,m}^T \right)^{\alpha_m^T} \tilde{r}^{1-\alpha_m^T} \right) \right)^{\alpha_m^T \left( 1 - \alpha_m^T \right)} Y_m. \tag{40}\]

\[
q_{j,l,m}^{TH} = Z_{j,m}^{\psi_l^T} \left( \mu_{j,m}^{TH} \right)^{\psi_l^T} \left( p_{j,m}^{TH} \right)^{-\psi_l^T} \left( \sum_i \left( \delta_{i,m}^T \right)^{\alpha_i^T} \tilde{P}_i^{1-\alpha_i^T} + \left( \delta_{H,m}^T \right)^{\alpha_m^T} \tilde{w}^{1-\alpha_m^T} + \left( \delta_{K,m}^T \right)^{\alpha_m^T} \tilde{r}^{1-\alpha_m^T} \right) \right)^{\alpha_m^T \left( 1 - \alpha_m^T \right)} Y_m. \tag{41}\]

The zero profit condition corresponding to the cost-minimizing allocation of freight trips by mode and link in eq (12) is:

\[
P_{j}^{TF} = \left( \sum_i \sum_m \left( \gamma_{j,l,m}^{TF} \right)^{\alpha_j^T} \left( p_{j,l,m}^{TF} \right)^{-\alpha_j^T} \right)^{1-\alpha_j^T} Y_j, \tag{42}\]

The associated conditional demands for freight trips by link, mode and commodity are:

\[
q_{j,l,m}^{TF} = \left( \gamma_{j,l,m}^{TF} \right)^{\alpha_j^T} \left( p_{j,l,m}^{TF} \right)^{-\alpha_j^T} \left( p_j \right)^{\alpha_j^T} Y_j. \tag{43}\]

Finally, using eq. (11), the consumer price of non-transport commodities, \( \tilde{P}_i \), is given by the following zero-profit condition:

\[
\tilde{P}_i = P_i^{TF} / \chi_i^{TF} + P_i, \tag{44}\]

whose first term indicates the transportation margin. The associated demands are

\[
Q_i^{TF} = \tilde{Y}_i / \chi_i^{TF}, \tag{45}\]
Y_i = \tilde{Y}_i. \quad (46)

7.2. Market Clearance Conditions

Substituting eqs. (35), (36) and (46) into (16) and (37), (38) and (46) into (17) yields the supply-demand balances for labor and capital

\[
\tilde{H} = \sum_j \left( \delta_{H,j}^{NT} \right)^{\alpha_j^{NT}} \tilde{w}^{-\alpha_j^{NT}} \left( \bar{P}_j \right)^{\alpha_j^{NT}} Y_j
\]

\[
+ \sum_m \left( \delta_{H,m}^{NT} \right)^{\alpha_j^{NT}} \tilde{w}^{-\alpha_j^{NT}} \left( \sum_i \left( \delta_{i,m}^{NT} \right)^{\alpha_j^{NT}} \tilde{P}_i^{1-\sigma_j^e} + \left( \delta_{H,m}^{NT} \right)^{\alpha_j^{NT}} \tilde{w}^{-\sigma_j^e} + \left( \delta_{K,m}^{NT} \right)^{\alpha_j^{NT}} \bar{r}^{-\sigma_j^e} \right) \right) Y_m, \quad \perp \tilde{w} \quad (47)
\]

\[
K = \sum_j \left( \delta_{K,j}^{NT} \right)^{\alpha_j^{NT}} \bar{r}^{-\alpha_j^{NT}} \left( \bar{P}_j \right)^{\alpha_j^{NT}} Y_j
\]

\[
+ \sum_m \left( \delta_{K,m}^{NT} \right)^{\alpha_j^{NT}} \bar{r}^{-\alpha_j^{NT}} \left( \sum_i \left( \delta_{i,m}^{NT} \right)^{\alpha_j^{NT}} \bar{P}_i^{1-\sigma_j^e} + \left( \delta_{K,m}^{NT} \right)^{\alpha_j^{NT}} \bar{r}^{-\sigma_j^e} \right) \right) Y_m, \quad \perp \bar{r} \quad (48)
\]

Substituting eqs. (33), (34) and (46) into (15) yields the market clearance condition for delivered non-transport commodities:

\[
\tilde{Y}_i = \sum_j \left( \delta_{i,j}^{NT} \right)^{\alpha_j^{NT}} \left( \bar{P}_i \right)^{\alpha_j^{NT}} \left( \bar{P}_j \right)^{\alpha_j^{NT}} Y_j
\]

\[
+ \sum_m \left( \delta_{i,m}^{NT} \right)^{\alpha_j^{NT}} \left( \bar{P}_i \right)^{\alpha_j^{NT}} \left( \bar{P}_m \right)^{\alpha_j^{NT}} Y_m + \left( \beta_i^e \right)^{\alpha_j^{NT}} \bar{P}_i^{1-\sigma_j^e} \tilde{C}_i, \quad \perp \bar{P}_i \quad (49)
\]

while the corresponding equation for non-transport firms’ outputs is given by (46):

\[
Y_i = \tilde{Y}_i. \quad \perp P_i \quad (46')
\]

We note that a similar condition for the services produced by transportation firms (Y_m) does not exist, as we assume that there are only markets for trips.

The balance between supply and demand for the final use of the commodity-retail transport aggregate is given by (19), and is complementary to the composite final commodity price:

\[
\tilde{C}_i = \alpha_i^{\sigma_i^e} \bar{P}_i^{-\sigma_i^e} \sigma_i^e U, \quad \perp \bar{P}_i \quad (19')
\]
which enables us to specify analogous conditions for the retail, commuting and freight mobility aggregates, given by (22), (25) and (45):

\[
Q^{TC}_i = (\beta^{TC}_i)^{\alpha^{TC}_i} \left( P^{TC}_i \right)^{-\alpha^{TC}_i} \tilde{P}^{TC}_i \tilde{C}_i, \quad \perp P^{TC} \quad (22')
\]

\[
Q^{TH} = (\beta^{TH})^{\alpha^{TH}} \left( P^{TH} \right)^{-\alpha^{TH}} \tilde{W}^{TH} \tilde{H}, \quad \perp P^{TH} \quad (25')
\]

\[
Q^{TF}_j = \bar{Y}_j / \chi^{TF}_j. \quad \perp P^{TF} \quad (45')
\]

Supply-demand equilibria for trips, which are complementary to mode- and link-specific marginal trip costs, are found by equating (27) and (39), (28) and (40), and (43) and (41):

\[
Z_{l,m}^{\psi^*} \left( P^{TF}_{l,j,m} \right)^{\psi^*} \left( P^{TF}_{j,l,m} \right)^{\psi^*} \left( \sum_i \left( \delta^{T}_l \right)^{\alpha^*} \tilde{P}^{T}_i + \left( \delta^{H}_{l,m} \right)^{\alpha^*} \tilde{W}^{H} \tilde{C}_i \right) Y_m
\]

\[
= \left( \gamma^{TF}_{l,j,m} \right)^{\alpha^*} \left( P^{TF}_{l,j,m} \right)^{-\alpha^*} \left( P^{TF}_{j,l,m} \right)^{-\alpha^*} \tilde{Q}^{TF}_j. \quad \perp P^{TF} \quad (50)
\]

\[
Z_{l,m}^{\psi^*} \left( \mu^{TC}_{l,j,m} \right)^{\psi^*} \left( \mu^{TC}_{j,l,m} \right)^{\psi^*} \left( \sum_i \left( \delta^{T}_l \right)^{\alpha^*} \tilde{P}^{T}_i + \left( \delta^{H}_{l,m} \right)^{\alpha^*} \tilde{W}^{H} \tilde{C}_i \right) Y_m
\]

\[
= \left( \gamma^{TC}_{l,j,m} \right)^{\alpha^*} \left( P^{TC}_{l,j,m} \right)^{-\alpha^*} \left( P^{TC}_{j,l,m} \right)^{-\alpha^*} \tilde{Q}^{TC}_j. \quad \perp P^{TC} \quad (51)
\]

\[
Z_{l,m}^{\psi^*} \left( \mu^{TH}_{l,m} \right)^{\psi^*} \left( \mu^{TH}_{j,m} \right)^{\psi^*} \left( \sum_i \left( \delta^{T}_l \right)^{\alpha^*} \tilde{P}^{T}_i + \left( \delta^{H}_{l,m} \right)^{\alpha^*} \tilde{W}^{H} \tilde{C}_i \right) Y_m
\]

\[
= \left( \gamma^{TH}_{l,m} \right)^{\alpha^*} \left( P^{TH}_{l,m} \right)^{-\alpha^*} \left( P^{TH}_{j,m} \right)^{-\alpha^*} \tilde{Q}^{TH}. \quad \perp P^{TH} \quad (52)
\]

A particularly attractive feature of the model is the fact that the value of time exhibits complementary slackness with respect to the representative agent’s time budget constraint. The associated market clearance condition is derived by substituting eqs. (19), (29) and (30) into the representative agent’s time budget constraint, (9):

\[
\sum_i \sum_m \tau^{TC}_{i,m} \left( \sum_i \left( \gamma^{TC}_{i,j,m} \right)^{\alpha^*} \left( P^{TC}_{i,j,m} \right)^{\alpha^*} \tilde{Q}^{TC}_i + \left( \gamma^{TH}_{i,m} \right)^{\alpha^*} \left( P^{TH}_{i,m} \right)^{-\alpha^*} \tilde{Q}^{TH} \right) + \left( \beta^{TH} \right)^{\alpha^*} \tilde{W}^{TH} \tilde{H} + \alpha^{TH}_e \theta^{\alpha^*} e^{\alpha^*} U \leq \tilde{T}. \quad \perp \theta \quad (53)
\]
Because $\theta$ is the Lagrange multiplier on a constraint which takes into account the fully endogenous price, supply and demand responses across the entire spectrum of markets in the economy (as opposed to just transportation), it represents the true general equilibrium value of time. It is also useful to observe that in this expression the mode- and link-specific time costs play the role of differentiated taxes, whose values are endogenous determined by trip volumes according to the capacity restraint formula (1)-(2).

The final market clearance condition is a placeholder equation that specifies the quantity of “utility goods” as the ratio of the representative agent’s aggregate income, $\Omega$, to the unit expenditure index. This expression is complementary to unit expenditure:

$$U = \frac{\Omega}{e}. \quad \perp e \quad (54)$$

7.3. Income Balance Conditions and Auxiliary Variables

Income-expenditure balance is defined by the representative agent’s money budget constraint, (8), which is complementary to aggregate income:

$$\sum_{i} \tilde{p}_i \tilde{C}_i \leq \theta H + rK. \quad \perp \Omega \quad (8')$$

The auxiliary variables in the model are the average trip times by mode and link ($\tau_{l,m}$) in eq. (53) and the congestion penalty parameter ($Z_{l,m}$) in eqs. (32) and (50)-(52). Assuming that $Z_{l,m}$ can be expressed as a parametric function of $\tau_{l,m}$ (e.g., as in Mayeres and Proost, 1997), we may specify two auxiliary equations that are complementary to these variables:

$$\tau_{l,m} = \tau_{l,m}^0 \left( 1 + 0.15 \left( \frac{\sum_{i} (d_{l,i,m}^{TC} + q_{l,i,m}^{TF}) + q_{l,m}^{TH}}{K_{l,m}} \right)^4 \right), \quad \perp \tau_{l,m} \quad (55)$$

$$Z_{l,m}(\tau_{l,m}). \quad \perp Z_{l,m} \quad (56)$$

7.4. General Equilibrium in Complementarity Format

Given the above, we can now specify the general equilibrium of the economy as follows:
3 + 5I + M zero profit equations (18), (21), (24), (27)-(28), (31)-(32), (42) and (44) in as many unknown activity variables \((U, \tilde{C}, \tilde{H}, Q_i^{TC}, Q_j^{TH}, Y_j, Y_m, \tilde{Y}_i, Q_f^{TP})\).

5 + 5I + (1 + 2I) \((L \times M)\) income balance equations (19'), (22'), (25'), (45')-(46'), and (47)-(54), in as many unknown price variables \((\tilde{P}_i, P_i^{TC}, P_j^{TH}, P_j, \tilde{\omega}, r, \tilde{P}_i, P_{j,l,m}^{TC}, P_{l,m}^{TH}, P_{l,m}^{TP}, \theta, \varepsilon)\).

A single income balance condition \((8')\) in one unknown income level \((\Omega)\), and

The \(2(L \times M)\) auxiliary constraints (55) and (56) in as many unknown auxiliary variables \((\tau_{l,m}, Z_{l,m})\).

The CGE model consists of the paired, stacked vectors of \(9 + 10I + M + (3 + 2I) \((L \times M)\) variables, \(b = \text{vec}[U, \tilde{C}, \tilde{H}, Q_i^{TC}, Q_j^{TH}, Y_j, Y_m, \tilde{Y}_i, Q_f^{TP}, \tilde{P}_i, P_i^{TC}, P_j^{TH}, P_j, \tilde{\omega}, r, \tilde{P}_i, P_{j,l,m}^{TC}, P_{l,m}^{TH}, P_{l,m}^{TP}, \theta, \varepsilon, \Omega, \tau_{l,m}, Z_{l,m}]\), and \(9 + 10I + M + (3 + 2I) \((L \times M)\) equations (18), (21), (24), (27)-(28), (31)-(32), (42)-(44), (19'), (22'), (25'), (45')-(46'), (47)-(54), (8'), (55)-(56), which we denote \(\Xi(b)\). The latter is the excess demand correspondence of the economy. By setting up the model in this way, the economy can be cast as a square system of nonlinear inequalities known as a mixed complementarity problem (Ferris and Pang, 1997; Ferris and Kanzow, 2002):

\[\Xi(b) \geq 0, \quad b \geq 0, \quad b' \Xi(b) = 0,\]

which is straightforward to express and solve using computational tools such as the MPSGE subsystem (Rutherford, 1999) for GAMS (Brooke et al, 1998) in conjunction with the PATH solver (Dirkse and Ferris, 1995).
REFERENCES


Figure 1. Nested CES Utility and Production Functions Used in the Model

\[ \begin{align*}
\sigma^U & \quad U \\
\Lambda & \quad \leftarrow \tilde{C}_i \rightarrow \\
\sigma^C_i & \quad Q_{i}^{TC} \\
\sigma^T_i & \quad q_{i,j,m}^{TC} \\
\end{align*} \]

A. Utility

\[ \begin{align*}
\hat{Y}_j & \\
\sigma = 0 & \\
Q_{j}^{TP} & \\
\sigma^T_j & \\
\leftarrow q_{j,j,m}^{TP} \rightarrow \\
\end{align*} \]

B. Production: Non-Transport Firms \((i \in I)\)

\[ \begin{align*}
\psi^T_m & \\
\leftarrow q_{j,j,m}^{TP} \rightarrow \\
\leftarrow q_{i,i,m}^{TC} \rightarrow \\
\leftarrow q_{i,m}^{TH} \rightarrow \\
Y_m & \\
\sigma^T_m & \\
\leftarrow \tilde{X}_{i,m} \rightarrow \\
\hat{h}_m & \\
k_m & \\
\end{align*} \]

C. Production: Transportation Firms \((m \in M)\)

\[ \begin{align*}
\leftarrow \tilde{X}_{i,m} \rightarrow \\
\sigma^T_m & \\
\end{align*} \]

Notes: \(U = \) utility; \(\Lambda = \) leisure; \(\tilde{C}_i = \) consumption goods-retail mobility composite; \(\tilde{C}_i = \) consumption goods-freight mobility composite; \(Q_{i}^{TC} = \) retail mobility; \(\sigma^U, \sigma^C_i = \) goods-leisure and transport-goods, substitution elasticities; \(\hat{Y}_j = \) delivered non-transport goods; \(Y_j = \) non-transport goods production; \(Y_m = \) transport services production; \(Q_{j}^{TP} = \) freight mobility; \(\tilde{X}_{i,j}, \tilde{X}_{i,m} = \) intermediate inputs; \(\tilde{h}_j, \tilde{h}_m = \) labor inputs; \(k_j, k_m = \) capital inputs; \(\sigma^T_j, \sigma^T_m = \) firm input substitution elasticities; \(q_{j,j,m}^{TP}, q_{i,i,m}^{TC}, q_{i,m}^{TH} = \) freight, retail and commuting trips; \(\psi^T_m = \) elasticity of transformation of transport services into trips; \(\sigma^T_j, \sigma^T_i, \sigma^{TH} = \) freight, retail and commuting trip mode-link substitution elasticities.