7

Computation of Standard Errors

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INTRODUCTION

As shown in Chapter 4, replicates have to be used for the computation of the standard error for any population estimate. This chapter will give examples of such computations.

For PISA, the Fay's variant of the Balanced Repeated Replication (BRR) is used. The general formula for computing the standard error with this method is:

$$\sigma^2_{(\hat{\theta})} = \frac{1}{G(1-k)^2}\sum_{i=1}^{G}(\hat{\theta}_{(i)} - \hat{\theta})^2$$

Since the PISA databases include 80 replicates and since the Fay coefficient was set to 0.5, the above formula can be simplified as follows:

$$\sigma^2_{(\hat{\theta})} = \frac{1}{G(1-k)^2}\sum_{i=1}^{G}(\hat{\theta}_{(i)} - \hat{\theta})^2 = \frac{1}{80(1-0.5)^2}\sum_{i=1}^{80}(\hat{\theta}_{(i)} - \hat{\theta})^2 = \frac{1}{20}\sum_{i=1}^{80}(\hat{\theta}_{(i)} - \hat{\theta})^2$$

THE STANDARD ERROR ON UNIVARIATE STATISTICS FOR NUMERICAL VARIABLES

To compute the mean and its respective standard error, it is necessary to first compute this statistic by weighting the data with the student final weight, *i.e.* `w_fstuwt`, and then to compute 80 other means, each of them by weighting the data with one of the 80 replicates, *i.e.* `w_fstr1` to `w_fstr80`.

Box 7.1 presents the SAS® syntax for computing these 81 means based on the social background index, denoted HISEI, for the PISA 2003 data for Germany. Table 7.1 presents the HISEI final estimates as well as the 80 replicate estimates.

Box 7.1 **SAS® syntax for computing 81 means (*e.g.* PISA 2003)**

```
data temp;
        set pisa2003.stud;
        if (cnt="DEU");
        keep cnt schoolid stidstd w_fstuwt w_fstr1-w_fstr80 hisei;
run;
proc means data=temp VARDEF=WGT;
        VAR hisei;
        WEIGHT w_fstuwt;
run;
proc means data=temp VARDEF=WGT;
        VAR hisei;
        WEIGHT w_fstr1;
run;
proc means data=temp VARDEF=WGT;
        VAR hisei;
        WEIGHT w_fstr2;
run;
.
.
.
proc means data=temp VARDEF=WGT;
        VAR hisei;
        WEIGHT w_fstr79;
run;
proc means data=temp VARDEF=WGT;
        VAR hisei;
        WEIGHT w_fstr80;
run;
```

Table 7.1
HISEI mean estimates

Weight	Mean estimate	Weight	Mean estimate
Final weight	**49.33**		
Replicate 1	49.44	Replicate 41	49.17
Replicate 2	49.18	Replicate 42	49.66
Replicate 3	49.12	Replicate 43	49.18
Replicate 4	49.46	Replicate 44	49.04
Replicate 5	49.24	Replicate 45	49.42
Replicate 6	49.34	Replicate 46	49.72
Replicate 7	49.13	Replicate 47	49.48
Replicate 8	49.08	Replicate 48	49.14
Replicate 9	49.54	Replicate 49	49.57
Replicate 10	49.20	Replicate 50	49.36
Replicate 11	49.22	Replicate 51	48.78
Replicate 12	49.12	Replicate 52	49.53
Replicate 13	49.33	Replicate 53	49.27
Replicate 14	49.47	Replicate 54	49.23
Replicate 15	49.40	Replicate 55	49.62
Replicate 16	49.30	Replicate 56	48.96
Replicate 17	49.24	Replicate 57	49.54
Replicate 18	48.85	Replicate 58	49.14
Replicate 19	49.41	Replicate 59	49.27
Replicate 20	48.82	Replicate 60	49.42
Replicate 21	49.46	Replicate 61	49.56
Replicate 22	49.37	Replicate 62	49.75
Replicate 23	49.39	Replicate 63	48.98
Replicate 24	49.23	Replicate 64	49.00
Replicate 25	49.47	Replicate 65	49.35
Replicate 26	49.51	Replicate 66	49.27
Replicate 27	49.35	Replicate 67	49.44
Replicate 28	48.89	Replicate 68	49.08
Replicate 29	49.44	Replicate 69	49.09
Replicate 30	49.34	Replicate 70	49.15
Replicate 31	49.41	Replicate 71	49.29
Replicate 32	49.18	Replicate 72	49.29
Replicate 33	49.50	Replicate 73	49.08
Replicate 34	49.12	Replicate 74	49.25
Replicate 35	49.05	Replicate 75	48.93
Replicate 36	49.40	Replicate 76	49.45
Replicate 37	49.20	Replicate 77	49.13
Replicate 38	49.54	Replicate 78	49.45
Replicate 39	49.32	Replicate 79	49.14
Replicate 40	49.35	Replicate 80	49.27

The mean that will be reported is equal to 49.33, *i.e.* the estimate obtained with the student final weight w_fstuwt. The 80 replicate estimates are just used to compute the standard error on the mean of 49.33.

There are three major steps for computing the standard error:

1. Each replicate estimate will be compared with the final estimate 49.33 and the difference will be squared. Mathematically, it corresponds to $(\hat{\theta}_{(i)} - \hat{\theta})^2$ or in this particular case, $(\hat{\mu}_{(i)} - \hat{\mu})^2$. For the first replicate, it will be equal to $(49.44 - 49.33)^2 = 0.0140$. For the second replicate, it corresponds to $(49.18 - 49.33)^2 = 0.0228$. Table 7.2 presents the squared differences.

2. The sum of the squared differences is computed, and then divided by 20. Mathematically, it corresponds to $\frac{1}{20}\sum_{i=1}^{80}(\hat{\mu}_{(i)} - \hat{\mu})^2$. In the example, the sum is equal to (0.0140 + 0.0228 + ... + 0.0354 + 0.0031- = 3.5195. The sum divided by 20 is therefore equal to $\frac{3.5195}{20} = 0.1760$. This value represents the sampling variance on the mean estimate for HISEI.

3. The standard error is equal to the square root of the sampling variance, *i.e.* $\sigma_{(\hat{\mu})} = \sqrt{\sigma^2_{(\hat{\mu})}} = \sqrt{0.1760} = 0.4195$.

This means that the sampling distribution on the HISEI mean for Germany has a standard error of 0.4195. This value also allows building a confidence interval around this mean. With a risk of type I error equal to 0.05, usually denoted α, the confidence interval will be equal to:

[49.33 – (1.96*0.4195);49.33 + (1.96*0.4195)]

[48.51;50.15]

In other words, there are 5 chances out of 100 that an interval formed in this way will fail to capture the population mean. It also means that the German population mean for HISEI is significantly different from, for example, a value of 51, as this number is not included in the confidence interval.

Chapter 11 will show how this standard error can be used for comparisons either between two or several countries, or between subpopulations within a particular country.

Table 7.2
Squared differences between replicate estimates and the final estimate

Weight	Squared difference	Weight	Squared difference
Replicate 1	0.0140	Replicate 41	0.0239
Replicate 2	0.0228	Replicate 42	0.1090
Replicate 3	0.0421	Replicate 43	0.0203
Replicate 4	0.0189	Replicate 44	0.0818
Replicate 5	0.0075	Replicate 45	0.0082
Replicate 6	0.0002	Replicate 46	0.1514
Replicate 7	0.0387	Replicate 47	0.0231
Replicate 8	0.0583	Replicate 48	0.0349
Replicate 9	0.0472	Replicate 49	0.0590
Replicate 10	0.0167	Replicate 50	0.0014
Replicate 11	0.0124	Replicate 51	0.3003
Replicate 12	0.0441	Replicate 52	0.0431
Replicate 13	0.0000	Replicate 53	0.0032
Replicate 14	0.0205	Replicate 54	0.0086
Replicate 15	0.0048	Replicate 55	0.0868
Replicate 16	0.0009	Replicate 56	0.1317
Replicate 17	0.0074	Replicate 57	0.0438
Replicate 18	0.2264	Replicate 58	0.0354
Replicate 19	0.0077	Replicate 59	0.0034
Replicate 20	0.2604	Replicate 60	0.0081
Replicate 21	0.0182	Replicate 61	0.0563
Replicate 22	0.0016	Replicate 62	0.1761
Replicate 23	0.0041	Replicate 63	0.1173
Replicate 24	0.0093	Replicate 64	0.1035
Replicate 25	0.0199	Replicate 65	0.0008
Replicate 26	0.0344	Replicate 66	0.0030
Replicate 27	0.0007	Replicate 67	0.0139
Replicate 28	0.1919	Replicate 68	0.0618
Replicate 29	0.0139	Replicate 69	0.0557
Replicate 30	0.0001	Replicate 70	0.0324
Replicate 31	0.0071	Replicate 71	0.0016
Replicate 32	0.0215	Replicate 72	0.0011
Replicate 33	0.0302	Replicate 73	0.0603
Replicate 34	0.0411	Replicate 74	0.0052
Replicate 35	0.0778	Replicate 75	0.1575
Replicate 36	0.0052	Replicate 76	0.0157
Replicate 37	0.0150	Replicate 77	0.0378
Replicate 38	0.0445	Replicate 78	0.0155
Replicate 39	0.0000	Replicate 79	0.0354
Replicate 40	0.0004	Replicate 80	0.0031
		Sum of squared differences	**3.5195**

THE SAS® MACRO FOR COMPUTING THE STANDARD ERROR ON A MEAN

Writing all the SAS® syntax to compute these 81 means and then transferring them into a Microsoft® Excel® spreadsheet to finally obtain the standard error would be very time consuming. Fortunately, SAS® macros simplify iterative computations. The software package will execute *N* times the commands included between the beginning command (DO I=1 TO N) and the ending command (END). Further, it also saves the results in a temporary file that can be used subsequently for the computation of the standard error.

Several SAS® macros have been written to simplify the main PISA computations as reported in the PISA initial reports. These macros have been saved in different files (with the extension .sas). Box 7.2 shows a SAS® syntax where a macro is called for computing the mean and standard error of the variable HISEI.

Box 7.2 **SAS® syntax for computing the mean of HISEI and its standard error (*e.g.* PISA 2003)**

```
libname PISA2003  “c:\pisa\2003\data\”;
options nofmterr notes;
run;

%include “c:\pisa\macro\proc_means_no_pv.sas”;

data temp1;
        set pisa2003.stud;
        if (cnt=“DEU”) ;
        w_fstr0=w_fstuwt;
        if (not missing (st03q01));
        keep  cnt schoolid stidstd hisei bsmj st01q01 st03q01
              w_fstr0-w_fstr80 ;
run;

%BRR_PROCMEAN(INFILE=temp1,
              REPLI_ROOT=w_fstr,
              BYVAR=cnt,
              VAR=hisei,
              STAT=mean,
              LIMIT=yes,
              LIMIT_CRITERIA=50 5 3 1,
              ID_SCHOOL=schoolid,
              OUTFILE=exercise1);
run;

proc print data=exercise1;
        var cnt stat sestat;
run;
```

After the definition of the SAS® library and a few options, the command (`%include“c:\pisa\macro\proc_means_no_pv.sas”;`) will create and save a new procedure for later use.

The `“data”` statement will create a temporary file by selecting from the PISA 2003 student database, the data for Germany (`if(cnt=“DEU”)`). To facilitate the iterative process, the final weight, `w_fstuwt`, is recoded with the same replicate root, *i.e.* `w_fstr`. The number 0 is added after this root to avoid any possible confusion with the 80 replicates.

As these iterative computations might be central-processing-unit (CPU) consuming, it is advised to reduce the size of the input database by selecting the variables requested to perform a set of analyses. This can be easily done using the `“keep”` statement. In the example:

- the three international identification variables are kept, *i.e.*
  - CNT for the alphanumerical country code,
  - SCHOOLID for the alphanumerical school code,
  - STIDSTD for the alphanumerical student code;
- the socio-economic index, denoted HISEI;
- the 81 final and replicate weights;
- a few other variables that will be used later in this chapter.

To ensure their efficiency, all SAS® macros start by creating a temporary data file that only includes the variables requested by the macro.

The next nine lines call the macro. Nine pieces of information need to be provided:

- The input data file (`INFILE=temp`).
- The root of the final and replicate weights (`REPLI_ROOT=w_fstr`).
- One or several breakdown variables (`BYVAR=cnt`).
- The variable on which an estimate and its respective standard error will be computed (VAR=hisei).
- The requested statistic (`STAT=mean`).
- The request of flagging statistics for not reaching one of the sample size minimal requirements (`LIMIT=yes`). The other alternative is no. If no sample size requirements are set *i.e.* (`LIMIT=no`), no additional information are required for the next two arguments.
- The sample size minimal requirements per cell as defined by the BYVAR statement:[1]
  - The minimal number of students.
  - The minimal number of schools.
  - The minimal percentage of the population.
  - The *N* first variables of the `BYVAR` statement used for the computation of the percentages; it is recommended that the percentages are computed at the country level. For that purpose, the first variable included in the `BYVAR` statement will be CNT and the last component of the `LIMIT_CRITERIA` will be 1, indicating that only the first variable of the BYVAR statement will be used as a basis for the computation of the percentages.[2]
  - As a minimal sample size can be specified on the number of schools, it is necessary to indicate the variable name of the school identification (`ID_SCHOOL=schoolid`).
- The output data file in which the estimates and their respective standard errors will be stored (`OUTFILE=exercise1`).

From the temporary input data file denoted `"temp"`, this macro will compute per country the mean of HISEI and its standard error by using the 81 final and replicate weights denoted w_fstr0 to w_fstr80. The results will be stored in a file that will be labelled `"exercise1"`. This macro will return exactly the same values for the mean estimate and its respective standard error as the ones obtained through Table 7.1 and Table 7.2.

The structure of the output data file is presented in Table 7.3.

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Table 7.3

Output data file exercise1 from Box 7.2

CNT	STAT	SESTAT	FLAG_STUD	FLAG_SCH	FLAG_PCT
DEU	49.33	0.42	0	0	0

If the dataset had not been reduced to the data for Germany, then the number of rows in the output data file would be equal to the number of countries in the database.

As the three flags are equal to 0, the mean estimate has been computed on at least on 50 students, 5 schools and represents at least 3% of the population, *i.e.* Germany in this case. This last result is useless as the mean estimate was requested at the country level. These sample size requirements are more useful when statistics are requested at a subnational level. With a required percentage set at 95, the FLAG_PCT would be equal to 1. It would indicate in that particular case that more than 5% of the data for HISEI are missing.

There are a few restrictions, as well as a few options, with this macro:

- Only one input data file can be specified.
- The final and the replicate weights need to have the same root. The final weight will be assigned the number 0 while the 80 replicates, as already defined in the dataset, will range from 1 to 80.
- Several breakdown variables can be specified. For instance, if results per gender are needed, then the breakdown variables will be CNT and ST03Q01[3] (`BYVAR=cnt st03q01`).
- Only one numerical variable can be specified in the VAR statement.
- Only one statistic can be specified. The available statistics are presented in Table 7.4.
- Only one output data file can be specified.

Table 7.4

Available statistics with the PROC_MEANS_NO_PV macro[a]

Statistics available	Meaning
SUMWGT	Sum of the weight
MEAN	Mean
VAR	Variance
STD	Standard deviation
CV	Coefficient of variation
MEDIAN	Median
Q1	First quartile
Q3	Third quartile
QRANGE	Range between Q1 and Q3
Px	Percentile, with P1, P5, P10, P25, P50, P75, P90, P95 and P99

a. Some other statistics are also available through the PROC MEANS procedure in SAS®, such as the minimum, the maximum, the range, the number of observations, and so on. Nevertheless, they are not included in the table, either because it does not make sense to apply these statistics to the PISA data, or because Fay's method cannot be applied on these statistics. For instance, as no weights are set to 0 in any replicates, the minimum or maximum value for a particular variable will always be the same. Therefore, the macro will return the value of 0, which is meaningless.

Box 7.3 presents the syntax for computing the standard deviation per gender and Table 7.5 sets out the structure of the output data file. As sample size limits are not requested, the output data file will not include the three flagging variables.

Box 7.3 **SAS® syntax for computing the standard deviation of HISEI and its standard error by gender (*e.g.* PISA 2003)**

```
%BRR_PROCMEAN(INFILE=temp1,
              REPLI_ROOT=w_fstr,
              BYVAR=cnt st03q01,
              VAR=hisei,
              STAT=std,
              LIMIT=no,
              LIMIT_CRITERIA=,
              ID_SCHOOL=,
              OUTFILE=exercise2);
run;
```

Table 7.5
Output data file exercise2 from Box 7.3

CNT	ST03Q01	STAT	SESTAT
DEU	1	16.12	0.29
DEU	2	16.34	0.23

THE STANDARD ERROR ON PERCENTAGES

For variables such as gender, the statistic of interest is usually the percentage per category. The procedure for estimating the standard error is identical to the procedure used for the estimation of the standard error on a mean or a standard deviation, *i.e.* per category of the variable, 81 percentages have to be computed.

Box 7.4 presents the SAS® syntax for running the macro that will compute the percentages and their respective standard errors for each category of the gender variable. The structure of the output data file is presented in Table 7.6.

Box 7.4 **SAS® syntax for computing the percentages and their standard errors for gender (*e.g.* PISA 2003)**

```
%include "c:\pisa\macro\proc_freq_no_pv.sas";

%BRR_FREQ( INFILE=temp1,
          REPLI_ROOT=w_fstr,
          BYVAR=cnt,
          VAR=st03q01,
          LIMIT=yes,
          LIMIT_CRITERIA=100 10 5 1,
          ID_SCHOOL=schoolid,
          OUTFILE=exercise3);
run;
```

Table 7.6
Output data file exercise3 from Box 7.4

CNT	ST03Q01	STAT	SESTAT	FLAG_STUD	FLAG_SCH	FLAG_PCT
DEU	1	49.66	1.04	0	0	0
DEU	2	50.34	1.04	0	0	0

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Table 7.7 presents the estimates of the percentage of females for the 81 weights and the squared differences. The percentage of females that will be reported is equal to 49.66, *i.e.* the percentage obtained with the final student weight.

As previously, there are three major steps for computing the standard error.

4. Each replicate estimate will be compared with the final estimate 49.66 and the difference will be squared. Mathematically, it corresponds to $(\hat{\pi}_{(i)} - \hat{\pi})^2$. For the first replicate, it will be equal to $(49.82 - 49.66)^2 = 0.0256$.

5. The sum of the squared differences is computed, and then divided by 20. Mathematically, it corresponds to $\frac{1}{20}\sum_{i=1}^{80}(\hat{\pi}_{(i)} - \hat{\pi})^2$. In the example, the sum is equal to (0.0252 + 0.1044 + ... + 0.3610 + 0.1313) = 21.4412. The sum divided by 20 is therefore equal to $\frac{21.4412}{20} = 1.07$. This value represents the sampling variance on the percentage estimate of females.

6. The standard error is equal to the square root of the sampling variance, *i.e.* $\sigma_{(\hat{\pi})} = \sqrt{\sigma^2_{(\hat{\pi})}} = \sqrt{1.07} = 1.035$.

Table 7.7
Percentage of girls for the final and replicate weights and squared differences

Weight	% estimate	Squared difference	Weight	% estimate	Squared difference
Final weight	**49.66**				
Replicate 1	49.82	0.03	Replicate 41	50.00	0.11
Replicate 2	49.98	0.10	Replicate 42	49.95	0.09
Replicate 3	49.44	0.05	Replicate 43	49.70	0.00
Replicate 4	49.32	0.11	Replicate 44	50.59	0.87
Replicate 5	49.39	0.07	Replicate 45	49.07	0.35
Replicate 6	49.06	0.36	Replicate 46	48.82	0.71
Replicate 7	48.59	1.14	Replicate 47	49.88	0.05
Replicate 8	48.85	0.66	Replicate 48	49.14	0.27
Replicate 9	49.06	0.36	Replicate 49	49.53	0.02
Replicate 10	49.72	0.00	Replicate 50	49.81	0.02
Replicate 11	50.05	0.15	Replicate 51	49.87	0.04
Replicate 12	49.31	0.13	Replicate 52	49.82	0.02
Replicate 13	49.29	0.13	Replicate 53	49.42	0.06
Replicate 14	49.47	0.04	Replicate 54	48.99	0.45
Replicate 15	49.90	0.06	Replicate 55	50.07	0.17
Replicate 16	50.82	1.35	Replicate 56	50.68	1.04
Replicate 17	49.11	0.30	Replicate 57	50.34	0.46
Replicate 18	49.51	0.02	Replicate 58	49.54	0.02
Replicate 19	49.79	0.02	Replicate 59	48.75	0.83
Replicate 20	50.75	1.18	Replicate 60	50.14	0.23
Replicate 21	50.24	0.33	Replicate 61	49.45	0.05
Replicate 22	49.79	0.02	Replicate 62	49.46	0.04
Replicate 23	49.87	0.04	Replicate 63	50.11	0.20
Replicate 24	49.37	0.08	Replicate 64	49.64	0.00
Replicate 25	49.50	0.02	Replicate 65	49.72	0.00
Replicate 26	49.82	0.02	Replicate 66	50.79	1.27
Replicate 27	49.92	0.07	Replicate 67	49.73	0.00
Replicate 28	49.55	0.01	Replicate 68	49.96	0.09
Replicate 29	50.22	0.31	Replicate 69	50.31	0.42
Replicate 30	49.16	0.25	Replicate 70	49.17	0.24
Replicate 31	50.51	0.73	Replicate 71	50.10	0.19
Replicate 32	49.98	0.10	Replicate 72	49.93	0.07
Replicate 33	50.67	1.02	Replicate 73	49.55	0.01
Replicate 34	49.29	0.13	Replicate 74	49.42	0.06
Replicate 35	48.96	0.49	Replicate 75	49.60	0.00
Replicate 36	49.98	0.10	Replicate 76	49.45	0.05
Replicate 37	50.23	0.33	Replicate 77	49.80	0.02
Replicate 38	48.25	1.99	Replicate 78	49.91	0.07
Replicate 39	49.56	0.01	Replicate 79	49.06	0.36
Replicate 40	49.66	0.00	Replicate 80	50.02	0.13
			Sum of squared differences		**21.44**

The same process can be used for the percentage of males. It should be noted that the standard error for males is equal to the one for females. Indeed, it can be mathematically shown that the standard error on π is equal to the standard error on 1-π, *i.e.* $\sigma_{(p)} = \sigma_{(1-p)}$. Nevertheless, if missing data for gender are kept in the data file, the standard error on the percentage of males can differ slightly from the standard error on the percentage of females.

Just as for the macro for numerical variables, more than one breakdown variable can be used. In PISA 2003, the first question in the student questionnaire provides the students' grade. German 15-year-olds are distributed from grade 7 to grade 11.

Box 7.5 presents the SAS® syntax for computing the percentage for each grade per gender and its standard error and Table 7.8 presents the output data file. The percentages within the VAR group variable add up to 100%. In this example, the percentages of pupils in grades 7 to 11 within gender and country add up to 100%.

Box 7.5 **SAS® syntax for computing the percentages and its standard errors for grades by gender (*e.g.* PISA 2003)**

```
%BRR_FREQ( INFILE=temp1,
           REPLI_ROOT=w_fstr,
           BYVAR=cnt st03Q01,
           VAR=st01q01,
           LIMIT=yes,
           LIMIT_CRITERIA=100 10 5 1,
           ID_SCHOOL=schoolid,
           OUTFILE=exercise4);
run;
```

As shown in Table 7.8, more males tend to be in lower grades than females and more females tend to be in upper grades in Germany. A few rows are flagged in Table 7.8. Grade 7 and grade 11 count less that 100 males and less than 100 females. Further, these four subpopulations (males in grades 7 and11 and females in grades 7 and 11) each represent less than 5% of the German population. Finally, the computations in grade 11 are based on less than ten schools both for males and females.

Table 7.8
Output data file exercise4 from Box 7.5

CNT	ST03Q01	ST01Q01	STAT	SESTAT	FLAG_STUD	FLAG_SCH	FLAG_PCT
DEU	1	7	1.15	0.26	1	0	1
DEU	1	8	13.09	0.83	0	0	0
DEU	1	9	59.33	1.00	0	0	0
DEU	1	10	26.28	1.08	0	0	0
DEU	1	11	0.17	0.08	1	1	1
DEU	2	7	2.28	0.45	1	0	1
DEU	2	8	16.92	1.04	0	0	0
DEU	2	9	60.32	1.06	0	0	0
DEU	2	10	20.41	0.79	0	0	0
DEU	2	11	0.08	0.05	1	1	1

THE STANDARD ERROR ON REGRESSION COEFFICIENTS

For any requested statistic, the computation of the estimate and its standard error will follow exactly the same procedure as the ones described for the mean of HISEI and for the percentages for gender. The remainder of this chapter will explain the use of two other SAS® macros developed for analysing PISA data.

The first macro is for simple linear regression analyses. Besides the seven arguments common to SAS® macros previously described in this manual, *i.e.* *(i)* INFILE=; *(ii)* REPLI_ROOT=; *(iii)* BYVAR=; *(iv)* OUTFILE=; *(v)* LIMIT=; *(vi)* LIMIT_CRITERIA=; and *(vii)* ID_SCHOOL=, two arguments need to be specified: the dependent variable and the independent variables. Only one dependent variable can be specified, whereas several independent variables can be specified.

Box 7.6 provides the syntax for running the simple linear regression macro. In this example, the dependent variable is the socio-economic index derived from the expected student job at the age of 30 (BSMJ) and the independent variables are the family socio-economic index (HISEI) and the student gender after recoding females into 1 and males into 0 (GENDER). Germany (DEU) and Austria (AUT) are selected.

Box 7.6 **SAS® syntax for computing regression coefficients, R^2 and its respective standard errors: Model 1 (*e.g.* PISA 2003)**

```
%include “c:\pisa\macro\proc_reg_no_pv.sas”;
data temp2;
   set pisa2003.stud;
   if (cnt in (“DEU”,“AUT”));
   w_fstr0=w_fstuwt;
   if (st03q01=1) then gender=1;
   if (st03q01=2) then gender=0;
   if (not missing(st03q01));
run;
%BRR_REG(  INFILE=temp2,
           REPLI_ROOT=w_fstr,
           VARDEP=bsmj,
           EXPLICA=hisei gender,
           BYVAR=cnt ,
           LIMIT=yes,
           LIMIT_CRITERIA=100 10 5 1,
           ID_SCHOOL=schoolid,
           OUTFILE=exercise5);
run;
```

Table 7.9 presents the structure of the output data file of the regression analysis in Box 7.6.

Table 7.9
Output data file exercise5 from Box 7.6

CNT	CLASS	STAT	SESTAT
AUT	Intercept	32.25	1.20
AUT	HISEI	0.36	0.02
AUT	GENDER	3.99	0.98
AUT	_RSQ_	0.13	0.02
DEU	Intercept	32.9	1.29
DEU	HISEI	0.37	0.03
DEU	GENDER	2.07	0.62
DEU	_RSQ_	0.12	0.02

There are two ways to determine whether the regression coefficients are significantly different from 0. The first method consists of building a confidence interval around the estimated regression coefficient. The confidence interval for the GENDER regression coefficient on BSMJ in Germany can be computed for a value of α equal to 0.05 as: [2.07 – (1.96*0.62);2.07 + (1.96*0.62)] = [0.85;3.29].

As the value 0 is not included in this confidence interval, the regression coefficient is significantly different from 0. As the value 0 was assigned to males and the value 1 to females in the GENDER variable, it can be concluded that on average, females have significantly higher job expectations in Germany.

Another way to test the null hypothesis of the regression coefficient consists of dividing the regression coefficient by its standard error. This procedure will standardise the regression coefficient. It also means that the sampling distribution of the standardised regression coefficient, under the null hypothesis, has an expected mean of 0 and a standard deviation of 1. Therefore, if the ratio of the regression coefficient to its standard error is lower than –1.96 or higher than 1.96, it will be considered as significantly different from 0.

Table 7.9 also includes the R^2 of the regression and its standard error. As several rows are necessary for reporting the results of the regression analysis, the outcomes of sample size requirements are included in another output file, denoted in this example exercise5_criteria. This file includes, per subpopulation defined by the variables included in the BYVAR statement, a row with the three flagging variables.

It should be mentioned that exercise6 will provide different results from exercise5. In exercise5, GENDER is considered as an explanatory variable. With exercise6, GENDER is used as a breakdown variable. In the second model, there is only one explanatory variable, *i.e.* HISEI.

Box 7.7 **SAS® syntax for computing regression coefficients, R^2 and its respective standard errors: Model 2 (*e.g.* PISA 2003)**

```
%BRR_REG(  INFILE=temp2,
           REPLI_ROOT=w_fstr,
           VARDEP=bsmj,
           EXPLICA=hisei,
           BYVAR=cnt gender,
           LIMIT=yes,
           LIMIT_CRITERIA=100 10 5 1,
           ID_SCHOOL=schoolid,
           OUTFILE=exercise6);
run;
```

Table 7.10
Output data file exercise6 from Box 7.7

CNT	GENDER	CLASS	STAT	SESTAT
AUT	0	Intercept	32.00	1.64
AUT	0	HISEI	0.37	0.03
AUT	0	_RSQ_	0.12	0.02
AUT	1	Intercept	36.49	1.52
AUT	1	HISEI	0.36	0.03
AUT	1	_RSQ_	0.11	0.02
DEU	0	Intercept	32.54	1.44
DEU	0	HISEI	0.37	0.03
DEU	0	_RSQ_	0.13	0.02
DEU	1	Intercept	35.33	1.66
DEU	1	HISEI	0.36	0.03
DEU	1	_RSQ_	0.10	0.02

Table 7.11
Output data file exercise6_criteria from Box 7.7

CNT	GENDER	FLAG_STUD	FLAG_SCH	FLAG_PCT
AUT	0	0	0	0
AUT	1	0	0	0
DEU	0	0	0	0
DEU	1	0	0	0

Table 7.10 presents the structure of the output data file for the second model in Box 7.7 and Table 7.11 presents the structure of the output file devoted to the sampling requirements.

THE STANDARD ERROR ON CORRELATION COEFFICIENTS

Box 7.8 and Table 7.12 present, respectively, the SAS® syntax and the structure of the output data file for running the macro devoted to computing a correlation between two and only two variables.

Box 7.8 **SAS® syntax for computing correlation coefficients and its standard errors (*e.g.* PISA 2003)**

```
%include "c:\pisa\macro\proc_corr_no_pv.sas";

%BRR_CORR(  INFILE=temp2,
            REPLI_ROOT=w_fstr,
            BYVAR=cnt,
            VAR1=hisei,
            VAR2=bsmj,
            LIMIT=yes,
            LIMIT_CRITERIA=100 10 5 1,
            ID_SCHOOL=schoolid,
            OUTFILE=exercise7);
run;
```

Table 7.12
Output data file exercise7 from Box 7.8

CNT	STAT	SESTAT	FLAG_STUD	FLAG_SCH	FLAG_PCT
AUT	0.34	0.02	0	0	0
DEU	0.34	0.02	0	0	0

CONCLUSION

This chapter described the computation of a standard error by using 80 replicates. For any given statistic, the procedure is the same.

Further, the SAS® syntax for running the SAS® macros, which were developed to facilitate the computation of the standard errors, has been provided in various examples.

However, all macros described in this chapter are for computing various statistics **without** using plausible values. Chapter 8 will describe how to conduct computation with plausible values.

Notes

1. The minimal numbers of students and schools are computed without weights and the minimal percentage of the population is computed with weights.

2. In general, PISA does not report estimates based on fewer than 30 students or less than 3% of students, unless otherwise stated.

3. In PISA 2006, the gender variable is ST04Q01.

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User's Guide

Preparation of data files

All data files (in text format) and the SAS® control files are available on the PISA website (*www.pisa.oecd.org*).

SAS® users

By running the SAS® control files, the PISA data files are created in the SAS® format. Before starting analysis, assigning the folder in which the data files are saved as a SAS® library.

For example, if the PISA 2000 data files are saved in the folder of "c:\pisa2000\data\", the PISA 2003 data files are in "c:\pisa2003\data\", and the PISA 2006 data files are in "c:\pisa2006\data\", the following commands need to be run to create SAS® libraries:

```
libname PISA2000 "c:\pisa2000\data\";
libname PISA2003 "c:\pisa2003\data\";
libname PISA2006 "c:\pisa2006\data\";
run;
```

SAS® syntax and macros

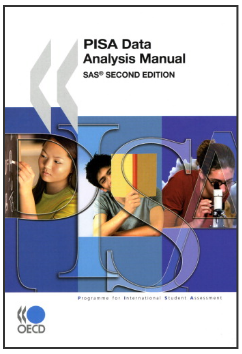
All syntaxes and macros in this manual can be copied from the PISA website (*www.pisa.oecd.org*). The 17 SAS® macros presented in Chapter 17 need to be saved under "c:\pisa\macro\", before staring analysis. Each chapter of the manual contains a complete set of syntaxes, which must be done sequentially, for all of them to run correctly, within the chapter.

Rounding of figures

In the tables and formulas, figures were rounded to a convenient number of decimal places, although calculations were always made with the full number of decimal places.

Country abbreviations used in this manual

AUS	Australia	FRA	France	MEX	Mexico
AUT	Austria	GBR	United Kingdom	NLD	Netherlands
BEL	Belgium	GRC	Greece	NOR	Norway
CAN	Canada	HUN	Hungary	NZL	New Zealand
CHE	Switzerland	IRL	Ireland	POL	Poland
CZE	Czech Republic	ISL	Iceland	PRT	Portugal
DEU	Germany	ITA	Italy	SVK	Slovak Republic
DNK	Denmark	JPN	Japan	SWE	Sweden
ESP	Spain	KOR	Korea	TUR	Turkey
FIN	Finland	LUX	Luxembourg	USA	United States



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