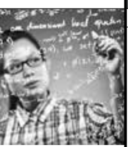
2

A Profile of Student Performance in Mathematics

This chapter compares student performance in mathematics across and within countries and economies. It discusses the PISA definition of literacy in mathematics and describes the tasks associated with each PISA proficiency level. The chapter then digs deep into the results of the mathematics assessment, showing gender differences in performance, trends in mathematics performance up to 2012, and differences in students' abilities to handle certain mathematics processes, such as formulating situations mathematically, and certain mathematics contents, such as *uncertainty and data*, and *space and shape*.



All adults, not just those with technical or scientific careers, now require adequate mathematics proficiency for personal fulfilment, employment and full participation in society. To one degree or another, mathematical concepts and processes are intrinsic to many daily tasks: from buying and selling goods and services, to cooking or planning a vacation, to explaining highly complex phenomena. Students about to leave compulsory education should thus have a solid understanding of these concepts and be able to apply them to solve problems that they encounter in their daily lives.

This chapter summarises the mathematics performance of students in PISA 2012. It describes how performance is defined, measured and reported, and then provides results from the paper-based assessment, showing what students are able to do in mathematics. After a summary of mathematics performance, it examines the ways in which this performance varies on subscales representing different aspects of mathematics. Annex B3 provides further results for 32 countries and economies that participated in the computer-based assessment, supplementing the paper-based scale with two others: the computer-based scale and the combined paper- and computer-based scale.

What the data tell us

- Of the 64 countries and economies with trend data up to 2012, 25 show an average annual improvement in mathematics performance, 25 show no change, and 14 show a deterioration in performance.
- Among countries and economies that have participated in every assessment since 2003, Brazil, Italy, Mexico, Poland, Portugal, Tunisia and Turkey show an average improvement in mathematics performance of more than 2.5 points per year.
- Germany, Hong Kong-China, Macao-China, Shanghai-China and Singapore improved in mathematics performance and their previous scores placed them at or above the OECD average.
- Between 2003 and 2012 Italy, Poland and Portugal reduced the proportion of low performers and increased the proportion of high performers. This was also observed in Israel, Qatar and Romania between 2006 and 2012, and in Ireland, Malaysia and the Russian Federation between 2009 and 2012.
- Boys perform better than girls in mathematics in 38 out of the 65 countries and economies that participated in PISA 2012, and girls outperform boys in 5 countries.

Box I.2.1. What does performance in PISA say about readiness for further education and a career?

To what extent is the performance of 15-year-olds in PISA predictive of further education and career readiness and success later in life? The transition from adolescence to early adulthood is a critical time in the social and intellectual development of young people. Once compulsory education is completed, adolescents have to make important decisions about post-secondary education, employment and other life choices that will have a major impact on their future learning and employment prospects as well as on their overall well-being. A decade-long study undertaken in Canada coupled data collected from the PISA assessment of 15-year-olds in 2000 with follow-ups conducted every two years through a national survey of those same students and parents (the Youth in Transition Survey). The results from this study show that having a solid foundation in the kinds of skills that PISA measures makes it much easier to advance in post-compulsory education. Reading scores in PISA, for example, are associated with the likelihood of students progressing from one grade level to another across grades 10 to 16. Some 37% of boys with a high reading score, i.e. in the top quintile of reading proficiency, attained grade 16 compared to just 3.4% of boys with low reading scores (bottom quintile). Similarly, 52.4% of girls with high reading scores attained grade 16 compared to 14.9% of girls with low reading scores. The results show that reading scores had a stronger association with grade progression during the post-secondary school years than with schooling up to grade 12, particularly for boys.

Equally important, the results also show that introducing a uniform increase of one standard deviation in reading scores results in a 17.4% reduction in the proportion of young men who leave formal education before completing secondary school and a 12.6% increase in the proportion of young men who attend post-secondary education.

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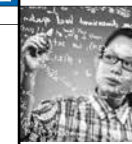


For girls, the effects of increased reading scores are also substantial. A one standard deviation increase in reading scores is associated with a 31.5% reduction in the proportion of girls who leave formal education before completing secondary school and an 11.4% increase in the share of young women who complete at least some post-secondary education. Even after adjusting for socio-economic status, both achievement in PISA and educational attainment are associated with a higher likelihood of continuing in education and a lower likelihood of proceeding to work or to a period of inactivity (OECD, 2010a).

To what extent are the differences in the performance of school systems, as observed in PISA, reflected in the skills of adults who have recently completed initial education and training? The Survey of Adult Skills, a product of the OECD Programme for the International Assessment of Adult Competencies (PIAAC), provides a way to assess this. Most adults aged 27 or under in participating countries correspond to the cohorts assessed in PISA in 2000, 2003, 2006 and 2009, when they were 15 years old.

The results from the Survey of Adult Skills show that, overall, there is a reasonably close correlation between countries' performance across the successive PISA assessments and the proficiency of the corresponding age cohorts in literacy and numeracy in the Skills Survey. Countries performing well in PISA in a given year (e.g. 2000) tend to show high performance among the corresponding age cohort (e.g. 27-year-olds) in the Survey of Adult Skills (PIAAC) and vice versa. This suggests that, at the country level, the reading and mathematics proficiency of an age cohort in PISA is a reasonably good predictor of the cohort's subsequent performance in literacy and numeracy as it moves through post-compulsory education and into the labour market. By implication, much of the difference in the literacy and numeracy proficiency of young adults today is likely related to the effectiveness of the instruction they received in primary and lower secondary school.

Of course, some caution is advised in comparing results of the two studies. The overlap between the target populations of the Survey of Adult Skills (PIAAC) and PISA is not complete; and while the concepts of literacy in the Skills Survey and reading literacy in PISA, and the concepts of numeracy in the Skills Survey and mathematical literacy in PISA are closely related, the measurement scales are not the same. In addition, the skills of 15-27 year-olds are subject to influences that vary across individuals and countries, including participation in post-secondary and tertiary education and the quality of these programmes, second-chance opportunities for low-skilled young adults, and characteristics of the labour market (OECD, 2013a and b).

A CONTEXT FOR COMPARING THE MATHEMATICS PERFORMANCE OF COUNTRIES AND ECONOMIES

Comparing mathematics performance, and educational performance more generally, poses numerous challenges. When teachers give a mathematics test in a classroom, students with varying abilities, attitudes and social backgrounds are required to respond to the same set of tasks. When educators compare the performance of schools, the same test is used across schools that may differ significantly in the structure and sequencing of their curricula, in the pedagogical emphases and instructional methods applied, and in the demographic and social contexts of their student populations. Comparing the performance of education systems across countries adds more layers of complexity, because students are given tests in different languages, and because the social, economic and cultural context of the countries that are being compared are often very different. However, while students within a country may learn in different contexts according to their home background and the school that they attend, their performance is measured against common standards, since, when they become adults, they will all face common challenges and have to compete for the same jobs. Similarly, in a global economy, the benchmark for success in education is no longer improvement by national standards alone, but increasingly, in relation to the best-performing education systems internationally. As difficult as international comparisons are, they are important for educators, and PISA goes to considerable lengths to ensure that such comparisons are valid and fair.

This section discusses countries' mathematics performance in the context of important economic, demographic and social factors that can influence assessment results. It provides a framework for interpreting the results that are presented later in the chapter.

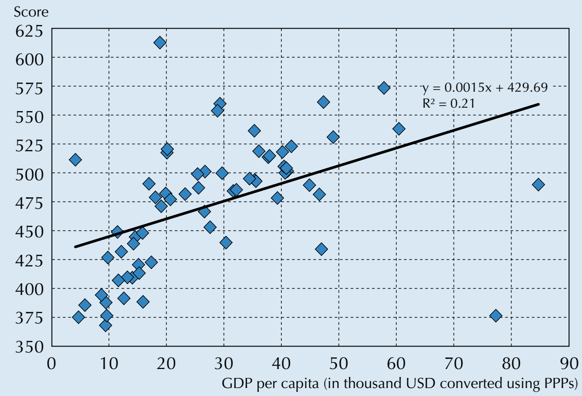
As shown in Volume II, *Excellence through Equity*, a family's wealth influences children's performance in school, but that influence varies markedly across countries. Similarly, the relative prosperity of some countries allows them to spend more on education, while other countries find themselves constrained by a lower national income. It is therefore important to keep the national income of countries in mind when comparing the performance of education systems across countries. Figure I.2.1 displays the relationship between national income as measured by per capita Gross Domestic Product (GDP) and students' average mathematics performance.[1] The figure also shows a trend line[2] that summarises the relationship between per capita GDP and mean student performance in mathematics among OECD countries. The relationship suggests that 21% of the variation in countries' mean scores can be predicted on the basis of their per capita GDP (12% of the variation in OECD countries). Countries with higher national incomes are thus at a relative advantage, even if the chart provides no indications about the causal nature of this relationship. This should be taken into account particularly when interpreting the performance of countries with comparatively low levels of national income, such as Viet Nam and Indonesia (Mexico and Turkey among OECD countries). Table I.2.27 shows an "adjusted" score that would be expected if the country had all of its present characteristics except that per capita GDP was equal to the average for OECD countries (Table I.2.27).

While per capita GDP reflects the potential resources available for education in each country, it does not directly measure the financial resources actually invested in education. Figure I.2.2 compares countries' actual spending per student, on average, from the age of 6 up to the age of 15, with average student performance in mathematics.[3] The results are expressed in USD using purchasing power parities (PPP). Figure I.2.2 shows a positive relationship between spending per student and mean mathematics performance among OECD countries. As expenditure on educational institutions per student increases, so does a country's mean performance. Expenditure per student explains 30% of the variation in mean performance between countries (17% of the variation in OECD countries). Relatively low spending per student needs to be taken into account when interpreting the performance of countries such as Viet Nam and Jordan (Turkey and Mexico among OECD countries). (For more details, see Figure IV.1.7 in Volume IV). At the same time, deviations from the trend line suggest that moderate spending per student cannot automatically be equated with poor performance. For example, the Slovak Republic, which spends around USD 53 000 per student, performs at the same level as the United States, which spends over USD 115 000 per student. Similarly, Korea, the highest-performing OECD country in mathematics, spends well below the average per-student expenditure (Table I.2.27).

Given the close interrelationship between a student's performance and his or her parents' level of education, it is also important to bear in mind the educational attainment of adult populations when comparing the performance of OECD countries, as countries with more highly educated adults are at an advantage over countries where parents have less education. Figure I.2.3 shows the percentage of 35-44 year-olds who have attained tertiary education. This group corresponds roughly to the age group of parents of the 15-year-olds assessed in PISA. Parents' level of education explains 27% of the variation in mean performance between countries (23% of the variation among OECD countries).

■ Figure I.2.1 ■

Mathematics performance and Gross Domestic Product



Source: OECD, PISA 2012 Database, Table I.2.27.
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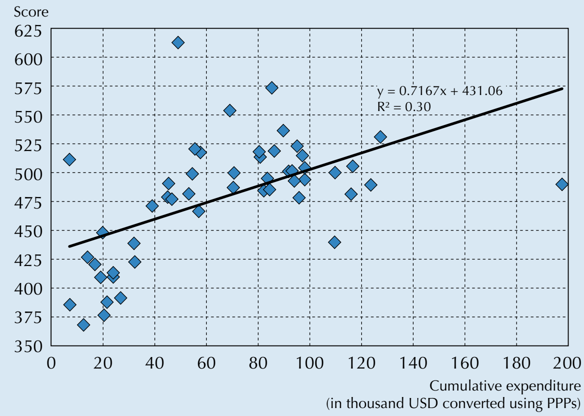
■ Figure I.2.2 ■

Mathematics performance and spending on education



Source: OECD, PISA 2012 Database, Table I.2.27.
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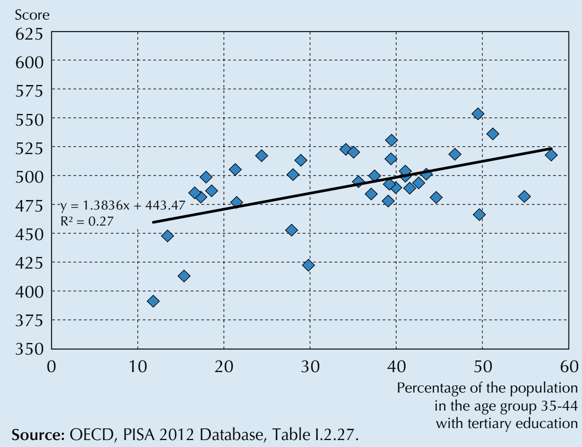
■ Figure I.2.3 ■

Mathematics performance and parents' education



Source: OECD, PISA 2012 Database, Table I.2.27.
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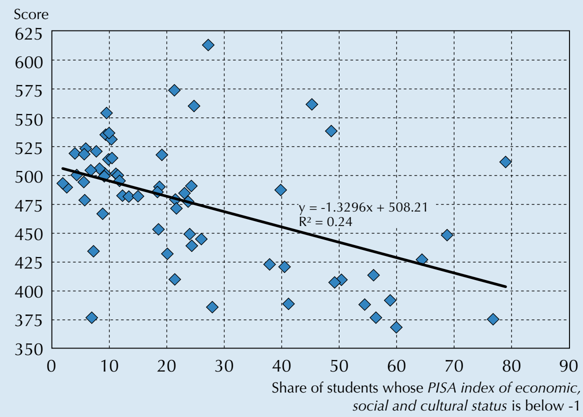
■ Figure I.2.4 ■

Mathematics performance and share of socio-economically disadvantaged students



Source: OECD, PISA 2012 Database, Table I.2.27.
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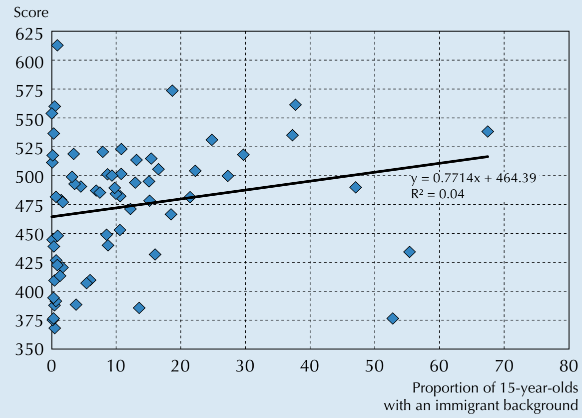
■ Figure I.2.5 ■

Mathematics performance and proportion of students from an immigrant background



Source: OECD, PISA 2012 Database, Table I.2.27.
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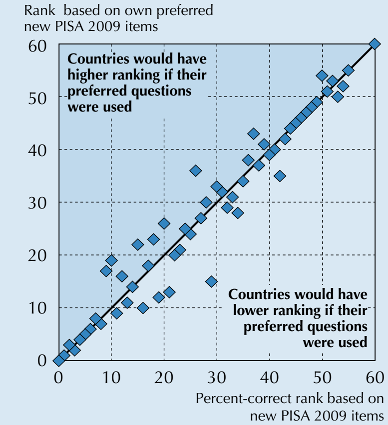
■ Figure I.2.6 ■

Equivalence of the PISA assessment across cultures and languages



Source: OECD, PISA 2009 Database, Table I.2.28.
StatLink http://dx.doi.org/10.1787/888932935572

Socio-economic heterogeneity in student populations poses another major challenge for teachers and education systems. As shown in Volume II, *Excellence through Equity*, teachers instructing socio-economically disadvantaged children are likely to face greater challenges than teachers teaching students from more advantaged backgrounds. Similarly, countries with larger proportions of disadvantaged children face greater challenges than countries with smaller proportions of these students. Figure I.2.4 shows the proportion of students at the lower end of an international scale of the economic, social and cultural status of students, which is described in detail in Volume II, and how this relates to mathematics performance. The relationship explains 24% of the performance variation among countries (46% of the variation among OECD countries). Among OECD countries, Turkey and Mexico, where 69% and 56% of students, respectively, belong to the most disadvantaged group, and Portugal, Chile, Hungary and Spain, where more than 20% of students belong to this group, face much greater challenges than, for example, Iceland, Norway, Finland and Denmark, where fewer than 5% of students are disadvantaged (Table I.2.27). These challenges are even greater in some partner countries like Viet Nam and Indonesia where 79% and 77% of students, respectively, are socio-economically disadvantaged.

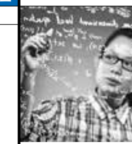
Integrating students with an immigrant background can also be challenging, and the level of performance of students who immigrated to the country in which they were assessed can be only partially attributed to their host country's education system. Figure I.2.5 shows the proportion of 15-year-olds from an immigrant background and how this relates to student performance. This proportion explains only 4% of the variation in mean performance among countries. Despite having large proportions of immigrant students, some countries, like Canada, perform above the OECD average (Table I.2.27).

When examining the results for individual countries, as shown in Table I.2.27, it is apparent that countries vary in their demographic, social and economic contexts. Table I.2.27 summarises in an index the different factors discussed above.[4] Among the countries with available data, the index shows Luxembourg, Norway, Japan, Finland, Iceland, Denmark, Ireland and the United States with the most advantaged demographic, social and economic contexts, and Turkey, Brazil, Mexico, Chile, Portugal, Hungary, the Slovak Republic, Poland and the Czech Republic with the most challenging contexts.

These differences need to be considered when interpreting PISA results. At the same time, the future economic and social prospects of both individuals and countries depend on the results they actually achieve, not on the performance they might have achieved under different social and economic conditions. That is why the results that are actually achieved by students, schools and countries are the focus of this volume.

Even after accounting for the demographic, economic and social context of education systems, the question remains: to what extent is an international test meaningful when differences in languages and cultures lead to very different ways in which subjects such as language, mathematics and science are taught and learned? It is inevitable that not all tasks on the PISA assessments are equally appropriate in different cultural contexts and equally relevant in different curricular and instructional contexts. To gauge this, in 2009 PISA asked every country to identify those tasks from the PISA tests that it considered most appropriate for an international test. Countries were advised to give an on-balance rating for each task with regard to its usefulness in indicating "preparedness for life", its authenticity, and its relevance for 15-year-olds. Tasks given a high rating by a country are referred to as that country's most preferred questions for PISA. PISA then scored every country on its own most preferred questions and compared the resulting performance with the performance on the entire set of PISA tasks (Figure I.2.6). It is clear that, generally, the proportion of questions answered correctly by students does not depend significantly on whether countries were only scored on their preferred questions or on the overall set of PISA tasks. This provides robust evidence that the results of the PISA assessments would not change markedly if countries had more influence in selecting texts that they thought might be "fairer" to their students.

Finally, when comparing student performance across countries, the extent to which student performance on international tests might be influenced by the effort that students in different countries invest in the assessment must be considered. In PISA 2003, students were asked to imagine an actual situation that was highly important to them, so that they could try their very best and invest as much effort as they could into doing well. They were then asked to report how much effort they had put into doing the PISA test compared to the situation they had just imagined; and how much effort they would have invested if their marks from PISA had been counted in their school marks. The students generally answered realistically, saying that they would expend more effort if the test results were to count towards their school marks; but the analysis also established that the reported expenditure of effort by students was fairly stable across countries. This finding counters the claim that systematic cultural differences in the effort expended by students invalidate international comparisons. The analysis also showed that within countries, the amount of effort invested was related to student achievement, with an effect size similar to variables such as single-parent family structure, gender and socio-economic background.[5]



THE PISA APPROACH TO ASSESSING STUDENT PERFORMANCE IN MATHEMATICS

The PISA definition of mathematical literacy

The focus of the PISA 2012 assessment was on measuring an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain and predict phenomena. It assists individuals in recognising the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens.

The definition asserts the importance of mathematics for full participation in society and it stipulates that this importance arises from the way in which mathematics can be used to describe, explain and predict phenomena of many types. The resulting insight into phenomena is the basis for informed decision making and judgements.

Literacy in mathematics described in this way is not an attribute that an individual has or does not have; rather, it can be acquired to a greater or lesser extent, and it is required in varying degrees in society. PISA seeks to measure not just the extent to which students can reproduce mathematical content knowledge, but also how well they can extrapolate from what they know and apply their knowledge of mathematics, in both new and unfamiliar situations. This is a reflection of modern societies and workplaces, which value success not by what people know, but by what people can do with what they know.

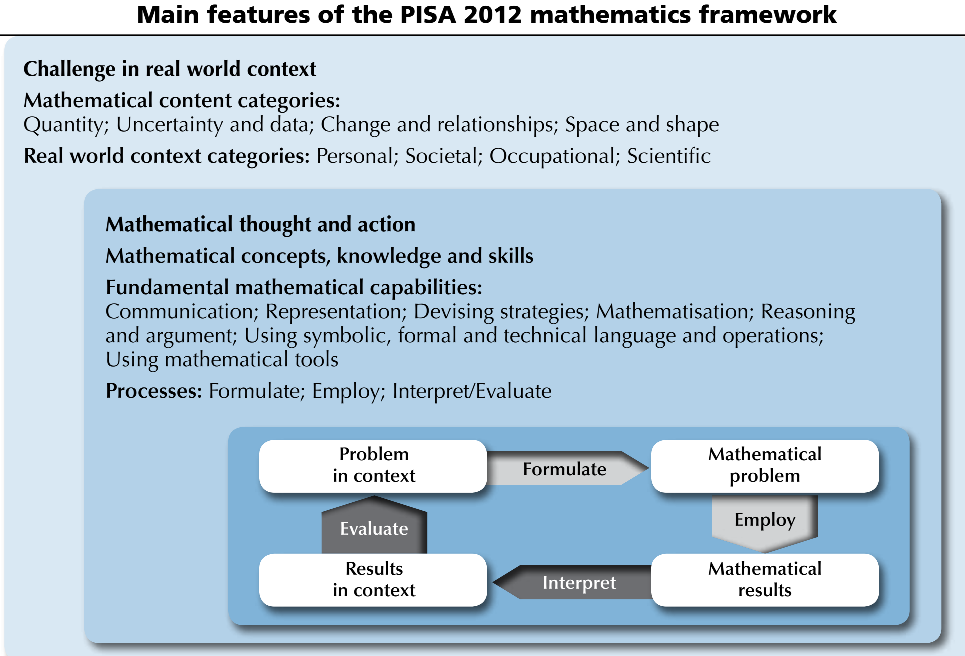
The focus on real-life contexts is also reflected in the reference to using "tools" that appears in the PISA 2012 definition of mathematical literacy. The word "tools" here refers to physical and digital equipment, software and calculation devices that have become ubiquitous in 21st century workplaces. Examples for this assessment include a ruler, a calculator, a spreadsheet, an online currency converter and specific mathematics software, such as dynamic geometry. Using these tools require a degree of mathematical reasoning that the PISA assessment is well-equipped to measure.

The PISA 2012 framework for assessing mathematics

Figure I.2.7 presents an overview of the main constructs of the PISA 2012 mathematics framework that was established and agreed by the participating countries, and how the constructs relate to each other. The largest box shows that mathematical literacy is assessed in the context of a challenge or problem that arises in the real world. The middle box highlights the nature of mathematical thought and action that can be used to solve the problem. The smallest box describes the processes that the problem solver uses to construct a solution.

■ Figure I.2.7 ■

Main features of the PISA 2012 mathematics framework



Context categories

Real-world challenges or situations are categorised in two ways: their context and the domain of mathematics involved. The four context categories identify the broad areas of life in which the problems may arise: personal, which is related to individuals' and families' daily lives; societal, which is related to the community – local, national or global – in which an individual lives; occupational, which is related to the world of work; or scientific, which is related to the use of mathematics in science and technology. According to the framework, these four categories are represented by equal numbers of items.

Content categories

As seen in Figure I.2.7, the PISA items also reflect four categories of mathematical content that are related to the problems posed. The four content categories are represented by approximately equal proportions of items. For the assessment of 15-year-olds, age-appropriate content was developed.

The content category *quantity* incorporates the quantification of attributes of objects, relationships, situations, and entities in the world, which requires an understanding of various representations of those quantifications, and judging interpretations and arguments based on quantity. It involves understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns, and employing number sense, multiple representations of numbers, mental calculation, estimation, and assessment of reasonableness of results.

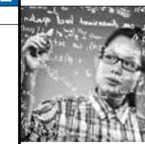
The content category *uncertainty and data* covers two closely related sets of issues: how to identify and summarise the messages that are embedded in sets of data presented in different ways, and how to appreciate the likely impact of the variability that is inherent in many real processes. Uncertainty is part of scientific predictions, poll results, weather forecasts and economic models; variation occurs in manufacturing processes, test scores and survey findings; and chance is part of many recreational activities that individuals enjoy. Probability and statistics, taught as part of mathematics, address these issues.

The content category *change and relationships* focuses on the multitude of temporary and permanent relationships among objects and circumstances, where changes occur within systems of interrelated objects or in circumstances where the elements influence one another. Some of these changes occur over time; some are related to changes in other objects or quantities. Being more literate in this content category involves understanding fundamental types of change and recognising when change occurs so that suitable mathematical models can be employed to describe and predict change.

The content category *space and shape* encompasses a wide range of phenomena that are encountered everywhere: patterns, properties of objects, positions and orientations, representations of objects, decoding and encoding of visual information, navigation, and dynamic interaction with real shapes and their representations. Geometry is essential to space and shape, but the category extends beyond traditional geometry in content, meaning and method, drawing on elements of other mathematical areas, such as spatial visualisation, measurement and algebra. Mathematical literacy in *space and shape* involves understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes.

Process categories

The smallest box of Figure I.2.7 shows a schema of the stages through which a problem-solver may move when solving PISA tasks. The action begins with the "problem in context." The problem-solver tries to identify the mathematics relevant to the problem situation, formulates the situation mathematically according to the concepts and relationships identified, and makes assumptions to simplify the situation. The problem-solver thus transforms the "problem in context" into a "mathematical problem" that can be solved using mathematics. The downward-pointing arrow in Figure I.2.7 represents the work undertaken as the problem-solver employs mathematical concepts, facts, procedures and reasoning to obtain the "mathematical results". This stage usually involves mathematical manipulation, transformation and computation, with and without tools. The "mathematical results" then need to be interpreted in terms of the original problem to obtain the "results in context". The problem solver thus must interpret, apply and evaluate mathematical outcomes and their reasonableness in the context of a real-world problem. The three processes – formulate, employ and interpret – each draw on fundamental mathematical capabilities, which, in turn, draw on the problem-solver's detailed mathematical knowledge.



However, not all PISA tasks engage students in every stage of the modelling cycle. Items are classified according to the dominant process and results are reported by these processes, formally named as:

- Formulating situations mathematically.
- Employing mathematical concepts, facts, procedures and reasoning.
- Interpreting, applying and evaluating mathematical outcomes.

Fundamental mathematical capabilities

Through a decade of experience in developing PISA items and analysing the ways in which students respond to them, a set of fundamental mathematical capabilities has been established that underpins performance in mathematics. These cognitive capabilities can be learned by individuals in order to understand and engage with the world in a mathematical way. Since the PISA 2003 framework was written, researchers (e.g. Turner, 2013) have examined the extent to which the difficulty of a PISA item can be understood, and even predicted, from how each of the fundamental mathematical capabilities is used to solve the item. Four levels describe the ways in which each of the capabilities is used, from simple to complex. For example, an item involving a low level of communication would be simple to read and require only a simple response (e.g. a word); an item involving a high level of communication might require the student to assemble information from various different sources to understand the problem, and the student might have to write a response that explains several steps of thinking through a problem. This research has resulted in sharper definitions of the fundamental mathematical capabilities at each of four levels. A composite score has been shown to be a strong predictor of PISA item difficulty. These fundamental mathematical capabilities are evident across the content categories, and are used to varying degrees in each of the three mathematical processes used in the reporting. The PISA framework (OECD, 2013c) describes this in detail.

The seven fundamental mathematical capabilities used in the PISA 2012 assessment are described as follows:

Communication is both receptive and expressive. Reading, decoding and interpreting statements, questions, tasks or objects enables the individual to form a mental model of the situation. Later, the problem-solver may need to present or explain the solution.

Mathematising involves moving between the real world and the mathematical world. It has two parts: formulating and interpreting. Formulating a problem as a mathematical problem can include structuring, conceptualising, making assumptions and/or constructing a model. Interpreting involves determining whether and how the results of mathematical work are related to the original problem and judging their adequacy. It directly relates to the *formulate* and *interpret* processes of the framework.

Representation entails selecting, interpreting, translating between and using a variety of representations to capture a situation, interact with a problem, or present one's work. The representations referred to include graphs, tables, diagrams, pictures, equations, formulae, textual descriptions and concrete materials.

Reasoning and argument is required throughout the different stages and activities associated with mathematical literacy. This capability involves thought processes rooted in logic that explore and link problem elements so as to be able to make inferences from them, check a justification that is given, or provide a justification of statements or solutions to problems.

Devising strategies for solving problems is characterised as selecting or devising a plan or strategy to use mathematics to solve problems arising from a task or context, and guiding and monitoring its implementation. It involves seeking links between diverse data presented so that the information can be combined to reach a solution efficiently.

Using symbolic, formal and technical language and operations involves understanding, interpreting, manipulating and making use of symbolic and arithmetic expressions and operations, using formal constructs based on definitions, rules and formal systems, and using algorithms with these entities.

Using mathematical tools involves knowing about and being able to use various tools (physical or digital) that may assist mathematical activity, and knowing about the limitations of such tools. The optional computer-based component of the PISA 2012 mathematics assessment has expanded the opportunities for students to demonstrate their ability to use mathematical tools.

Paper-based and computer-based media

PISA 2012 supplemented the paper-based assessment with an optional computer-based assessment, in which specially designed PISA units were presented on a computer and students responded on the computer. Thirty-two of the 65 participating countries and economies participated in this computer-based assessment. For these countries and economies, results are reported for the paper-based assessment scale and supplemented with a computer-based scale and a combined paper-and-computer scale (see Annex B3).

The design of the computer-based assessment ensures that mathematical reasoning and processes take precedence over mastery of using the computer as a tool. Each computer-based item involves three aspects:

- the mathematical demand (as for paper-based items);
- the general knowledge and skills related to information and communication technologies (ICT) that are required (e.g. using keyboard and mouse, and knowing common conventions, such as arrows to move forward). These are intentionally kept to a minimum;
- competencies related to the interaction of mathematics and ICT, such as making a pie chart from data using a simple "wizard", or planning and implementing a sorting strategy to locate and collect desired data in a spreadsheet.

Response types

The response types distinguish between selected response items and constructed response items. Selected response items include simple multiple choice, complex multiple choice, in which students must select correct answers to a series of multiple-choice items, and, for computer-based items, "selected response variations", such as selecting from options in a drop-down box. Constructed response items include those that can be scored routinely (such as a single number or simple phrase, or, for computer-based items, those for which the response can be captured and processed automatically), and others that need expert scoring (e.g. responses that include an explanation or a long calculation).

Examples of items representing the different framework categories

Figure I.2.8 summarises the six categories constructed to create a balanced assessment. Three of the six – process, content and medium – are reporting categories. As noted before, PISA 2012 reports scores separately for the three process categories. Since PISA questions are set in real contexts, they usually involve multiple processes, contents and contexts. It is necessary to make judgements about the major source of demand in order to allocate items to just one of the categories for process, content and context, even though the items are multi-faceted. The items are allocated to the category that reflects the highest cognitive focus of the item.

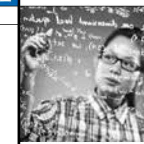
■ Figure I.2.8 ■

Categories describing the items constructed for the PISA 2012 mathematics assessment

<table>
<tr><th colspan="3">Reporting categories</th><th colspan="3">Further categories to ensure balanced assessment</th></tr>
<tr><th>Process categories</th><th>Content categories</th><th>Medium categories</th><th>Context categories</th><th>Response types</th><th>Cognitive demand</th></tr>
<tr><td rowspan="2">Formulating situations mathematically</td><td>Quantity</td><td rowspan="3">Paper-based</td><td>Personal</td><td rowspan="2">Multiple choice</td><td rowspan="3">Empirical difficulty (continuum)</td></tr>
<tr><td rowspan="2">Uncertainty and data</td><td rowspan="2">Societal</td></tr>
<tr><td rowspan="2">Employing mathematical concepts, facts, procedures, and reasoning</td><td rowspan="2">Complex multiple choice</td></tr>
<tr><td rowspan="2">Change and relationships</td><td rowspan="3">Computer-based</td><td rowspan="2">Occupational</td><td rowspan="3">Across fundamental mathematical capabilities</td></tr>
<tr><td rowspan="2">Interpreting, applying and evaluating mathematical outcomes</td><td rowspan="2">Constructed response (simple, elaborated)</td></tr>
<tr><td>Space and shape</td><td>Scientific</td></tr>
</table>

The PISA 2012 mathematics assessment includes the same proportion of items from each of the categories content, context and response type. A quarter of the items in the assessment reflect the process *formulating*, half reflect the process *employing*, and a quarter reflect the process *interpreting*. To measure the full range of student performance, the set of items reflects all levels of difficulty.

Figure I.2.9 summarises how several sample items (see at the end of this chapter) are categorised.



■ Figure I.2.9 ■

Classification of sample items, by process, context and content categories and response type

Item/Question (position on PISA scale)	Process category	Content category	Context category	Response type
WHICH CAR? – Question 01 (327.8)	Interpret	Uncertainty and data	Personal	Simple Multiple Choice
WHICH CAR? – Question 02 (490.9)	Employ	Quantity	Personal	Simple Multiple Choice
WHICH CAR? – Question 03 (552.6)	Employ	Quantity	Personal	Constructed Response Manual
CHARTS – Question 01 (347.7)	Interpret	Uncertainty and data	Societal	Simple Multiple Choice
CHARTS – Question 02 (415.0)	Interpret	Uncertainty and data	Societal	Simple Multiple Choice
CHARTS – Question 05 (428.2)	Employ	Uncertainty and data	Societal	Simple Multiple Choice
GARAGE – Question 01 (419.6)	Interpret	Space and shape	Occupational	Simple Multiple Choice
GARAGE – Question 02 (687.3)	Employ	Space and shape	Occupational	Constructed Response Expert
HELEN THE CYCLIST – Question 01 (440.5)	Employ	Change and relationships	Personal	Simple Multiple Choice
HELEN THE CYCLIST – Question 02 (510.6)	Employ	Change and relationships	Personal	Simple Multiple Choice
HELEN THE CYCLIST – Question 03 (696.6)	Employ	Change and relationships	Personal	Constructed Response Manual
CLIMBING MOUNT FUJI – Question 01 (464.0)	Formulate	Quantity	Societal	Simple Multiple Choice
CLIMBING MOUNT FUJI – Question 02 (641.6)	Formulate	Change and relationships	Societal	Constructed Response Expert
CLIMBING MOUNT FUJI – Question 03 (610.0)	Employ	Quantity	Societal	Constructed Response Manual
REVOLVING DOOR – Question 01 (512.3)	Employ	Space and shape	Scientific	Constructed Response Manual
REVOLVING DOOR – Question 02 (840.3)	Formulate	Space and shape	Scientific	Constructed Response Expert
REVOLVING DOOR – Question 03 (561.3)	Formulate	Quantity	Scientific	Simple Multiple Choice

Example 1: WHICH CAR?

The unit, “WHICH CAR?”, (Figure I.2.10) consists of three questions. It presents a table of data that a person might use to choose a car and make sure that she can afford it.

Context: Because buying a car is an experience that many people might have during their lifetimes, all three questions were allocated to the personal context category.

Response type: Question 1 and Question 2 are simple multiple-choice questions; Question 3, which asks for a single number, is a constructed response item that does not require expert scoring.

Content: Question 1 was allocated to the *uncertainty and data* content category. The item requires knowledge of the basic row-column conventions of a table, as well as co-ordinated data-handling ability to identify where the three conditions are simultaneously satisfied. While the solution also requires basic knowledge of large whole numbers, that knowledge is unlikely to be the main source of difficulty in the item. In contrast, Question 2 has been allocated to the *quantity* content category because it is well known that even at age 15, many students have misconceptions about the base ten and place value ideas required to order “ragged” decimal numbers. Question 3 is also allocated to the *quantity* content category because the calculation of 2.5% is expected to require more cognitive effort from students than identifying the correct data in the table. The difficulty for this age group in dealing with decimal numbers and percentages is reflected in the empirical results: Question 1 is considered an easy item, Question 2 is close to the international average, and Question 3 is of above-average difficulty.

Process: In allocating the items to process categories, their relation to "real-world" problems has been taken into consideration. The primary demand in items in the formulate category is the transition from the real-world problem to the mathematical problem; in the employ category, the primary demand is within the mathematical world; and in the interpret category, an item's primary demand is in using mathematical information to provide a real-world solution. Questions 2 and 3 are allocated to the *employ* category. This is because in both of these items, the main cognitive effort is made within mathematics: decimal notation and the calculation of a percentage. In Question 1, the construction of a table of data, including the need to identify key variables, is a mathematisation of a real situation. Question 1 is allocated to the *interpret* category because it requires these mathematical entities to be interpreted in relation to the real world.

■ Figure I.2.10 ■

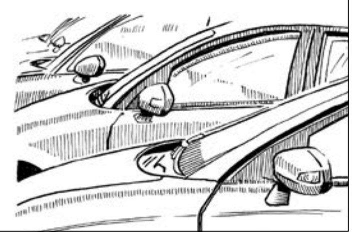
WHICH CAR? – a unit from the PISA 2012 main survey

WHICH CAR?

Chris has just received her car driving licence and wants to buy her first car.

This table below shows the details of four cars she finds at a local car dealer.

Model:	Alpha	Bolte	Castel	Dezal
Year	2003	2000	2001	1999
Advertised price (zeds)	4 800	4 450	4 250	3 990
Distance travelled (kilometres)	105 000	115 000	128 000	109 000
Engine capacity (litres)	1.79	1.796	1.82	1.783



WHICH CAR? – *QUESTION 1*

Chris wants a car that meets **all** of these conditions:

- The distance travelled is **not** higher than 120 000 kilometres.
- It was made in the year 2000 or a later year.
- The advertised price is **not** higher than 4 500 zeds.
- Which car meets Chris's conditions?

A. Alpha
B. Bolte
C. Castel
D. Dezal

WHICH CAR? – *QUESTION 2*

Which car's engine capacity is the smallest?

A. Alpha
B. Bolte
C. Castel
D. Dezal

WHICH CAR? – *QUESTION 3*

Chris will have to pay an extra 2.5% of the advertised cost of the car as taxes.

How much are the extra taxes for the Alpha?

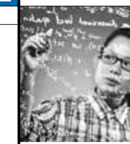
Extra taxes in zeds: ....

Example 2: CLIMBING MOUNT FUJI

Context: The unit "CLIMBING MOUNT FUJI", containing three questions, as shown in Figure I.2.11, was allocated to the societal context category. Question 1 goes beyond the personal concerns of a walker to wider community issues – in this case, concerns about use of the public trail. Items classified as societal involve such things as voting systems, public transport, government, public policies, demographics, advertising, national statistics and economics. Although individuals can be personally involved in these, the focus of the problem is more on the community perspective.

Response: Question 1 is simple multiple choice (choose one out of four). Question 2 requires the answer 11 a.m. and as such, is a constructed response with expert scoring to ensure that all equivalent ways of writing the time are considered. Question 3 requires the number 40 for full score, or the number 0.4 (answering in metres) for partial credit. It, too, is a constructed response with expert scoring.

Content: Question 1 requires calculating the number of days open using the given dates, and then calculating an average. The question was allocated to the *quantity* content category because it involves quantification of time and of an average. While the formula for average is required, and this is indeed a relationship, since this question requires use of an average to calculate the number of people per day, rather than focus on the relationship, this question is not allocated to the *change and relationships* category. Question 3 has similar characteristics, involving units of length. Question 2 is allocated to the *change and relationships* category because the relationship between distance and time, encapsulated as



speed, is paramount. From information about distances and speed, the time to go up and the time to come down have to be quantified, and then used in combination with the finishing time to get the starting time. Had the time needed to go up and down been given directly, rather than indirectly through distance and speed, then the question could have been allocated to the *quantity* category.

■ Figure I.2.11 ■

CLIMBING MOUNT FUJI – a unit from the field trial

CLIMBING MOUNT FUJI

Mount Fuji is a famous dormant volcano in Japan

CLIMBING MOUNT FUJI – *QUESTION 1*

Mount Fuji is only open to the public for climbing from 1 July to 27 August each year. About 200 000 people climb Mount Fuji during this time.

On average, about how many people climb Mount Fuji each day?

A. 340

B. 710

C. 3 400

D. 7 100

E. 7 400

CLIMBING MOUNT FUJI – *QUESTION 2*

The Gotemba walking trail up Mount Fuji is about 9 kilometres (km) long.

Walkers need to return from the 18 km walk by 8 p.m.

Toshi estimates that he can walk up the mountain at 1.5 kilometres per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times.

Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 p.m.?

....

CLIMBING MOUNT FUJI – *QUESTION 3*

Toshi wore a pedometer to count his steps on his walk along the Gotemba trail.

His pedometer showed that he walked 22 500 steps on the way up.

Estimate Toshi's average step length for his walk up the 9 km Gotemba trail. Give your answer in centimetres (cm).

Answer: .... cm

Process: Question 1 was allocated to the *formulating* category because most of the cognitive effort in this relatively easy item requires taking two pieces of real-world information (open season and total number of climbers) and establishing a mathematical problem to be solved: find the length of the open season from the dates and use it with the information about the total number of climbers to find the average number of climbers each day. Expert judgement is that the major cognitive demand for 15-year-olds lies in this movement from the real world problem to the mathematical relationships, rather than in the ensuing whole number calculations. Question 2 was also allocated to the *formulating* process category for the same reason: the main cognitive effort required is to translate real-world data into a mathematical problem and identify all the relationships involved, rather than calculate or interpret the answer as a starting time of 11 a.m. In this difficult item, the mathematical structure involves multiple relationships: starting time = finishing time – duration; duration = time up + time down; time up (down) = distance/speed (or equivalent proportional reasoning); time down = half time up; and appreciating the simplifying assumptions that average speeds already include consideration of variable speed during the day and that no further allowance is required for breaks.

By contrast, Question 3 was allocated to the *employing* category. There is one main relationship involved: the distance walked = number of steps × average step length. There are two obstacles to using this relationship to solve the problem: rearranging the formula (which is probably done by students informally rather than formally using the written relationship) so that the average step length can be found from distance and number of steps; and making appropriate unit conversions. The main cognitive effort required for this question is in carrying out these steps, rather than identifying the relationships and assumptions to be made (the *formulating* process) or *interpreting* the answer in real-world terms.

How the PISA 2012 mathematics results are reported

How the PISA 2012 mathematics tests were designed, analysed and scaled

The test material had to meet several requirements:

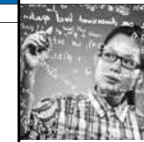
- Test items had to meet the requirements and specifications of the framework for PISA 2012 that was established and agreed upon by the participating countries. The content, processes and contexts of the items had to be deemed appropriate for a test of 15-year-olds.
- Items had to be of interest and of curricular relevance for 15-year-olds in participating countries and economies.
- Items had to meet stringent standards of technical quality and international comparability.

Items for the assessment were selected from a pool of diverse material with a diverse range of sources (authors in almost 30 different countries, with the contributions from national teams, members of the PISA mathematics expert group and the PISA Project Consortium) that reflected content, context and approaches relevant to a large number of PISA-participating countries and economies. Wordings and other features of the items were reviewed by experts, then the items were tested among classes of 15-year-old students, and finally the items underwent extensive field trials in all countries and economies that would ultimately use the material. Each participating country and economy provided detailed feedback on the curricular relevance, appropriateness and potential interest for 15-year-olds, by local mathematics experts. At each development stage, material was considered for rejecting, revising or keeping in the pool of potential items. Finally, the international mathematics expert group formulated recommendations as to which items should be included in the survey instruments and those recommendations were considered by the PISA Governing Board, in which governments of all participating countries are represented. The final selection of test items was balanced across the various categories specified in the mathematics framework and spanned a range of levels of difficulty, so that the entire pool of items could measure performance across a broad range of content, processes and contexts, and across a wide range of student abilities (for further details, see the *PISA 2012 Technical Report* [OECD, forthcoming]).

Test items were generally developed within "units" that included some stimulus material and one or more questions related to the stimulus. In many cases, students were required to construct a response to questions, based on their analysis, calculations and mathematical thinking. Some constructed-response items were relatively open-ended, requiring students to present an extended response that may have included presenting the steps of their solution or some explanation of their result, which thus revealed aspects of the methods and thought processes they had used to answer the question. In general, these items could not be machine scored; rather they required the professional judgement of trained coders to assign the responses to defined response categories. To ensure that the response coding process yielded reliable and cross-nationally comparable results, detailed guidelines and training were provided. All the procedures ensuring the consistency of the coding within and between countries are detailed in *PISA 2012 Technical Report* (OECD, forthcoming).

In other cases requiring students to construct their response, only a very simple response was required, such as a value read from a graph or table, or writing a word, short phrase or the numerical result of a calculation. The evaluation of these answers was restricted to the response itself and did not take into account an explanation of how the response was derived. Responses could often be processed without the intervention of a coding expert. The use of computer-delivered test forms also allowed for a number of response formats such that responses could be captured relatively easily by computer without any additional intervention.

Other items were presented in a format that required students to select one or more responses from a set of given response options. This format category includes both standard multiple-choice items, for which students were required to select one correct response from a number of given response options; and complex multiple choice items, for which students were required to select a response from given optional responses to each of a number of propositions or questions. Responses to these items could be processed automatically, with no intervention by an expert coder needed.



The final PISA 2012 survey included 36 paper-based items linking to previous PISA survey instruments, 74 new paper-based items and 41 new computer-based items. Each student completed a fraction of the paper-based items – a minimum of 12 items, up to a maximum of 37 items, depending on which test booklet they were randomly assigned from the booklet rotation design. The mathematics questions selected for inclusion in the paper-based component of the survey were arranged into half-hour clusters of 12-13 items. These, along with clusters of reading and science questions, were assembled into test booklets, each containing four clusters. Each participating student was assigned a test booklet to be completed in two hours. In the computer-based survey, students completed a one-hour test composed of two half-hour components selected from a rotated design of mathematics, reading and problem-solving item clusters.

The test design, similar to those used in previous PISA assessments, makes it possible to construct a single scale of proficiency in mathematics, so that each question is associated with a particular point on the scale that indicates its difficulty, and each test-taker's performance is associated with a particular point on the same scale that indicates his or her estimated mathematical proficiency. A description of the modelling technique used to construct this scale can be found in the *PISA 2012 Technical Report* (OECD, forthcoming).

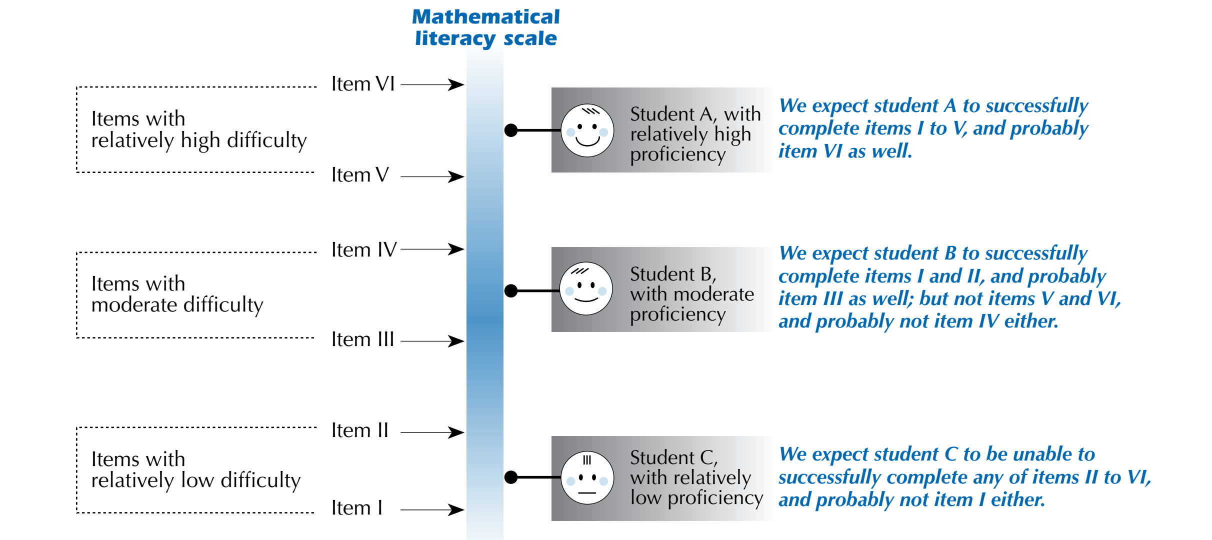
The relative difficulty of tasks in a test is estimated by considering the proportion of test-takers who answer each question correctly; and the relative proficiency of individuals taking a particular test can be estimated by considering the proportion of test questions they answer correctly. A single continuous scale shows the relationship between the difficulty of questions and the proficiency of test-takers. By constructing a scale that shows the difficulty of each question, it is possible to locate the level of mathematics that the question demands. By showing the proficiency of each test-taker on the same scale, it is possible to describe the level of mathematics that each test taker possesses.

The location of different described levels of mathematical proficiency on this scale is set in relation to the particular group of questions used in the assessment; but just as the sample of students who sat the PISA test in 2012 was drawn to represent all 15-year-old students in the participating countries and economies, so the individual test questions used in the assessment were designed to represent the definition of literacy in mathematics adequately. Estimates of student proficiency reflect the kinds of tasks students would be expected to perform successfully. This means that students are likely to be able to successfully complete questions located at or below the difficulty level associated with their own position on the scale. Conversely, they are unlikely to be able to successfully complete questions above the difficulty level associated with their position on the scale. Figure I.2.12 illustrates how this probabilistic model works.

The higher an individual's proficiency level is located above a given test question, the more likely is he or she to successfully complete the question (and other questions of similar difficulty); the further the individual's proficiency is located below a given question, the less likely is he or she to be able to successfully complete the question and other questions of similar difficulty.

■ Figure I.2.12 ■

The relationship between questions and student performance on a scale



How mathematics proficiency levels are defined in PISA 2012

PISA 2012 provides an overall mathematics scale, which draws on all of the mathematics questions in the assessment, as well as scales for the three mathematical processes and the four mathematical content categories defined above. The metric for the overall mathematics scale is based on a mean for OECD countries of 500 points and a standard deviation of 100 points that were set in PISA 2003 when the first PISA mathematics scale was first developed. The items that were common to both the 2003 and 2012 test instruments enable a link to be made with the earlier scale. To help users interpret what student scores mean in substantive terms, the scale is divided into proficiency levels. For PISA 2012, the range of difficulty of the tasks is represented by six levels of mathematical proficiency that are aligned with the levels used in describing the outcomes of PISA 2003. The levels range from the lowest, Level 1, to the highest, Level 6. Descriptions of each of these levels have been generated, based on the framework-related cognitive demands imposed by tasks that are located within each level, to describe the kinds of knowledge and skills needed to successfully complete those tasks, and which can then be used as characterisations of the substantive meaning of each level.

Individuals with proficiency within the range of Level 1 are likely to be able to complete Level 1 tasks, but are unlikely to be able to complete tasks at higher levels. Level 6 reflects tasks that pose the greatest challenge in terms of the mathematical knowledge and skills needed to complete them successfully. Individuals with scores in this range are likely to be able to complete tasks located at that level, as well as all the other PISA mathematics tasks (see section *Students at the different levels of proficiency in mathematics* for a detailed description of the proficiency levels in mathematics).

STUDENT PERFORMANCE IN MATHEMATICS

PISA outcomes are reported in a variety of ways. This section gives the country results and shows the location of items on the overall PISA mathematics scale described above, how the different levels of proficiency in PISA mathematics can be characterised, and how these proficiency levels are represented by mathematics questions used in the survey. In subsequent sections, mathematical performance will be examined in more detail in relation to: the process categories referred to as *formulating*, *employing* and *interpreting*; and the content categories of *space and shape*, *quantity*, *change and relationships*, and *uncertainty and data*.

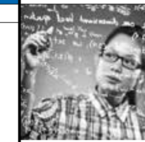
Average in mathematics performance

This section compares the countries and economies on the basis of their average mathematics scores. In addition, changes in the relative standing of countries since the 2003 survey – the most recent assessment in which mathematics was the major PISA domain – are presented.

The country results are estimates because they are obtained from samples of students, rather than from a census of all students, and they are obtained using a limited set of assessment tasks, not a population of all possible assessment tasks. When the sampling and assessment are done with scientific rigour it is possible to determine the magnitude of the probable uncertainty associated with the estimates. This uncertainty needs to be taken into account when making comparisons so that differences that could reasonably arise simply due to the sampling of students and items are not interpreted as differences that actually hold for the populations. A difference is called statistically significant if it is very unlikely that such a difference could be observed by chance, when in fact no true difference exists.

When interpreting mean performance, only those differences among countries and economies that are statistically significant should be taken into account. Figure I.2.13 shows each country's/economy's mean score and also for which groups of countries/economies the differences between the means are statistically significant. For each country/economy shown in the middle column, the countries/economies whose mean scores are not statistically significantly different are listed in the right column. In all other cases, country/economy A scores higher than country/economy B if country/economy A is situated above country/economy B in the middle column, and scores lower if country/economy A is situated below country/economy B. Figure I.2.13 lists each participating country and economy in descending order of its mean mathematics score (left column). The values range from a high of 613 points for the partner economy Shanghai-China to a low of 368 points for the partner country Peru.

Countries and economies are also divided into three broad groups: those whose mean scores are statistically around the OECD mean (highlighted in dark blue), those whose mean scores are above the OECD mean (highlighted in pale blue), and those whose mean scores are below the OECD mean (highlighted in medium blue). Across OECD countries, the average score in mathematics is 494 points (see Table I.2.3a). To gauge the magnitude of score differences, 41 score points corresponds to the equivalent of one year of formal schooling (see Annex A1, Table A1.2).



■ Figure I.2.13 ■

Comparing countries' and economies' performance in mathematics

Statistically significantly **above** the OECD average
Not statistically significantly different from the OECD average
Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
613	Shanghai-China	
573	Singapore	
561	Hong Kong-China	Chinese Taipei, Korea
560	Chinese Taipei	Hong Kong-China, Korea
554	Korea	Hong Kong-China, Chinese Taipei
538	Macao-China	Japan, Liechtenstein
536	Japan	Macao-China, Liechtenstein, Switzerland
535	Liechtenstein	Macao-China, Japan, Switzerland
531	Switzerland	Japan, Liechtenstein, Netherlands
523	Netherlands	Switzerland, Estonia, Finland, Canada, Poland, Viet Nam
521	Estonia	Netherlands, Finland, Canada, Poland, Viet Nam
519	Finland	Netherlands, Estonia, Canada, Poland, Belgium, Germany, Viet Nam
518	Canada	Netherlands, Estonia, Finland, Poland, Belgium, Germany, Viet Nam
518	Poland	Netherlands, Estonia, Finland, Canada, Belgium, Germany, Viet Nam
515	Belgium	Finland, Canada, Poland, Germany, Viet Nam
514	Germany	Finland, Canada, Poland, Belgium, Viet Nam
511	Viet Nam	Netherlands, Estonia, Finland, Canada, Poland, Belgium, Germany, Austria, Australia, Ireland
506	Austria	Viet Nam, Australia, Ireland, Slovenia, Denmark, New Zealand, Czech Republic
504	Australia	Viet Nam, Austria, Ireland, Slovenia, Denmark, New Zealand, Czech Republic
501	Ireland	Viet Nam, Austria, Australia, Slovenia, Denmark, New Zealand, Czech Republic, France, United Kingdom
501	Slovenia	Austria, Australia, Ireland, Denmark, New Zealand, Czech Republic
500	Denmark	Austria, Australia, Ireland, Slovenia, New Zealand, Czech Republic, France, United Kingdom
500	New Zealand	Austria, Australia, Ireland, Slovenia, Denmark, Czech Republic, France, United Kingdom
499	Czech Republic	Austria, Australia, Ireland, Slovenia, Denmark, New Zealand, France, United Kingdom, Iceland
495	France	Ireland, Denmark, New Zealand, Czech Republic, United Kingdom, Iceland, Latvia, Luxembourg, Norway, Portugal
494	United Kingdom	Ireland, Denmark, New Zealand, Czech Republic, France, Iceland, Latvia, Luxembourg, Norway, Portugal
493	Iceland	Czech Republic, France, United Kingdom, Latvia, Luxembourg, Norway, Portugal
491	Latvia	France, United Kingdom, Iceland, Luxembourg, Norway, Portugal, Italy, Spain
490	Luxembourg	France, United Kingdom, Iceland, Latvia, Norway, Portugal
489	Norway	France, United Kingdom, Iceland, Latvia, Luxembourg, Portugal, Italy, Spain, Russian Federation, Slovak Republic, United States
487	Portugal	France, United Kingdom, Iceland, Latvia, Luxembourg, Norway, Italy, Spain, Russian Federation, Slovak Republic, United States, Lithuania
485	Italy	Latvia, Norway, Portugal, Spain, Russian Federation, Slovak Republic, United States, Lithuania
484	Spain	Latvia, Norway, Portugal, Italy, Russian Federation, Slovak Republic, United States, Lithuania, Hungary
482	Russian Federation	Norway, Portugal, Italy, Spain, Slovak Republic, United States, Lithuania, Sweden, Hungary
482	Slovak Republic	Norway, Portugal, Italy, Spain, Russian Federation, United States, Lithuania, Sweden, Hungary
481	United States	Norway, Portugal, Italy, Spain, Russian Federation, Slovak Republic, Lithuania, Sweden, Hungary
479	Lithuania	Portugal, Italy, Spain, Russian Federation, Slovak Republic, United States, Sweden, Hungary, Croatia
478	Sweden	Russian Federation, Slovak Republic, United States, Lithuania, Hungary, Croatia
477	Hungary	Spain, Russian Federation, Slovak Republic, United States, Lithuania, Sweden, Croatia, Israel
471	Croatia	Lithuania, Sweden, Hungary, Israel
466	Israel	Hungary, Croatia
453	Greece	Serbia, Turkey, Romania
449	Serbia	Greece, Turkey, Romania, Bulgaria
448	Turkey	Greece, Serbia, Romania, Cyprus[1, 2], Bulgaria
445	Romania	Greece, Serbia, Turkey, Cyprus[1, 2], Bulgaria
440	Cyprus[1, 2]	Turkey, Romania, Bulgaria
439	Bulgaria	Serbia, Turkey, Romania, Cyprus[1, 2], United Arab Emirates, Kazakhstan
434	United Arab Emirates	Bulgaria, Kazakhstan, Thailand
432	Kazakhstan	Bulgaria, United Arab Emirates, Thailand
427	Thailand	United Arab Emirates, Kazakhstan, Chile, Malaysia
423	Chile	Thailand, Malaysia
421	Malaysia	Thailand, Chile
413	Mexico	Uruguay, Costa Rica
410	Montenegro	Uruguay, Costa Rica
409	Uruguay	Mexico, Montenegro, Costa Rica
407	Costa Rica	Mexico, Montenegro, Uruguay
394	Albania	Brazil, Argentina, Tunisia
391	Brazil	Albania, Argentina, Tunisia, Jordan
388	Argentina	Albania, Brazil, Tunisia, Jordan
388	Tunisia	Albania, Brazil, Argentina, Jordan
386	Jordan	Brazil, Argentina, Tunisia
376	Colombia	Qatar, Indonesia, Peru
376	Qatar	Colombia, Indonesia
375	Indonesia	Colombia, Qatar, Peru
368	Peru	Colombia, Indonesia

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Source: OECD, PISA 2012 Database.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.14 [Part 1/3] ■

Mathematics performance among PISA 2012 participants, at national and regional levels

	Mathematics scale				
		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Shanghai-China	613			1	1
Singapore	573			2	2
Hong Kong-China	561			3	5
Chinese Taipei	560			3	5
Korea	554	1	1	3	5
Macao-China	538			6	8
Japan	536	2	3	6	9
Liechtenstein	535			6	9
Switzerland	531	2	3	7	9
Flemish community (Belgium)	531				
Trento (Italy)	524				
Friuli Venezia Giulia (Italy)	523				
Netherlands	523	3	7	9	14
Veneto (Italy)	523				
Estonia	521	4	8	10	14
Finland	519	4	9	10	15
Canada	518	5	9	11	16
Australian Capital Territory (Australia)	518				
Poland	518	4	10	10	17
Lombardia (Italy)	517				
Navarre (Spain)	517				
Western Australia (Australia)	516				
Belgium	515	7	10	13	17
Germany	514	6	10	13	17
Massachusetts (United States)	514				
Viet Nam	511			11	19
German-speaking community (Belgium)	511				
New South Wales (Australia)	509				
Castile and Leon (Spain)	509				
Bolzano (Italy)	506				
Connecticut (United States)	506				
Austria	506	10	14	17	22
Basque Country (Spain)	505				
Australia	504	11	14	17	21
Madrid (Spain)	504				
Queensland (Australia)	503				
La Rioja (Spain)	503				
Ireland	501	11	17	18	24
Slovenia	501	12	16	19	23
Victoria (Australia)	501				
Emilia Romagna (Italy)	500				
Denmark	500	12	18	19	25
New Zealand	500	12	18	19	25
Asturias (Spain)	500				
Czech Republic	499	12	19	19	26
Piemonte (Italy)	499				
Scotland (United Kingdom)	498				
Marche (Italy)	496				
Aragon (Spain)	496				
Toscana (Italy)	495				
England (United Kingdom)	495				
France	495	16	21	23	29
United Kingdom	494	16	23	23	31
French community (Belgium)	493				
Catalonia (Spain)	493				
Iceland	493	18	22	25	29
Umbria (Italy)	493				
Valle d'Aosta (Italy)	492				
Cantabria (Spain)	491				
Latvia	491			25	32
Luxembourg	490	20	23	27	31
Norway	489	19	25	26	33
South Australia (Australia)	489				

Notes: OECD countries are shown in bold black. Partner countries are shown in bold blue. Participating economies and subnational entities that are not included in national results are shown in bold blue italics. Regions are shown in black italics (OECD countries) or blue italics (partner countries).
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2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.
Countries, economies and subnational entities are ranked in descending order of mean mathematics performance.
Source: OECD, PISA 2012 Database.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.14 [Part 2/3] ■

Mathematics performance among PISA 2012 participants, at national and regional levels

	Mathematics scale				
		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Alentejo (Portugal)	489				
Galicia (Spain)	489				
Liguria (Italy)	488				
Portugal	487	19	27	26	36
Northern Ireland (United Kingdom)	487				
Italy	485	22	27	30	35
Spain	484	23	27	31	36
Perm Territory region (Russian Federation)	484				
Russian Federation	482			31	39
Slovak Republic	482	23	29	31	39
United States	481	23	29	31	39
Lithuania	479			34	40
Sweden	478	26	29	35	40
Puglia (Italy)	478				
Tasmania (Australia)	478				
Hungary	477	26	30	35	40
Abruzzo (Italy)	476				
Balearic Islands (Spain)	475				
Lazio (Italy)	475				
Andalusia (Spain)	472				
Croatia	471			38	41
Wales (United Kingdom)	468				
Florida (United States)	467				
Israel	466	29	30	40	41
Molise (Italy)	466				
Basilicata (Italy)	466				
Dubai (United Arab Emirates)	464				
Murcia (Spain)	462				
Extremadura (Spain)	461				
Sardegna (Italy)	458				
Greece	453	31	32	42	44
Campania (Italy)	453				
Northern Territory (Australia)	452				
Serbia	449			42	45
Turkey	448	31	32	42	46
Sicilia (Italy)	447				
Romania	445			43	47
Cyprus [1, 2]	440			45	47
Sharjah (United Arab Emirates)	439				
Bulgaria	439			45	49
Aguascalientes (Mexico)	437				
Nuevo León (Mexico)	436				
Jalisco (Mexico)	435				
Querétaro (Mexico)	434				
United Arab Emirates	434			47	49
Kazakhstan	432			47	50
Calabria (Italy)	430				
Colima (Mexico)	429				
Chihuahua (Mexico)	428				
Distrito Federal (Mexico)	428				
Thailand	427			49	52
Durango (Mexico)	424				
Chile	423	33	33	50	52
Morelos (Mexico)	421				
Abu Dhabi (United Arab Emirates)	421				
Malaysia	421			50	52
Coahuila (Mexico)	418				
Ciudad Autónoma de Buenos Aires (Argentina)	418				
Mexico (Mexico)	417				
Federal District (Brazil)	416				
Ras Al Khaimah (United Arab Emirates)	416				
Santa Catarina (Brazil)	415				
Puebla (Mexico)	415				

Notes: OECD countries are shown in bold black. Partner countries are shown in bold blue. Participating economies and subnational entities that are not included in national results are shown in bold blue italics. Regions are shown in black italics (OECD countries) or blue italics (partner countries).

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Countries, economies and subnational entities are ranked in descending order of mean mathematics performance.

Source: OECD, PISA 2012 Database.

StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.14 [Part 3/3] ■

Mathematics performance among PISA 2012 participants, at national and regional levels

	Mathematics scale				
		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Baja California (Mexico)	415				
Baja California Sur (Mexico)	414				
Espírito Santo (Brazil)	414				
Nayarit (Mexico)	414				
Mexico	413	34	34	53	54
San Luis Potosí (Mexico)	412				
Guanajuato (Mexico)	412				
Tlaxcala (Mexico)	411				
Tamaulipas (Mexico)	411				
Sinaloa (Mexico)	411				
Fujairah (United Arab Emirates)	411				
Quintana Roo (Mexico)	411				
Yucatán (Mexico)	410				
Montenegro	410			54	56
Uruguay	409			53	56
Zacatecas (Mexico)	408				
Mato Grosso do Sul (Brazil)	408				
Rio Grande do Sul (Brazil)	407				
Costa Rica	407			54	56
Hidalgo (Mexico)	406				
Manizales (Colombia)	404				
São Paulo (Brazil)	404				
Paraná (Brazil)	403				
Ajman (United Arab Emirates)	403				
Minas Gerais (Brazil)	403				
Veracruz (Mexico)	402				
Umm Al Quwain (United Arab Emirates)	398				
Campeche (Mexico)	396				
Paraíba (Brazil)	395				
Albania	394			57	59
Medellin (Colombia)	393				
Bogota (Colombia)	393				
Brazil	391			57	60
Rio de Janeiro (Brazil)	389				
Argentina	388			57	61
Tunisia	388			57	61
Jordan	386			59	62
Piauí (Brazil)	385				
Sergipe (Brazil)	384				
Rondônia (Brazil)	382				
Rio Grande do Norte (Brazil)	380				
Goiás (Brazil)	379				
Cali (Colombia)	379				
Tabasco	378				
Ceará (Brazil)	378				
Colombia	376			62	64
Qatar	376			62	64
Indonesia	375			62	65
Bahia (Brazil)	373				
Chiapas (Mexico)	373				
Mato Grosso (Brazil)	370				
Peru	368			64	65
Guerrero (Mexico)	367				
Tocantins (Brazil)	366				
Pernambuco (Brazil)	363				
Roraima (Brazil)	362				
Amapá (Brazil)	360				
Pará (Brazil)	360				
Acre (Brazil)	359				
Amazonas (Brazil)	356				
Maranhão (Brazil)	343				
Alagoas (Brazil)	342				

Notes: OECD countries are shown in bold black. Partner countries are shown in bold blue. Participating economies and subnational entities that are not included in national results are shown in bold blue italics. Regions are shown in black italics (OECD countries) or blue italics (partner countries).

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Countries, economies and subnational entities are ranked in descending order of mean mathematics performance.

Source: OECD, PISA 2012 Database.

StatLink http://dx.doi.org/10.1787/888932935572

Figure I.2.14 shows how participating countries and economies compare in mathematics performance. Since a country's score is based on an estimate of scores obtained from a sample of students, there is some degree of uncertainty associated with the estimates. Thus countries/economies are shown with the range of ranks they could occupy given this uncertainty. A number of countries designed their PISA samples so that it is possible to calculate performance averages for subnational entities as well. These subnational averages are also included in Figure I.2.14.

Shanghai-China ranks first in mathematics performance followed by Singapore. Given the uncertainty inherent in the score estimates, Hong Kong-China could rank third, fourth or fifth among all participating countries and economies. Korea is the top ranking OECD country, but when all participating countries are taken into consideration, it could rank either third, fourth or fifth. Japan is the second listed OECD country (seventh among all countries and economies) with a rank of 2 or 3 among OECD countries (from 6 to 9 among all countries and economies); and Switzerland is the third listed OECD country (ninth among all countries and economies) with a rank also of 2 or 3 among OECD countries (and from 7 to 9 among all countries and economies). For entities other than those for which full samples were drawn, namely Chinese Taipei, Hong Kong-China, Macao-China and Shanghai-China, it is not possible to calculate a rank order; but the mean score provides the possibility of comparing subnational entities against the performance of countries and economies. For example, the Flemish Community of Belgium matches the performance of top-performer Switzerland. Similarly, the performance of the Italian provinces of Trento and Friuli Venezia Giulia, which is similar to that of the Netherlands, a high performer, is higher than the performance of the Italian province of Sicilia, which is similar to Turkey's performance, by the equivalent of almost two full years of schooling.

Trends in average mathematics performance

Trends in average performance provide an indicator of how school systems are improving. Trends in mathematics are available for 64 countries and economies that participated in PISA 2012. Thirty-eight of these have mathematics performance for 2012 and the three remaining PISA assessments (2003, 2006 and 2009); seventeen have information for 2012 and two additional assessments and nine countries and economies have information for 2012 and one previous assessment.[6] To better understand a country or economy's trend and maximise the number of countries in the comparisons, this report focuses on the *annualised change* in student performance. The annualised change is the average annual change in the observed period, taking into account all observations. For countries and economies that have participated in all four PISA assessments, the annualised change takes into account all four time points, and for those countries that have valid data for fewer assessments it only takes into account the valid and available information.

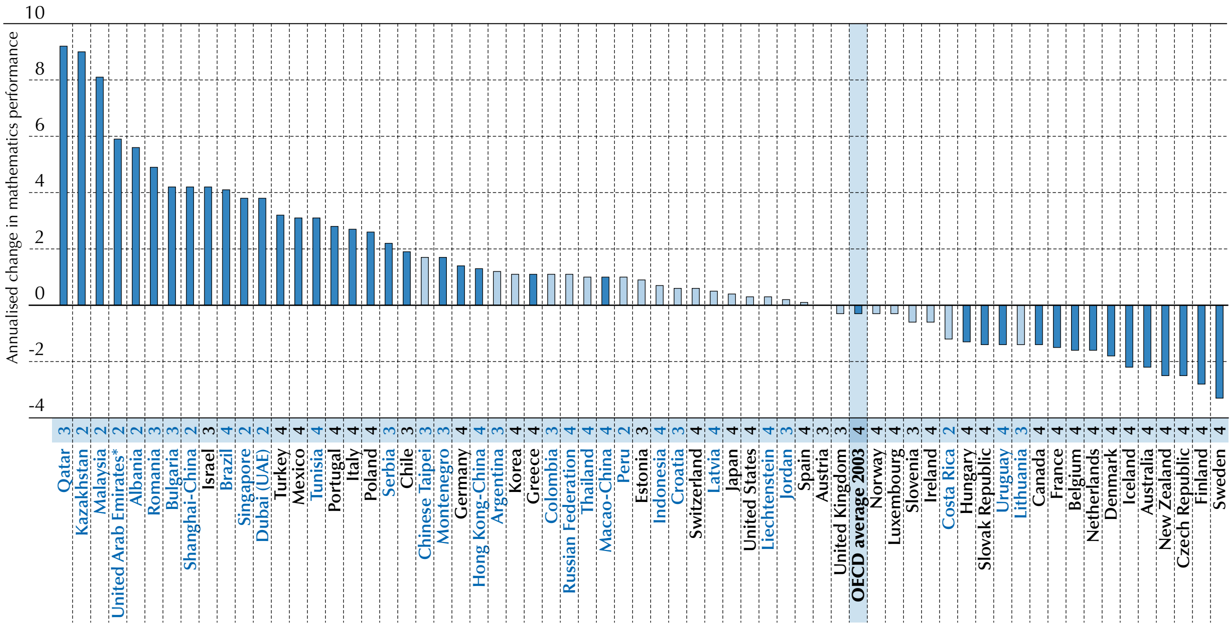
The annualised change is a more robust measure of trends in performance because it is based on all the available information (as opposed to the difference between one particular year and 2012). It is scaled by years, so it is interpreted as the average annual change in performance over the observed period and allows for comparisons of mathematics performance of countries that have participated in at least two PISA assessments since 2003 (for further details on the estimation of the annualised change, see Box I.2.2 and Annex A5).[7]

On average across OECD countries with comparable data in PISA 2003 and PISA 2012, performance has remained broadly similar, but there have been markedly more countries with increasing than with declining mathematics performance (see Box I.2.2 for details on interpreting trends in PISA). Of the 64 countries and economies with trend data up to 2012, 25 show an average annual improvement in mathematics performance; by contrast, 14 countries and economies show an average deterioration in performance between 2003 and 2012. For the remaining 25 countries and economies, there is no change in mathematics performance during the period. Figure I.2.15 illustrates that Albania, Kazakhstan, Malaysia, Qatar and the United Arab Emirates, except Dubai (United Arab Emirates, excluding Dubai), show an average improvement in mathematics performance of more than five score points per year. Among OECD countries, improvements in mathematics performance are observed in Israel (with an average improvement of more than four score points per year), Mexico, Turkey (more than three score points per year), Italy, Poland, Portugal (more than two score points per year), and Chile, Germany and Greece (more than one score point per year). Among countries that have participated in every assessment since 2003, Brazil, Italy, Mexico, Poland, Portugal, Tunisia and Turkey, show an average improvement in mathematics performance of more than 2.5 points per year. Box I.2.4 and Box I.2.5 highlight Brazil's and Turkey's improvement in PISA, and provides insight on the education policies and programmes implemented in the last decade. Other chapters of this volume and other volumes of this series highlight other country's improvements in PISA and outline their recent policy trajectories (e.g. Estonia and Korea in Chapters 4 and 5 of this volume, Mexico and Germany in Volume II, Japan and Portugal in Volume III, and Colombia, Israel, Poland and Tunisia in Volume IV).

■ Figure I.2.15 ■

Annualised change in mathematics performance throughout participation in PISA

Mathematics score-point difference associated with one calendar year



* United Arab Emirates excluding Dubai.

Notes: Statistically significant score point changes are marked in a darker tone (see Annex A3).

The number of comparable mathematics scores used to calculate the annualised change is shown next to the country/economy name.

The annualised change is the average annual change in PISA score points from a country's/economy's earliest participation in PISA to PISA 2012. It is calculated taking into account all of a country's/economy's participation in PISA. For more details on the calculation of the annualised change, see Annex A5.

OECD average 2003 compares only OECD countries with comparable mathematics scores since 2003.

Countries and economies are ranked in descending order of the annualised change in mathematics performance.

Source: OECD, PISA 2012 Database, Table I.2.3b.

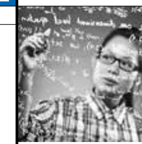
StatLink http://dx.doi.org/10.1787/888932935572

Box I.2.2. Measuring trends in PISA

PISA 2012 is the fifth round of PISA since the programme was launched in 2000. Every PISA assessment assesses students' reading, mathematics and science literacy, and in each round, one of these subjects is the main domain and the other two are minor domains. The first full assessment of reading was conducted in 2000 (when it was a major domain), while the first full assessment of mathematics was conducted in 2003 and science in 2006. In 2009, the assessment returned to reading as a major domain, which allowed for observations of trends in reading performance since PISA 2000. Mathematics is the major domain of PISA 2012, as it was in PISA 2003, allowing for observations of trends in mathematics performance since PISA 2003. The first full assessment of each domain sets the scale for future comparisons.

The methodologies underpinning performance trends in international studies of education are complex (Gebhardt and Adams, 2007). In order to ensure the comparability of successive PISA results, a number of conditions must be met. First, while successive assessments include a number of common assessment items, the limited number of such items increases measurement errors. Therefore, the confidence band for comparisons over time is wider than for single-year data, and only changes that are indicated as statistically significant should be considered robust.[8] Second, the sample of students must represent an equivalent population (that of 15-year-olds enrolled in school), and only results from samples that meet the strict standards set by PISA can be compared over time. Third, the conditions in which the assessment is conducted must also remain constant across the rounds that are to be compared.

...



Even though they participate in successive PISA assessment, some countries and economies cannot compare all their PISA results over time. For example, the PISA 2000 sample for the Netherlands did not meet the PISA response-rate standards, so the Netherland's PISA 2000 results are not comparable to those of subsequent assessments. In Luxembourg, the testing conditions changed substantially between 2000 and 2003, so PISA 2000 results are not comparable with those of subsequent assessments. The PISA 2000 and 2003 samples for the United Kingdom did not meet the PISA response-rate standards, so data from the United Kingdom cannot be used for comparisons including these years. In the United States, no results for reading literacy are available for 2006. In 2009, a dispute between teachers' unions and the education minister of Austria led to a boycott of PISA, which was only lifted after the first week of testing. The boycott required the OECD to remove identifiable cases from the dataset. Although the Austrian dataset met the PISA 2009 technical standards after these cases were removed, the negative reaction to education assessments has affected the conditions under which the PISA survey was conducted and could have adversely affected student motivation to respond to the PISA tasks. Therefore, the comparability of 2009 data with data from earlier PISA assessments cannot be ensured, and data for Austria have been excluded from trend comparisons.

In addition, not all countries have participated in all PISA assessments. Among OECD countries, the Slovak Republic and Turkey joined PISA in 2003. Chile and Israel did not participate in the PISA 2003 assessment, and Estonia and Slovenia began participation in 2006.

When comparing trends in mathematics, reading and science, only those countries with valid data to compare between assessments are included. As a result, comparisons between the 2000 and 2012 assessments use data on reading performance and include only 38 countries and economies. Comparisons between the 2003 and 2012 assessments use data on reading and mathematics performance and include 39 countries and economies. Comparisons between the 2006 and 2012 assessments use data on reading, mathematics and science performance and include 55 countries and economies (54 countries in the case of reading). Comparisons between 2009 and 2012 use data on all domains and include 63 countries and economies. In all, 64 countries and economies have valid trend information when their PISA 2012 data and all their previous valid data are used.

The annualised change in performance

Trends in a country's/economy's average mathematics, reading and science performance are presented as the annualised change. The annualised change is the average rate of change at which a country's/economy's average mathematics, reading and science scores has changed throughout their participation in PISA assessments. Thus, a positive annualised change of *x* points indicates that the country/economy has improved in performance by *x* points per year since its earliest comparable PISA results. For countries and economies that have participated in only two assessments, the annualised change is equal to the difference between the two assessments, divided by the number of years that passed between the assessments.

The annualised change is a more robust measure of a country's/economy's progress in education outcomes as it is based on information available from all assessments. It is thus less sensitive to abnormal measurements that may alter a country's/economy's PISA trends if results are compared only between two assessments. The annualised change is calculated as the best-fitting line throughout a country's/economy's participation in PISA. The year that individual students participated in PISA is regressed on their PISA scores, yielding the annualised change. The annualised change also takes into account the fact that, for some countries and economies, the period between PISA assessments is less than three years. This is the case for those countries and economies that participated in PISA 2000 or PISA 2009 as part of PISA+: they conducted the assessment in 2001, 2002 or 2010 instead of 2000 or 2009.

Annex B4 presents the average performance in mathematics, reading and science (circles) for each country and economy as well as the annualised change (slope of the dotted/solid line). Tables I.2.3b, I.4.3b and I.5.3b present the annualised change in average mathematics, reading and science performance, respectively. Tables I.2.3d, I.4.3d and I.5.3d present the annualised change for the 10th, 25th, 75th and 90th percentile in mathematics, reading and science performance. Annex A5 provides further details on the calculation of the annualised change and other trends measures.

The average improvement over time shows only one aspect of a country's/economy's trajectory; it does not indicate whether a country's/economy's improvement is steady, accelerating or decelerating. To evaluate the degree to which a country's improvement is accelerating or decelerating, only the 55 countries and economies that have participated in PISA 2012 and at least two other assessments have been considered. Annualised linear improvement in mathematics is observed for 18 countries and economies that have participated in PISA 2012 as well as two other assessments. The rate of improvement in the mathematics performance of the average student has accelerated in Macao-China and Poland, meaning that the rate of improvement observed in the 2009 to 2012 period is higher than that observed in the 2003 to 2006 period, for example. In Poland, this means that while scores improved by five score points (not statistically significant) between 2003 and 2006 and maintained that level between 2006 and 2009, between 2009 and 2012 there is a much faster improvement, at 23 points. Similarly, while mathematics scores in Macao-China did not change between 2003 and 2009, they improved by 13 score points between 2009 and 2012. The rate of improvement has remained steady in 13 countries and economies (Brazil, Bulgaria, Chile, Germany, Hong Kong-China, Israel, Italy, Montenegro, Portugal, Romania, Serbia, Tunisia and Turkey); the observed linear annualised change is similar to the rate of change observed throughout a country's/economy's participation in successive PISA assessments. By contrast, Qatar, Mexico and Greece show decelerating rates of improvement: the rate of improvement observed in the first assessments of PISA is slower in the later assessments. In Mexico, for example, between 2003 and 2006 the average mathematics score improved from 385 to 406 score points (a change of more than 20 points), then improved again in 2009 to 419 points, but decreased (not significantly) to 413 points in 2012 (Figure I.2.16 and Table I.2.3b).

Among the 25 countries that have no positive annualised change, 23 have participated in at least two assessments in addition to PISA 2012, and all those that show deteriorating performance participated in at least two assessments prior to PISA 2012. Among these, Chinese Taipei, Croatia, Ireland and Japan show signs of moving from no change to improvement, or from initial deterioration towards no change in mathematics performance. Although Chinese Taipei, Croatia, Ireland and Japan showed no change in mathematics performance during their participation in earlier rounds of PISA, there are signs of improvement in more recent years. Between PISA 2003 and 2006 assessments, France showed a deterioration in its average annual performance, but later assessments did not show any further deterioration (Figure I.2.16 and Table I.2.3b).

At any point in time, countries and economies share similar performance levels with other countries and economies. But as time passes and school systems evolve, some countries and economies improve their performance changing the group of countries with which they share similar performance levels. Figure I.2.17 shows, for each country and economy with comparable results in 2003 and 2012, those other countries and economies with similar performance in 2003 but higher or lower level performance in 2012. In 2003, Poland, for example, was similar in performance to the United States, Latvia, the Slovak Republic, Luxembourg, Hungary, Spain and Norway; but as a result of improvements during the period, it performed better than all those countries in 2012. In 2003, Poland scored below Finland, Germany, Austria, Canada, Belgium and the Netherlands; but by 2012, its performance was similar to this group of countries. Turkey was similar in performance to Uruguay and Thailand in 2003 but, in 2012, its score was higher than those of these two countries, and was at the same level as that of Greece. In 2003, Portugal scored lower than the United States, Latvia, the Slovak Republic, Luxembourg, the Czech Republic, France, Sweden, Hungary, Spain, Iceland and Norway; but by 2012 the country had caught up to those countries.

Figure I.2.18 shows the relationship between each country and economy's average mathematics performance in 2003 and their average rate of change over the 2003 to 2012 period. Countries and economies that show the strongest improvement throughout the various assessments (top half of the graph) are more likely to be those that had comparatively low performance in the initial years. The correlation between a country's/economy's earliest comparable mathematics score and the annualised rate of change is -0.60; this means that 35% of the variance in the rate of change can be explained by a country's/economy's initial score and that countries with a lower initial score tend to improve at a faster rate.

But this relationship is, by no means, a given. Although countries that improve the most are more likely to be those that had lower performance in 2003, some countries and economies that had average or high performance in 2003 saw improvements in their students' performance over time. Such was the case in the high-performing countries and economies of Hong Kong-China, Macao-China and Germany, all of which saw annualised improvements in mathematics performance even after PISA 2003 mathematics scores placed them at or above the OECD average (results for countries and economies that began their participation in PISA after PISA 2003 are in Table I.2.3b).

■ Figure I.2.16 ■

Curvilinear trajectories of average mathematics performance across PISA assessments

Rate of acceleration or deceleration in performance (quadratic term)

Notes: Figures are for illustrative purposes only. Countries and economies are grouped according to the direction and significance of their annualised change and their rate of acceleration.

Countries and economies with data from only one PISA assessments other than 2012 are excluded.

Source: OECD, PISA 2012 Database, Table I.2.3b.

StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.17 [Part 1/2] ■

Multiple comparisons of mathematics performance between 2003 and 2012

	Mathematics performance in 2003	Mathematics performance in 2012	Countries/economies with similar performance in 2003 but lower performance in 2012	Countries/economies with similar performance in 2003 and similar performance in 2012	Countries/economies with similar performance in 2003 but higher performance in 2012
Hong Kong-China	550	561	Finland, Japan, Netherlands, Liechtenstein	Korea	
Korea	542	554	Finland, Japan, Canada, Netherlands, Liechtenstein	Hong Kong-China	
Macao-China	527	538	New Zealand, Czech Republic, Australia, Canada, Belgium, Netherlands	Japan, Switzerland, Liechtenstein	
Japan	534	536	New Zealand, Finland, Australia, Canada, Belgium	Macao-China, Netherlands, Switzerland, Liechtenstein	Hong Kong-China, Korea
Liechtenstein	536	535	New Zealand, Finland, Australia, Canada, Belgium	Japan, Macao-China, Netherlands, Switzerland	Hong Kong-China, Korea
Switzerland	527	531	New Zealand, Czech Republic, Australia, Canada, Belgium	Japan, Macao-China, Netherlands, Liechtenstein	
Netherlands	538	523		Finland, Japan, Canada, Belgium, Switzerland, Liechtenstein	Hong Kong-China, Macao-China, Korea
Finland	544	519		Netherlands	Hong Kong-China, Japan, Liechtenstein, Korea
Canada	532	518		Belgium, Netherlands	Japan, Macao-China, Switzerland, Liechtenstein, Korea
Poland	490	518	United States, Latvia, Slovak Republic, Luxembourg, Hungary, Spain, Norway		
Belgium	529	515	New Zealand, Australia	Canada, Netherlands	Japan, Macao-China, Switzerland, Liechtenstein
Germany	503	514	Slovak Republic, France, Sweden, Ireland, Denmark, Norway	Austria	
Austria	506	506	Slovak Republic, France, Sweden, Norway	Germany, Czech Republic, Ireland, Denmark	
Australia	524	504		New Zealand, Czech Republic	Japan, Macao-China, Belgium, Switzerland, Liechtenstein
Ireland	503	501	Slovak Republic, Sweden, Norway	Austria, France	Germany
Denmark	514	500	Sweden	New Zealand, Austria, Czech Republic, France, Iceland	Germany
New Zealand	523	500		Czech Republic, Australia, Denmark	Japan, Macao-China, Belgium, Switzerland, Liechtenstein
Czech Republic	516	499	Sweden	New Zealand, Austria, France, Australia, Denmark, Iceland	Macao-China, Switzerland
France	511	495	Sweden	Czech Republic, Ireland, Denmark, Iceland	Germany, Austria
Iceland	515	493	Sweden	Czech Republic, France, Denmark	
Latvia	483	491	Hungary	United States, Spain, Norway, Russian Federation	Poland
Luxembourg	493	490	Hungary	Slovak Republic, Norway	Poland
Norway	495	489	Hungary	Latvia, Slovak Republic, Luxembourg	Poland, Germany, Austria, Ireland
Portugal	466	487		Russian Federation, Italy	
Italy	466	485		Portugal, Russian Federation	
Spain	485	484		United States, Latvia, Hungary	Poland
Russian Federation	468	482		Latvia, Portugal, Italy	
Slovak Republic	498	482		Luxembourg, Sweden, Hungary, Norway	Poland, Germany, Austria, Ireland
United States	483	481		Latvia, Hungary, Spain	Poland
Sweden	509	478		Slovak Republic	Germany, Austria, Czech Republic, France, Ireland, Denmark, Iceland
Hungary	490	477		United States, Slovak Republic, Spain	Poland, Latvia, Luxembourg, Norway
Greece	445	453			
Turkey	423	448	Uruguay, Thailand		
Thailand	417	427	Uruguay		Turkey
Mexico	385	413			
Uruguay	422	409			Thailand, Turkey
Brazil	356	391	Indonesia	Tunisia	
Tunisia	359	388		Brazil, Indonesia	
Indonesia	360	375		Tunisia	Brazil

Note: Only countries and economies that participated in the PISA 2003 and PISA 2012 assessments are shown.
Countries and economies are ranked in descending order of their mean mathematics performance in PISA 2012.
Source: OECD, PISA 2012 Database, Table I.2.3b.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.17 [Part 2/2] ■

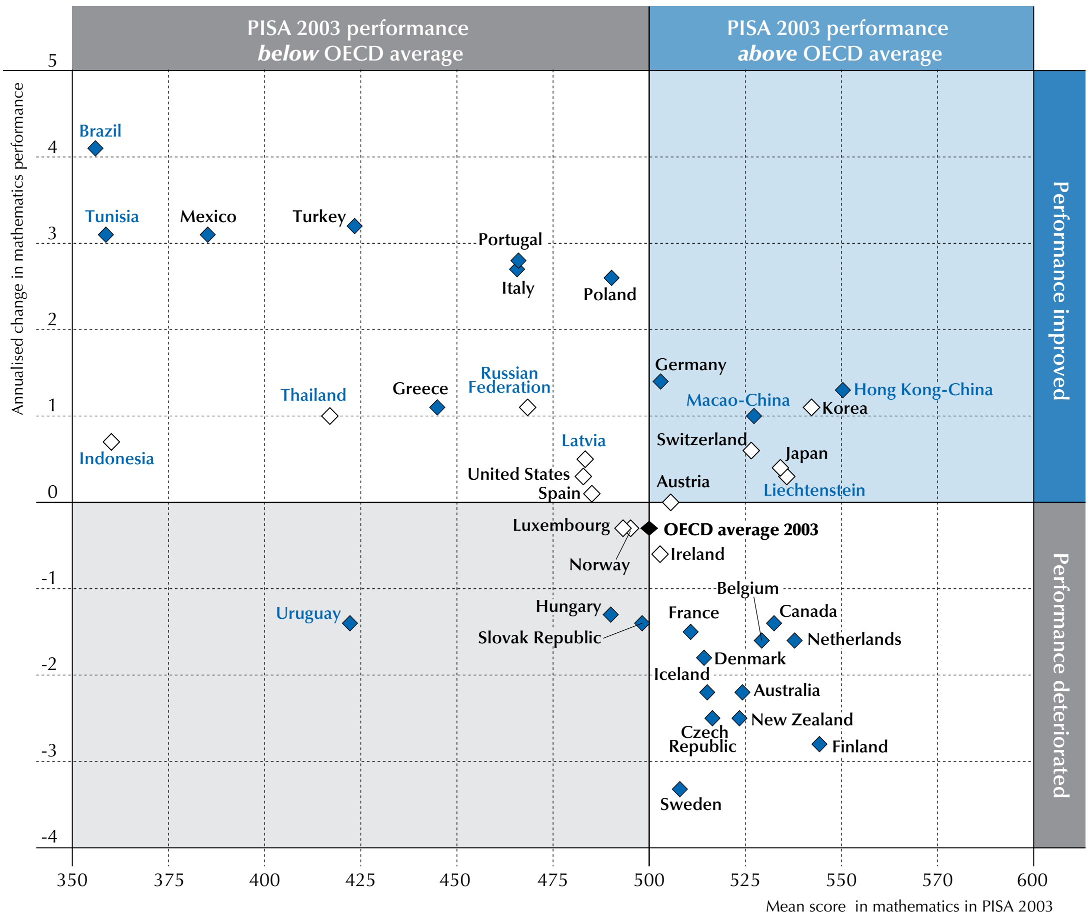
Multiple comparisons of mathematics performance between 2003 and 2012

Countries/economies with lower performance in 2003 but similar performance in 2012	Countries/economies with lower performance in 2003 but higher performance in 2012	Countries/economies with higher performance in 2003 but with similar performance in 2012	Countries/economies with higher performance in 2003 but lower performance in 2012	Mathematics performance in 2012	Mathematics performance in 2003	
				561	550	Hong Kong-China
				554	542	Korea
			Finland	538	527	Macao-China
				536	534	Japan
				535	536	Liechtenstein
			Finland	531	527	Switzerland
Poland, Germany				523	538	Netherlands
Poland, Germany, Canada, Belgium	Macao-China, Switzerland			519	544	Finland
Poland, Germany		Finland		518	532	Canada
		Finland, Germany, Austria, Canada, Belgium, Netherlands	New Zealand, Czech Republic, France, Sweden, Australia, Ireland, Denmark, Iceland	518	490	Poland
Poland, Germany, Austria		Finland		515	529	Belgium
Poland		Finland, Canada, Belgium, Netherlands	New Zealand, Czech Republic, Australia, Iceland	514	503	Germany
Poland		New Zealand, Australia, Belgium	Iceland	506	506	Austria
Austria, Ireland, Denmark	Poland, Germany			504	524	Australia
	Poland	New Zealand, Czech Republic, Australia, Denmark	Iceland	501	503	Ireland
Latvia, Ireland	Poland	Australia		500	514	Denmark
Latvia, Austria, France, Ireland, Iceland	Poland, Germany			500	523	New Zealand
Latvia, Ireland, Portugal, Norway	Poland, Germany			499	516	Czech Republic
Latvia, Luxembourg, Portugal, Norway	Poland	New Zealand		495	511	France
Latvia, Luxembourg, Portugal, Norway	Poland, Germany, Austria, Ireland	New Zealand		493	515	Iceland
Portugal, Italy		New Zealand, Slovak Republic, Luxembourg, Czech Republic, France, Denmark, Iceland	Sweden	491	483	Latvia
United States, Latvia, Spain, Portugal, Russian Federation, Italy		France, Iceland	Sweden	490	493	Luxembourg
United States, Spain, Portugal, Russian Federation, Italy		Czech Republic, France, Iceland	Sweden	489	495	Norway
		United States, Latvia, Slovak Republic, Luxembourg, Czech Republic, France, Sweden, Hungary, Spain, Iceland, Norway		487	466	Portugal
		United States, Latvia, Slovak Republic, Luxembourg, Sweden, Hungary, Spain, Norway		485	466	Italy
Portugal, Russian Federation, Italy		Slovak Republic, Luxembourg, Sweden, Norway		484	485	Spain
		United States, Slovak Republic, Luxembourg, Sweden, Hungary, Spain, Norway		482	468	Russian Federation
United States, Latvia, Spain, Portugal, Russian Federation, Italy				482	498	Slovak Republic
Portugal, Russian Federation, Italy		Slovak Republic, Luxembourg, Sweden, Norway		481	483	United States
United States, Hungary, Spain, Portugal, Russian Federation, Italy	Poland, Latvia, Luxembourg, Norway			478	509	Sweden
Portugal, Russian Federation, Italy		Sweden		477	490	Hungary
Turkey				453	445	Greece
		Greece		448	423	Turkey
				427	417	Thailand
		Uruguay		413	385	Mexico
Mexico				409	422	Uruguay
				391	356	Brazil
				388	359	Tunisia
				375	360	Indonesia

Note: Only countries and economies that participated in the PISA 2003 and PISA 2012 assessments are shown.
Countries and economies are ranked in descending order of their mean mathematics performance in PISA 2012.
Source: OECD, PISA 2012 Database, Table I.2.3b.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.18 ■

Relationship between annualised change in performance and average PISA 2003 mathematics scores



Notes: Annualised score point change in mathematics that are statistically significant are indicated in a darker tone (see Annex A3).
The annualised change is the average annual change in PISA score points from a country's/economy's earliest participation in PISA to PISA 2012. It is calculated taking into account all of a country's/economy's participation in PISA. For more details on the calculation of the annualised change, see Annex A5.
The correlation between a country's/economy's mean score in 2003 and its annualised performance is -0.60.
OECD average 2003 considers only those countries with comparable data since PISA 2003.
Source: OECD, PISA 2012 Database, Tables I.2.3b.
StatLink http://dx.doi.org/10.1787/888932935572

Other high-performing countries and economies that began their participation in PISA after the 2003 assessment, like Shanghai-China and Singapore, also show improvements in performance. In addition, there are many countries and economies that performed similarly in 2003 but evolved differently. As shown in Table I.2.3b, Bulgaria, Chile, Romania and Thailand began their participation in PISA with a mathematics performance of around 410 score points; but while Thailand showed no annual improvement between 2003 and 2012, Chile, Bulgaria and Romania showed an annual improvement between 2006 and 2012 of 1.9, 4.2 and 4.9 score points, respectively (Figure I.2.18 and Table I.2.3b).

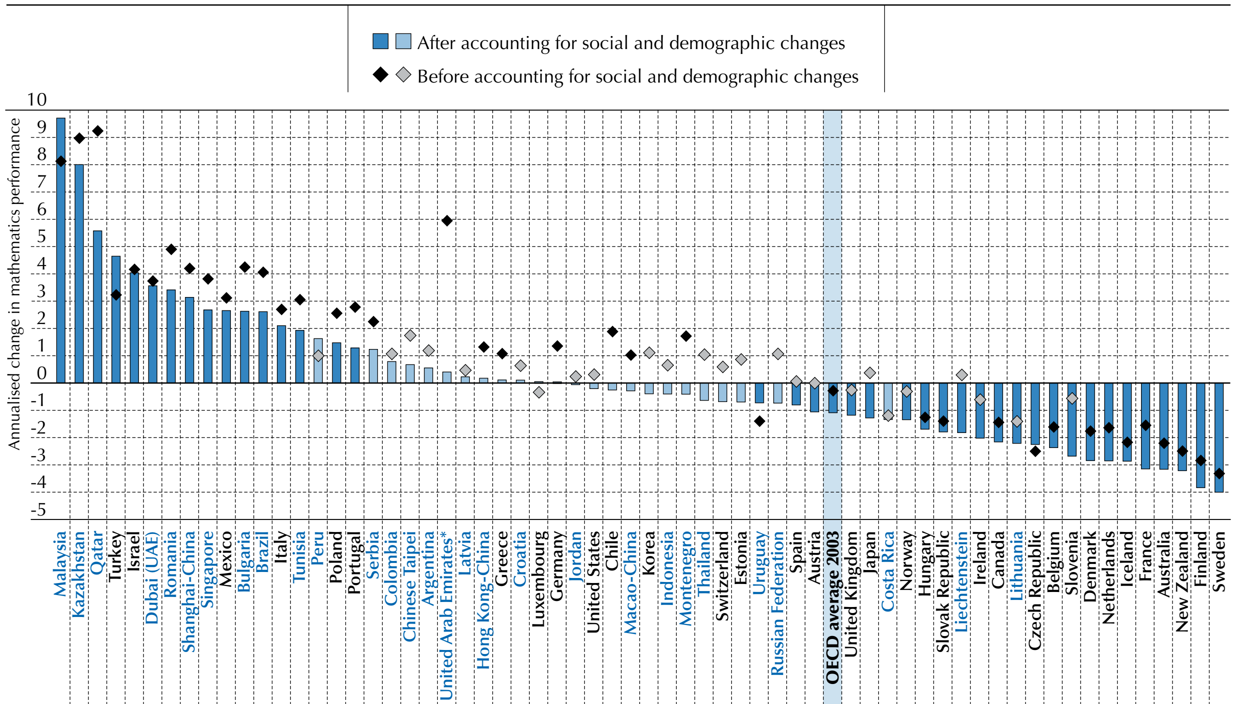
Trends in mathematics performance adjusted for sampling and demographic changes

Changes in a country's or economy's mathematics performance can have many sources. While improvements may result from improved education services, they can also result from demographic changes that have shifted the country's population profile. By following strict sampling and methodological standards PISA ensures that all countries and economies are measuring the mathematics performance of their 15-year-olds enrolled in school; but because of migration or other demographic and social trends, the characteristics of this reference population may change. Annex A5 provides details on the calculation of the adjusted trends.

Figure I.2.19 presents annualised changes after adjusting for changes in the age, gender, socio-economic status, migration background and language spoken at home of the population of students in each country or economy.[9] On average across OECD countries, and assuming that the 2003, 2006 and 2009 population of 15-year-old students had the same demographic profile as the population in 2012, scores in mathematics dropped by around one point per year. The observed trend shows no change since 2006. This difference in trends before and after accounting for demographic changes means that were it not for these demographic and socio-economic changes, average mathematics performance across OECD countries would have deteriorated since 2006.

■ Figure I.2.19 ■

Adjusted and observed annualised performance change in average PISA mathematics scores



* United Arab Emirates excluding Dubai.

Notes: Statistically significant values are marked in a darker tone (see Annex A3).

The annualised change is the average annual change in PISA score points. It is calculated taking into account all of a country's/economy's participation in PISA. For more details on the calculation of the annualised change, see Annex A5.

The annualised change adjusted for demographic changes assumes that the average age and *PISA index of social, cultural and economic status,* as well as the percentage of female students, those with an immigrant background and those who speak a language other than the assessment at home is the same in previous assessments as those observed in 2012. For more details on the calculation of the adjusted annualised change, see Annex A5.

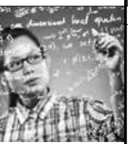
OECD average 2003 considers only those countries with comparable mathematics scores since PISA 2003.

Countries and economies are ranked in descending order of the annualised change after accounting for demographic changes.

Source: OECD, PISA 2012 Database, Tables I.2.3b and I.2.4.

StatLink http://dx.doi.org/10.1787/888932935572

As shown in Figure I.2.19, of the 25 countries and economies that saw an overall improvement in mathematics performance, 16 show this improvement after accounting for demographic changes in their student population.[10] In these countries and economies, changes in the age, immigrant background and language spoken at home of the student population do not explain all of the observed improvement in mathematics performance. Of the 14 countries and economies that show deteriorating performance during their participation in PISA, in no country or economy does this trend lose statistical significance after accounting for demographic changes in the student population. Of the 25 countries and economies that did not see an annualised change in mathematics performance, 9 would show a deterioration in performance had their student populations in previous assessments shared the same profile as students who were assessed in PISA 2012.



Comparing the results of the adjusted and unadjusted trends in mathematics performance, shown in Figure I.2.19, Costa Rica, the Czech Republic, Dubai (United Arab Emirates), Israel, Kazakhstan, Malaysia and Mexico, have less than a 20% difference between unadjusted and adjusted annualised trends, meaning that the characteristics of the student population have not changed much between 2003 and 2012, that changes in the characteristics of the student population are unrelated to average student performance, or that education services have adapted to the changes in the student population so that any of those changes that may have an impact on student performance have been compensated for by adaptations made in education service. Similarly, in Colombia, Hungary, Jordan, Latvia, Luxembourg and the Slovak Republic, the difference between the unadjusted and adjusted annualised trends is less than 0.5 score points per year. Large differences in adjusted and unadjusted performance are observed in Chile, Liechtenstein, Montenegro, Qatar, Slovenia and the United Arab Emirates, excluding Dubai. In these countries and economies, the difference between adjusted and unadjusted annualised trends is greater than two score points, signalling that demographic changes have had a considerable impact on trends in mathematics performance.

Informative as they may be, adjusted trends are merely hypothetical scenarios that help to understand the source of changes in students' performance over time. Observed (unadjusted) trends depicted in Figure I.2.19 and throughout this chapter summarise the overall evolution of a school system, highlighting the challenges that countries and economies face in improving students' and schools' mathematics performance. To better understand the observed trends in performance, Chapters 2 and 3 of Volume II analyses in greater detail, how the student population has changed through migration and in socio-economic background, and how these characteristics are related to mathematics performance. Volume III explores students' engagement with and at school, drive and self-beliefs towards learning and mathematics. Volume IV, in turn, explores how attributes of school organisation and educational resources are related to changes in performance, providing further insight into the policies and practices that may explain the trends observed in mathematics performance.

Students at the different levels of proficiency in mathematics

Figure I.2.20 shows the location of some of these items on the PISA 2012 scale. A selection of items used in the 2012 survey is presented at the end of the chapter. Since PISA is a triennial assessment, it is useful to retain a sufficient number of questions over successive PISA assessments in order to generate trend data over time.

■ Figure I.2.20 ■

Map of selected mathematics questions, by proficiency level

Level	Lower score limit	Questions (position on PISA scale)
6	669	**REVOLVING DOOR** – Question 2 (840.3)
		HELEN THE CYCLIST – Question 3 (696.6)
		GARAGE – Question 2, FULL CREDIT (687.3)
5	607	**GARAGE** – Question 2, PARTIAL CREDIT (663.2)
		CLIMBING MOUNT FUJI – Question 2 (641.6)
		CLIMBING MOUNT FUJI – Question 3, FULL CREDIT (610.0)
4	545	**CLIMBING MOUNT FUJI** – Question 3, PARTIAL CREDIT (591.3)
		REVOLVING DOOR – Question 3 (561.3)
		WHICH CAR? – Question 3 (552.6)
3	482	**REVOLVING DOOR** – Question 1 (512.3)
		HELEN THE CYCLIST – Question 2 (510.6)
		WHICH CAR? – Question 2 (490.9)
2	420	**CLIMBING MOUNT FUJI** – Question 1 (464.0)
		HELEN THE CYCLIST – Question 1 (440.5)
		CHARTS – Question 5 (428.2)
1	358	**GARAGE** – Question 1 (419.6)
		CHARTS – Question 2 (415.0)
Below Level 1		**CHARTS** – Question 1 (347.7)
		WHICH CAR? – Question 1 (327.8)

The six mathematics proficiency levels are defined in the same way as the corresponding levels of the PISA 2003 scale, with the highest level labelled "Level 6", and the lowest labelled "Level 1". However, their descriptions have been updated to reflect the new mathematical process categories in the PISA 2012 framework and the large number of new items developed for PISA 2012. Figure I.2.21 provides descriptions of the mathematical skills, knowledge and understanding required at each level of the mathematical literacy scale and the average proportion of students at each of these proficiency levels across OECD countries.

Figure I.2.22 shows the distribution of students on each of these six proficiency levels. The percentage of students performing below Level 2 is shown on the left side of the vertical axis.

■ Figure I.2.21 ■

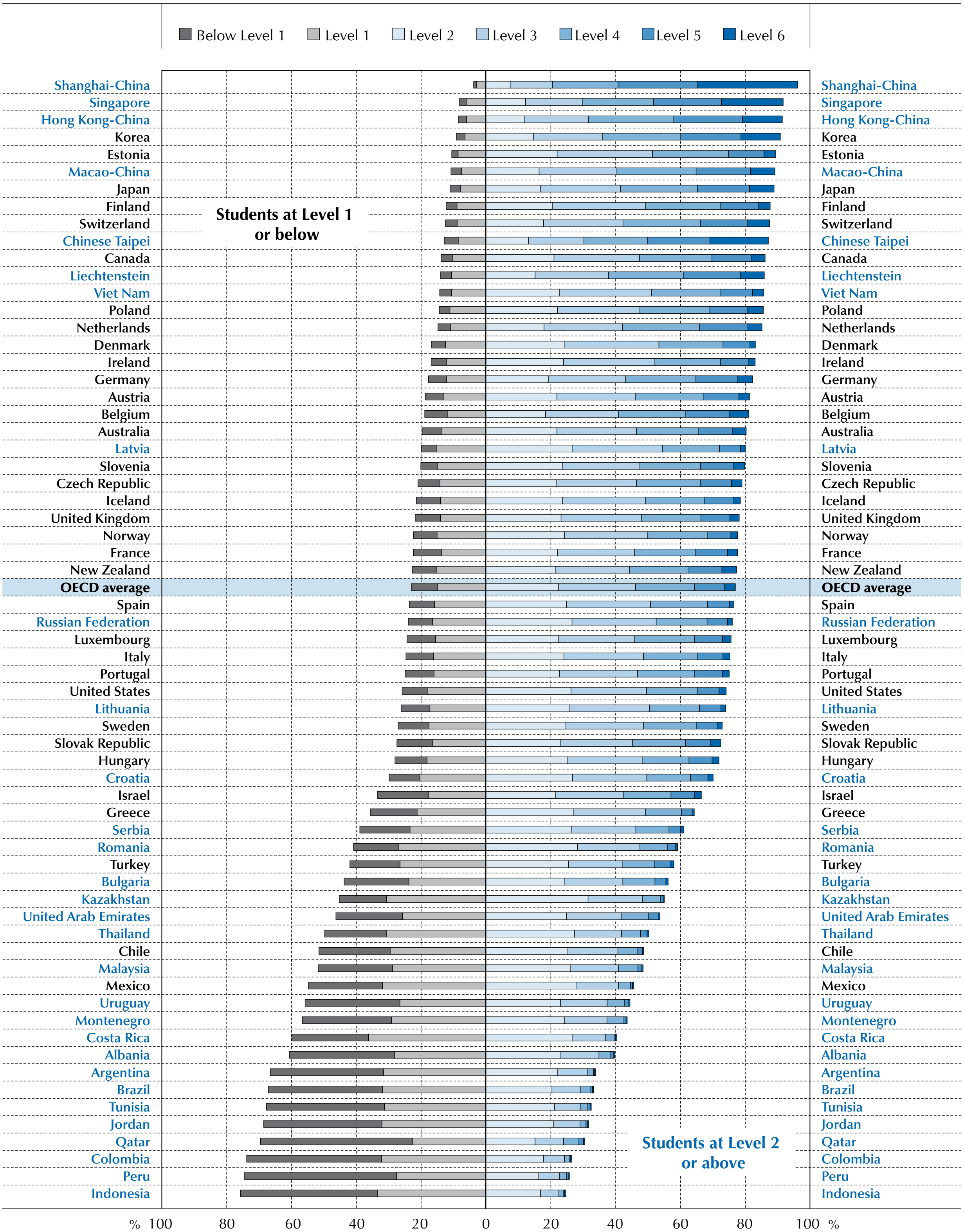
Summary descriptions for the six levels of proficiency in mathematics

Level	Lower score limit	Percentage of students able to perform tasks at each level or above (OECD average)	What students can typically do
6	669	3.3%	At Level 6, students can conceptualise, generalise and utilise information based on their investigations and modelling of complex problem situations, and can use their knowledge in relatively non-standard contexts. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for attacking novel situations. Students at this level can reflect on their actions, and can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situation.
5	607	12.6%	At Level 5, students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare, and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insight pertaining to these situations. They begin to reflect on their work and can formulate and communicate their interpretations and reasoning.
4	545	30.8%	At Level 4, students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations. Students at this level can utilise their limited range of skills and can reason with some insight, in straightforward contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments, and actions.
3	482	54.5%	At Level 3, students can execute clearly described procedures, including those that require sequential decisions. Their interpretations are sufficiently sound to be a base for building a simple model or for selecting and applying simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They typically show some ability to handle percentages, fractions and decimal numbers, and to work with proportional relationships. Their solutions reflect that they have engaged in basic interpretation and reasoning.
2	420	77.0%	At Level 2, students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions to solve problems involving whole numbers. They are capable of making literal interpretations of the results.
1	358	92.0%	At Level 1, students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are almost always obvious and follow immediately from the given stimuli.

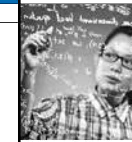
■ Figure I.2.22 ■

Proficiency in mathematics

Percentage of students at each level of mathematics proficiency



Countries and economies are ranked in descending order of the percentage of students at Levels 2, 3, 4, 5 and 6.
Source: OECD, PISA 2012 Database, Table I.2.1a.
StatLink http://dx.doi.org/10.1787/888932935572



Proficiency at Level 6 (scores higher than 669 points)

Students at Level 6 of the PISA mathematics assessment are able to successfully complete the most difficult PISA items. At Level 6, students can conceptualise, generalise and use information based on their investigations and modelling of complex problem situations, and can use their knowledge in relatively non-standard contexts. They can link different information sources and representations and move flexibly among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for addressing novel situations. Students at this level can reflect on their actions, and can formulate and precisely communicate their actions and reflections regarding their findings, interpretations and arguments, and can explain why they were applied to the original situation.

Question 3 in the example HELEN THE CYCLIST (Figure I.2.55) requires Level 6 proficiency. It requires a deeper understanding of the meaning of average speed, appreciating the importance of linking total time with total distance. Average speed cannot be obtained just by averaging the speeds, even though in this specific case the incorrect answer (28.3 km/hr) obtained by averaging the speeds (26.67 km/hr and 30 km/hr) is not much different from the correct answer of 28 km/hr. There are both mathematical and real world understandings of this phenomenon, leading to high demands on the fundamental mathematical capabilities of *mathematisation* and *reasoning and argumentation* and also *using symbolic, formal and technical language and operations*.

For students who know to work from total time (9 + 6 = 15 minutes) and total distance (4 + 3 = 7 km), the answer can be obtained simply by proportional reasoning (7 km in ¼ hour is 28 km in 1 hour), or by more complicated formula approaches (e.g. distance / time = 7 / (15/60) = 420 / 15 = 28). This question has been classified as an *employing* process because the greatest part of the demand arises from the mathematical definition of average speed and possibly also the unit conversion, especially for students using speed–distance–time formulas. It is one of the more difficult tasks of the item pool, and sits in Level 6 on the proficiency scale.

On average across OECD countries, 3.3% of students attain Level 6. The partner economy Shanghai-China has by far the largest proportion of students (30.8%) who score at this level in mathematics. Indeed, Shanghai-China has more students at this level of mathematics proficiency than at any other level, and is the only PISA participant where this is the case. Between 10% and 20% of students in four other Asian countries and economies – the three partner countries and economies Singapore (19.0%), Chinese Taipei (18.0%), Hong Kong-China (12.3%) and the OECD country Korea (12.1%) score at this level. Between 5% and 10% of students in Japan (7.6%), the partner economy Macao-China (7.6%), the partner country Liechtenstein (7.4%), Switzerland (6.8%) and Belgium (6.1%) attain Level 6 in mathematics. Thirty-three participating countries and economies show between 1% and 5% of their students at this level, while in 22 others, fewer than 1% of students score at the highest level, including the three OECD countries Mexico, Chile and Greece (Figure I.2.20 and Table I.2.1a).

Proficiency at Level 5 (scores higher than 607 but lower than or equal to 669 points)

At Level 5, students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insights pertaining to these situations. They begin to reflect on their work and can formulate and communicate their interpretations and reasoning.

Typical questions for Level 5 are exemplified by Question 3 from the unit CLIMBING MOUNT FUJI (Figure I.2.56). This question has been allocated to the *employing* category. There is one main relationship involved: the distance walked = number of steps x average step length. To use this relationship to solve the problem, there are two obstacles: rearranging the formula (which is probably done by students informally rather than formally using the written relationship) so that the average step length can be found from distance and number of steps, and making appropriate unit conversions. For this question, it was judged that the major cognitive demand comes from carrying out these steps; hence it has been categorised in the *employing* process, rather than identifying the relationships and assumptions to be made (the *formulating* process) or *interpreting* the answer in real world terms.

Box I.2.3. Top performers and all-rounders in PISA

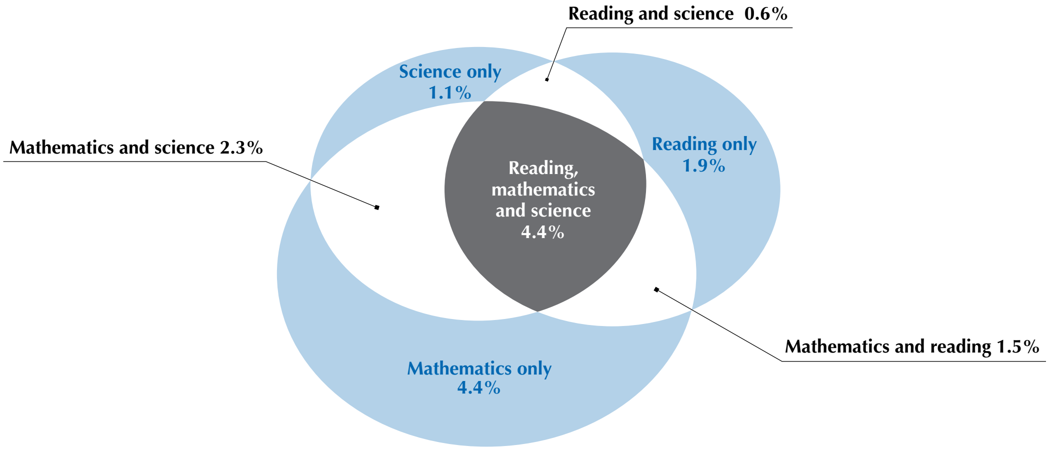
Performance in PISA refers to particular and increasingly complex tasks students are able to complete. A small proportion of students attains the highest levels and can be called top performers in mathematics, reading or science. Even fewer are the academic all-rounders, those students who achieve proficiency Level 5 or higher in mathematics, reading and science simultaneously. These students will be at the forefront of a competitive, knowledge-based global economy. They are able to draw on and use information from multiple and indirect sources to solve complex problems.

Results from the PISA 2012 assessment show that nurturing top performance and tackling low performance need not be mutually exclusive. Some high-performing countries in PISA 2012, like Estonia and Finland, have also low variation in student scores. Equally important, since their first participation in PISA, France, Hong Kong-China, Italy, Japan, Korea, Luxembourg, Macao-China, Poland, Portugal and the Russian Federation have been able to increase the share of top performers in mathematics, reading or science.

Figure I.2.a shows the proportion of top performers and all-rounders across OECD countries. Parts in the diagram shaded blue represent the percentage of 15-year-old students who are top performers in just one of the three subject areas assessed, that is, either in mathematics, reading or science. The parts in blue show the percentage of students who are top performers in two of the subject areas, while the grey part in the centre of the diagram shows the percentage of 15-year-old students who are top performers in all three subject areas.

■ Figure I.2.a ■

Overlapping of top performers in mathematics, reading and science on average across OECD countries



Note: Non-top performers in any of the three domains: 83.8%.
Source: OECD, PISA 2012 Database, Table I.2.29.

On average across OECD countries, 16.2% of students are top performers in at least one of the three subject areas; but only 4.4% of 15-year-old students are top performers in all three. This shows that excellence is not simply strong performance in all areas, but rather that it can be found among a wide range of students in various subjects.

About 1.5% of students are top performers in both mathematics and reading but not in science, 2.3% are top performers in both mathematics and science but not in reading, and fewer than 1% of students (0.6%) are top performers in both reading and science but not in mathematics. The percentage of students who are top performers in both mathematics and science is greater than the percentages who are top performers in mathematics and reading or in reading and science.

There is substantial variation among countries in the percentages of top performers in the three subjects (Table I.2.29).

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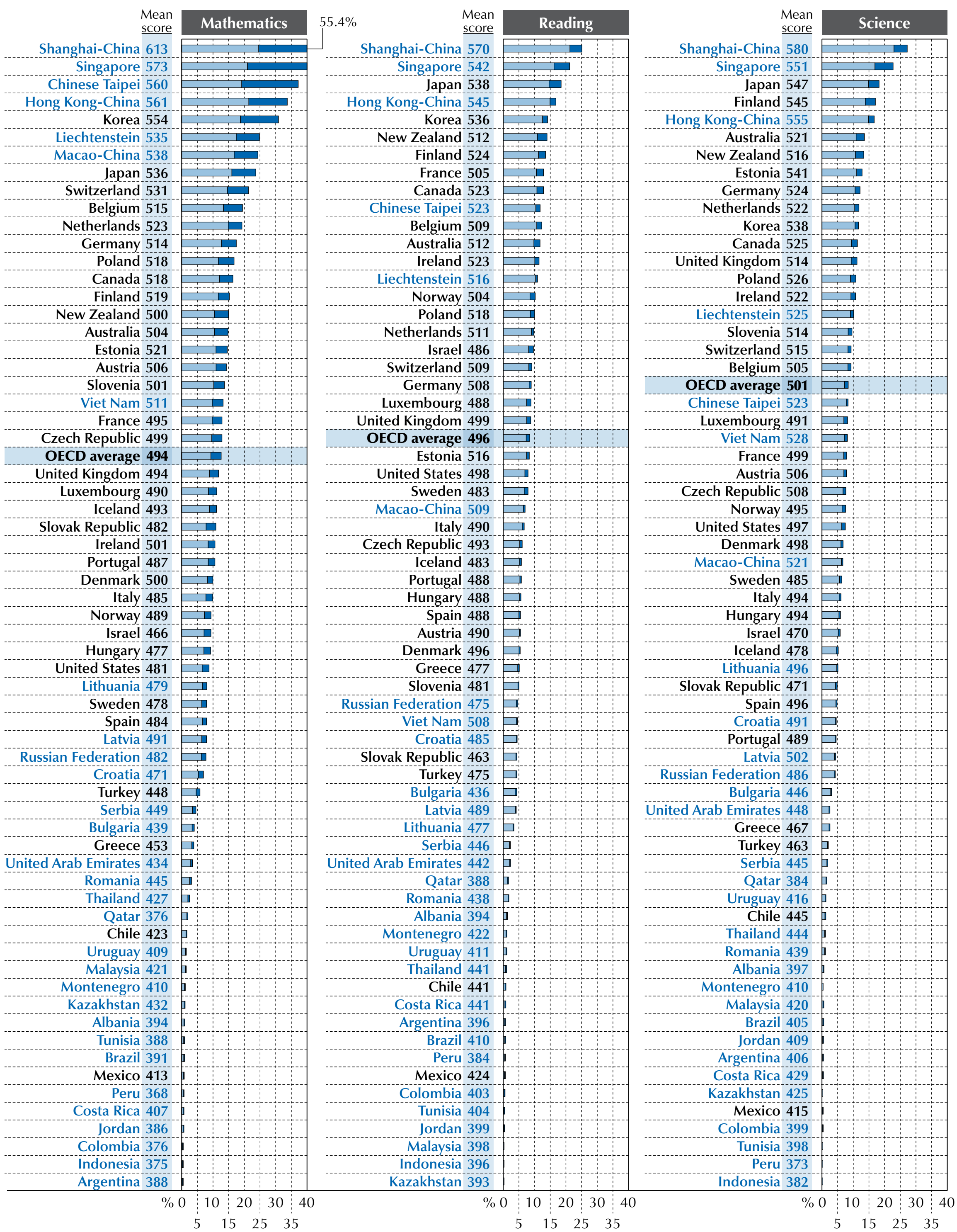
■ Figure I.2.b ■

Top performers in mathematics, reading and science

Percentage of students reaching the two highest levels of proficiency



Countries and economies are ranked in descending order of the percentage of top performers (Levels 5 and 6).
Source: OECD, PISA 2012 Database, Tables I.2.1a, I.2.3a, I.4.1a, I.4.3a, I.5.1a and I.5.3a.
StatLink http://dx.doi.org/10.1787/888932935572

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All-rounders, or top performers in all three subjects, comprise between 6% and just over 8% of 15-year-old students in Korea (8.1%), New Zealand (8.0%), Australia (7.6%), Finland (7.4%), Canada (6.5%), Poland (6.1%), Belgium (6.1%), the Netherlands (6.0%) and the partner economy Chinese Taipei (6.1%), and even larger proportions are found in the countries and economies Shanghai-China (19.6%), Singapore (16.4%), Japan (11.3%) and Hong Kong-China (10.9%). Conversely, in two OECD countries and 17 partner countries and economies, fewer than 1% of students are top performers in all three subjects.

Figure I.2.b shows the proportions of top performers in mathematics, reading and science for each country. Although on average across OECD countries, 9.3% and 3.3% of 15-year-olds reach Level 5 and Level 6 in mathematics, respectively, these proportions vary substantially across countries. For example, among OECD countries, Korea, Japan and Switzerland have at least 20% of top performers in mathematics, whereas Mexico and Chile have fewer than 1% and 2%, respectively. Among partner countries and economies, the overall proportion of these top performers also varies considerably from country to country; in some countries, no student achieves Level 6 in mathematics. At the same time, Shanghai-China, Singapore, Chinese Taipei and Hong Kong-China have the highest proportion of students performing at Level 5 or 6. Similar variations are shown in reading and science, with only slight differences in the patterns of these results among countries.

Among countries with similar mean scores in PISA, there are remarkable differences in the percentage of top-performing students. For example, Denmark has a mean score of 500 points in mathematics in PISA 2012 and 10% of students perform at high proficiency levels in mathematics, which is less than the average of around 13%. New Zealand has a similar mean mathematics score of 500 points, but 15% of its students attain the highest levels of proficiency, which is above the average. Although only a small percentage of students in Denmark perform at the lowest levels (see Table I.2.1a), these results could signal the absence of a highly educated talent pool for the future.

Having a large proportion of top performers in one subject is no guarantee of having a large proportion of top performers in the others. For example, Switzerland has one of the 10 largest shares of top performers in mathematics, but only a slightly-above-average share of top performers in reading and science.

Across the three subjects and across all countries, girls are as likely to be top performers as boys. On average across OECD countries, 4.6% of girls and 4.3% of boys are top performers in all three subjects, and 15.6% of girls and 16.8% of boys are top performers in at least one subject (Table I.2.30). However, while the gender gap among students who are top performers only in science is small (0.9% of girls and 1.3% of boys), it is large among top performers in mathematics only (2.9% of girls and 5.9% of boys) and in reading only (3.2% of girls and 0.6% of boys).

To increase the share of top-performing students, countries and economies need to look at the barriers posed by social background (examined in Volume II of this series), the relationship between performance and students' attitudes towards learning (examined in Volume III), and schools' organisation, resources and learning environment (examined in Volume IV).

On average across OECD countries, 12.6% of students are top performers, meaning that they are proficient at Level 5 or 6. Among all participants in PISA 2012, the partner economy Shanghai-China (55.4%) has the largest proportion of students performing at Level 5 or 6, followed by Singapore (40.0%), Chinese Taipei (37.2%) and Hong Kong-China (33.7%). In Korea 30.9% of students are top performers in mathematics. Between 15% and 25% of students in Liechtenstein, Macao-China, Japan, Switzerland, Belgium, the Netherlands, Germany, Poland, Canada, Finland and New Zealand perform at Level 5 or above in mathematics. By contrast, in 36 countries, 10% of students or fewer perform at these levels. These include the OECD countries Denmark (10.0%), Italy (9.9%), Norway (9.4%), Israel (9.4%), Hungary (9.3%), the United States (8.8%), Sweden (8.0%), Spain (8.0%), Turkey (5.9%), Greece (3.9%) and Chile (1.6%). In Kazakhstan, Albania, Tunisia, Brazil, Mexico, Peru, Costa Rica, Jordan, Colombia, Indonesia and Argentina, fewer than 1% of students are top performers in mathematics (Figure I.2.22 and Table I.2.1a).

Proficiency at Level 4 (scores higher than 545 but lower than or equal to 607 points)

At Level 4, students can work effectively with explicit models on complex, concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic representations, linking them directly to aspects of real-world situations. Students at this level can use their limited range of skills and can reason with some insight, in straightforward contexts. They can construct and communicate explanations and arguments based on their interpretations, reasoning and actions.

Question 3 in REVOLVING DOOR (Figure I.2.57) involves rates and proportional reasoning, and it sits within Level 4 on the mathematics proficiency scale. In one minute, the door revolves 4 times bringing $4 \times 3 = 12$ sectors to the entrance, which enables $12 \times 2 = 24$ people to enter the building. In 30 minutes, $24 \times 30 = 720$ people can enter (hence, the correct answer is response option D). The high frequency of PISA items that involve proportional reasoning highlights its centrality to mathematical literacy, especially for students whose mathematics has reached a typical stage for 15-year-olds. Many real contexts involve direct proportion and rates, which as in this case are often used in chains of reasoning. Coordinating such a chain of reasoning requires *devising a strategy* to bring the information together in a logical sequence.

This item also makes considerable demand on the *mathematisation* fundamental mathematical capability, especially in the *formulating* process. A student needs to understand the real situation, perhaps visualising how the doors rotate, presenting one sector at a time, making the only way for people to enter the building. This understanding of the real world problem enables the data given in the problem to be assembled in the right way. The questions in this unit have been placed in the *scientific* context category, even though they do not explicitly involve scientific or engineering concepts, as do many of the other items in this category. The scientific category includes items explaining why things are as they are in the real world.

On average across OECD countries, 30.8% of students perform at proficiency Level 4, 5 or 6. More than three out of four students in Shanghai-China perform at one of these levels (75.6%), and more than one in two students in Singapore, Hong Kong-China, Chinese Taipei and Korea do. Countries and economies where more than one in three students are proficient at proficiency Level 4, 5 or 6 are Macao-China (48.8%), Liechtenstein (48.0%), Japan (47.4%), Switzerland (45.3%), the Netherlands (43.1%), Belgium (40.2%), Germany (39.1%), Canada (38.8%), Finland (38.4%), Poland (38.1%), Estonia (38.0%), Austria (35.3%), Viet Nam (34.6%) and Australia (33.8%). Yet in 17 participating countries and economies, fewer than 10% of students attain Level 4 or above. In Indonesia, Colombia, Argentina, Jordan, Peru, Tunisia, Costa Rica, Brazil, Mexico and Albania, fewer than 5% of students attain Level 4 or above (Figure I.2.22 and Table I.2.1a).

Proficiency at Level 3 (scores higher than 482 but lower than or equal to 545 points)

At Level 3, students can execute clearly described procedures, including those that require sequential decisions. Their interpretations are sufficiently sound to be the basis for building a simple model or for selecting and applying simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They typically show some ability to handle percentages, fractions and decimal numbers, and to work with proportional relationships. Their solutions reflect that they have engaged in basic interpretation and reasoning.

Question 1 in REVOLVING DOOR (Figure I.2.57) requires Level 3 proficiency. This question may appear very simple: finding the angle of 120 degrees between the two door wings, but the student responses indicate it is at Level 3. This is probably because of the demand arising from *communication*, *representation* and *mathematisation* as well as the specific knowledge of circle geometry that is needed. The context of three-dimensional revolving doors has to be understood from the written descriptions. It also needs to be understood that the three diagrams in the initial stimulus provide different two-dimensional information about just one revolving door (not three doors) – first the diameter, then the directions in which people enter and exit from the door, and thirdly connecting the wings mentioned within the text with the lines of the diagrams. The fundamental mathematical capability of *representation* is required at a high level to interpret these diagrams mathematically. They give the view from above, but students also need to visualise real revolving doors especially in answering Questions 2 and 3.

On average across OECD countries, 54.5% of students are proficient at Level 3 or higher (that is, at Level 3, 4, 5 or 6). More than three out of four students in Shanghai-China (88.7%), Singapore (79.5%), Hong Kong-China (79.5%) and Korea (76.2%) attain Level 3 or above. More than two out of three students are proficient at these levels in Chinese Taipei (74.0%), Macao-China (72.8%), Japan (72.0%), Liechtenstein (70.7%), Switzerland (69.8%), Estonia (67.5%), the Netherlands (67.3%) and Finland (67.2%). By contrast, in 22 participating countries, fewer than one in three students attains these levels. In Peru, Colombia and Indonesia, fewer than 10% of students perform at those levels (Figure I.2.22 and Table I.2.1a).

Proficiency at Level 2 (scores higher than 420 but lower than or equal to 482 points)

At Level 2, students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures or conventions to solve problems involving whole numbers. They are capable of making literal interpretations of the results.

Results from longitudinal studies in Australia, Canada, Denmark and Switzerland show that students who perform below Level 2 often face severe disadvantages in their transition into higher education and the labour force in subsequent years. The proportion of students who perform below this baseline proficiency level thus indicates the degree of difficulty countries face in providing their populations with a minimum level of competencies (OECD, 2012).

Question 1 in the unit HELEN THE CYCLIST (Figure I.2.55) is typical of Level 2 tasks. Question 1, a simple multiple choice item, requires comparison of speed when travelling 4 km in 10 minutes versus 2 km in 5 minutes. It is been classified within the *employing* process category because it requires the precise mathematical understanding that speed is a rate and that proportionality is the key. This question can be solved by recognising the doubles involved (2 km – 4 km; 5 km – 10 km), which is the very simplest notion of proportion. Consequently, with this Level 2 question, successful students demonstrate a very basic understanding of speed and of proportion calculations. If distance and time are in the same proportion, the speed is the same. Of course, students could correctly solve the problem in more complicated ways (e.g. calculating that both speeds are 24 km per hour) but this is not necessary. PISA results for this question do not incorporate information about the solution method used. The correct response option here is B (Helen's average speed was the same in the first 10 minutes and in the next 5 minutes).

Level 2 is considered the baseline level of mathematical proficiency that is required to participate fully in modern society. More than 90% of students in the four top-performing countries and economies in PISA 2012, Shanghai-China, Singapore, Hong Kong-China and Korea, meet this benchmark. Across OECD countries, an average of 77% of students attains Level 2 or higher: more than one in two students perform at these levels in all OECD countries except Chile (48.5%) and Mexico (45.3%). Only around one in four students in the partner countries Colombia, Peru and Indonesia attains this benchmark (Figure I.2.22 and Table I.2.1a).

Proficiency at Level 1 (scores higher than 358 but lower than or equal to 420 points) or below

At Level 1 students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are almost always obvious and follow immediately from the given stimuli.

Students below Level 1 may be able to perform very direct and straightforward mathematical tasks, such as reading a single value from a well-labelled chart or table where the labels on the chart match the words in the stimulus and question, so that the selection criteria are clear and the relationship between the chart and the aspects of the context depicted are evident, and performing arithmetic calculations with whole numbers by following clear and well-defined instructions.

Question 1 in GARAGE (Figure I.2.60) is a task that corresponds to the top of Level 1 in difficulty, very close to the Level 1/Level 2 boundary on the proficiency scale. It asks students to identify a picture of a building from the back, given the view from the front. The diagrams must be interpreted in relation to the real world positioning of "from the back", so this question is classified in the *interpreting* process. The correct response is C. Mental rotation tasks such as this are solved by some people using intuitive spatial visualisation. Other people need explicit reasoning processes. They may analyse the relative positions of multiple features (door, window, nearest corner), discounting the multiple choice alternatives one by one. Others might draw a bird's eye view, and then physically rotate it. This is just one example of how different students may use quite different methods to solve PISA questions: in this case explicit reasoning for some students is intuitive for others.

Question 1 in CHARTS (Figure I.2.59), with a difficulty of 347.7, is a task below Level 1 on the mathematical proficiency scale, being one of the easiest tasks in the PISA 2012 item pool. It requires the student to find the bars for April, select the correct bar for the Metafolkies, and read the height of the bar to obtain the required response selection B (500). No scale reading or interpolation is required.

All PISA participating countries and economies show students at Level 1 or below; but the largest proportions of students who attain only these levels are found in the lowest-performing countries.

Across OECD countries, an average of 23.0% of students is proficient only at or below Level 1. In Shanghai-China, Singapore, Hong Kong-China and Korea, fewer than 10% of students perform at or below Level 1. Fewer than 15% do in Estonia, Macao-China, Japan, Finland, Switzerland, Chinese Taipei, Canada, Liechtenstein, Viet Nam, Poland and the Netherlands. By contrast, in 31 participating countries and economies more than one out of four students perform at these levels. In 15 countries the proportion of students who attain only Level 1 or below exceeds 50% (Figure I.2.22 and Table I.2.1a).

Trends in the percentage of low- and top-performers in mathematics

Changes in a country's or economy's average performance can result from changes at different levels of the performance distribution. For example, for some countries and economies, average improvement is driven by improvements among low-achieving students, where the share of students scoring below Level 2 is reduced. In other countries and economies, average improvement is driven mostly by changes among high-achieving students, where the share of students who perform at or above Level 5 increases. On average across OECD countries with comparable data, between 2003 and 2012 there was an increase of 0.7 percentage points in the share of students who do not meet the baseline proficiency level in mathematics and a reduction of 1.6 percentage points in the share of students at or above proficiency Level 5 (Figure I.2.23 and Table I.2.1b).

However, these trends vary across countries. Some countries and economies saw a reduction in the proportion of low-performing students and a concurrent increase in the proportion of top-performing students. These are school systems that have seen improvements in performance both at the bottom and the top ends of the performance distribution. There are other countries where improvements are limited to reducing the share of low-performing students or increasing the share of top-performing students.

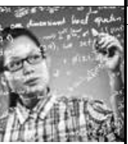
Countries and economies can be grouped into categories based on whether they have: simultaneously reduced the share of low performers and increased the share of top performers between previous PISA assessments and PISA 2012; reduced the share of low performers but not increased the share of top performers between any previous PISA assessment and PISA 2012; increased the share of top performers but not reduced the share of low performers; and reduced the share of top performers or increased the share of low performers between PISA 2012 and any previous PISA assessment. The following section groups countries along these categories, first identifying those that have simultaneously reduced the share of low performers and increased the share of top performers between PISA 2003 and PISA 2012, between PISA 2006 and PISA 2012 or between PISA 2009 and PISA 2012. The remaining countries and economies are categorised as those that reduced the share of low performing students, increased the share of top performing students, or that saw an increase in the share of low performers or a reduction in the share of top performers.

Moving everyone up: Reductions in the share of low performers and increases in that of top performers

Countries and economies that have reduced the proportion of students scoring below Level 2 and increased the proportion of students scoring above Level 5 are ones that have been able to spread the improvements in their education systems across all levels of performance. Between 2003 and 2012 this was observed in Italy, Poland and Portugal. This reduction in the share of low-performers and increase in the share of high-performers was observed in Israel, Romania and Qatar between PISA 2006 and PISA 2012, and in Ireland, Malaysia and the Russian Federation between PISA 2009 and PISA 2012 (Figure I.2.23 and Table I.2.1b).

Poland, for example, reduced the share of students scoring below Level 2 by eight percentage points while increasing the share of high achievers by seven percentage points between 2003 and 2012. A large part of this change is concentrated in the 2009 to 2012 period. In 2003, 2006 and 2009 about 20% of students were low-performers and around 10% were top-performers; by 2012 the share of students scoring below Level 2 dropped to 14% and the share of students scoring at or above Level 5 increased to 17%. Similarly, Portugal reduced the share of students scoring below Level 2 by five percentage points and increased the share of students scoring at or above Level 5 also by five percentage points during the period, with most of this change taking place between 2006 and 2009. Italy saw an overall reduction of seven percentage points in the share of students performing below Level 2 and an increase of three percentage points in the share of students scoring at or above Level 5, with most of this change taking place between 2006 and 2009 (Figure I.2.23 and Table I.2.1b).

Annex B4 illustrates, for each country and economy, how mathematics performance at the 10th, 25th, 75th and 90th percentiles has evolved since 2003. Like the trends in the share of low- and top-performing students, it shows that average improvement in Poland and Italy, for example, is observed among low-, average and high-achieving students alike.

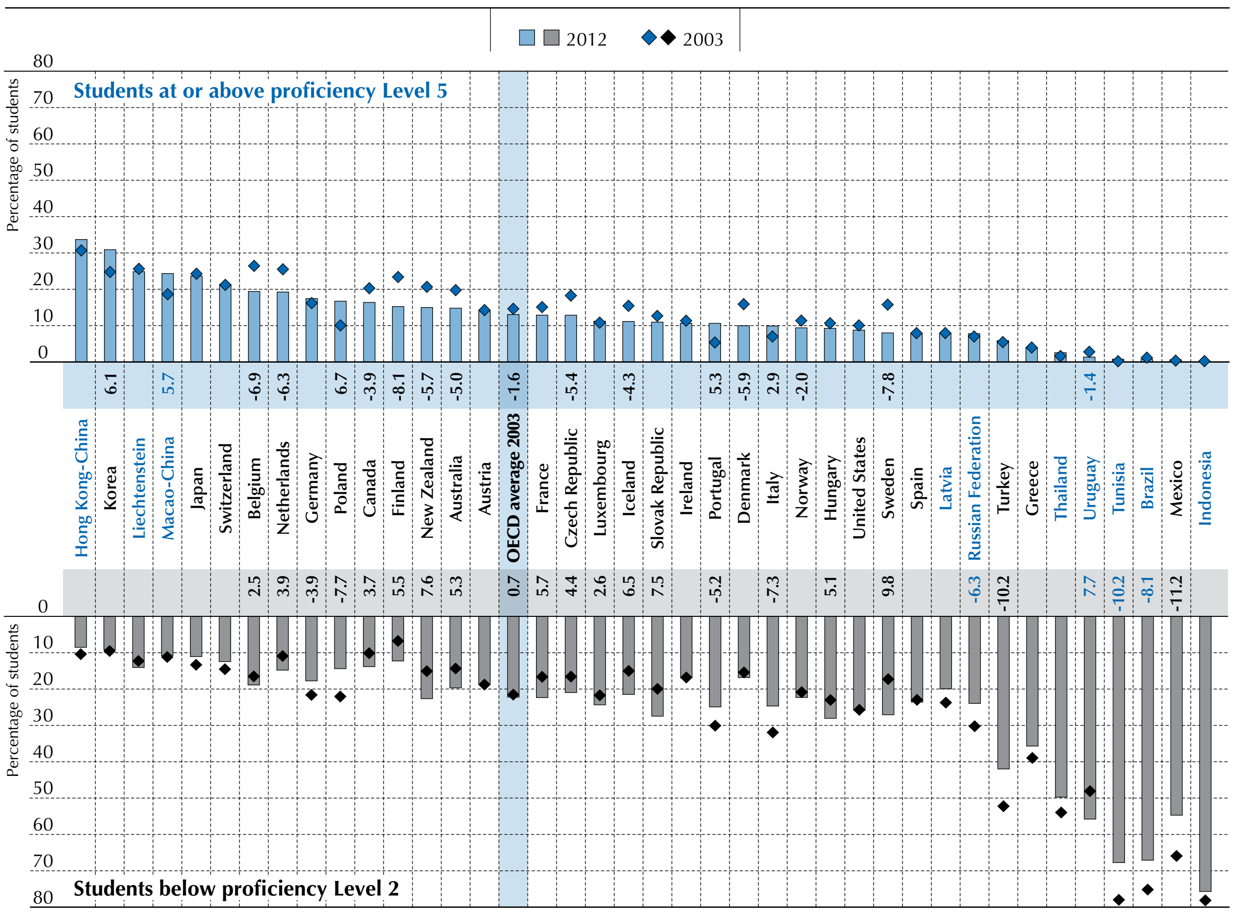


Reducing underperformance: Reductions in the share of low performers but no change in that of top performers

Other countries and economies have concentrated change among those students who did not meet the baseline proficiency level. These countries and economies saw significant improvements in the performance of students who need it most and who now have basic skills and competencies to fully participate in society. Between 2003 and 2012, Brazil, Mexico, Tunisia and Turkey saw a reduction of more than five percentage points in the share of students scoring below proficiency Level 2 in mathematics. Germany also saw significant reductions in the proportion of students at proficiency Level 2, but no change in the proportion of those scoring at or above Level 5. Similarly, Bulgaria and Montenegro, both of which began participating in PISA after 2003, showed significant reductions in the proportion of students scoring at Level 2 between 2006 and 2012, as did Albania, Dubai (United Arab Emirates) and Kazakhstan between 2009 and 2012 (Figure I.2.23 and Table I.2.1b). Annex B4 shows the performance trajectories of these countries and economies, highlighting how the performance of their lowest achievers (those in the 10th percentile of performance) improved more than that of the highest-achieving students (those in the 90th percentile). By lifting the performance of their lowest-achieving students, these countries and economies have narrowed the gap between high- and low-achieving students and, in some cases, increased equity as well, as many low-achieving students are also from disadvantaged backgrounds (see Volume II, Chapter 2).

■ Figure I.2.23 ■

Percentage of low-performing students and top performers in mathematics in 2003 and 2012



Notes: The chart shows only countries/economies that participated in both PISA 2003 and PISA 2012 assessments.

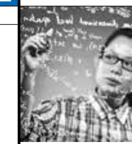
The change between PISA 2003 and PISA 2012 in the share of students performing below Level 2 in mathematics is shown below the country/economy name. The change between PISA 2003 and PISA 2012 in the share of students performing at or above Level 5 in mathematics is shown above the country/economy name. Only statistically significant changes are shown (see Annex A3).

OECD average 2003 compares only OECD countries with comparable mathematics scores since 2003.

Countries and economies are ranked in descending order of the percentage of students at or above proficiency Level 5 in mathematics in 2012.

Source: OECD, PISA 2012 Database, Table I.2.1b.

StatLink http://dx.doi.org/10.1787/888932935572



Nurturing top performance: Increase in the share of top performers but no change in that of low performers

Some countries and economies increased the proportion of students performing at or above Level 5. These are students who can handle complex mathematical content and processes. Higher proportions of these students signal a school system's capacity to promote student performance at the highest level. Between 2003 and 2012, Korea and Macao-China saw around a six percentage-point increase in the share of students performing at this level. Other increases in the proportion of students scoring at or above Level 5 were observed in Chinese Taipei, Hong Kong-China, Japan, Serbia and Thailand (between 2006 and 2012) and in Estonia, Latvia, Shanghai-China and Singapore (between 2009 and 2012) (Figure I.2.23 and Table I.2.1b). As shown in Annex B4, the trajectories of these countries' and economies' low- and high-achieving students point to greater increases among the high achievers than among the low achievers. When comparing Korea's mathematics scores in 2012 with those of 2003, for example, students in the 90th percentile improved by 20 scores points, and those at the 75th percentile improved by 18 points; however, there was no change in mathematics performance among those students in the 10th and 25th percentiles. That is, if those students at the bottom of the distribution performed at similar levels in 2003 and 2012, those at the top attained higher levels in 2012 than they did in 2003.

Increase in the share of low performers or decrease in that of top performers

There are 17 countries and economies, however, where the proportion of students who do not reach the baseline proficiency level increased or the proportion of students who reach the highest levels of proficiency decreased between a previous PISA assessment and PISA 2012. In these countries and economies there were fewer students performing at the top levels and more students who did not show the baseline level of mathematical literacy in 2012 than there were in a previous assessment (Figure I.2.23 and Table I.2.1b).

Variation in student performance in mathematics

The standard deviation in PISA scores, the difference between the top and bottom 5% of sampled students and the difference between the top and bottom 10%, or between the top and bottom quarters are all measures of the extent to which student performance varies among 15-year-olds. In fact, each of these measures gives more or less the same picture. Table I.2.3a shows the mean, standard deviation and percentiles of PISA mathematics scores for all participating countries and economies.

As shown in Figure I.2.24, the ten PISA participants with the widest spread in scores (score-point difference between the top and bottom 10% of students) are Israel, Belgium, the Slovak Republic, New Zealand, France and Korea as well as the partner countries and economies Chinese Taipei, Singapore, Shanghai-China and Qatar. This group includes four of the highest-performing countries and economies (Chinese Taipei, Singapore, Shanghai-China and Korea), one of the lowest performers (Qatar) as well as two OECD countries that perform close to the OECD average (France, which is at the OECD average, and New Zealand, which is just above the OECD average) (Table I.2.3a).

The ten participating countries/economies with the narrowest spread are Mexico and the partner countries Costa Rica, Indonesia, Kazakhstan, Colombia, Jordan, Argentina, Tunisia, Brazil and Thailand. All of these countries are among the 20 lowest-performing countries; seven of them are among the 10 lowest-performing countries. Less variation in performance is observed among the very lowest-performing countries, largely because there are fewer scores at the highest proficiency levels and, as a result, scores tend to be concentrated at the lower proficiency levels (Figure I.2.24 and Table I.2.3a).

It is noteworthy that the relationship between average performance and the spread in student scores is weak, suggesting that high mean performance does not inevitably lead to large disparities in student performance. It is possible to combine a relatively narrow spread of scores and a relatively high average score, as does, for example, Estonia.

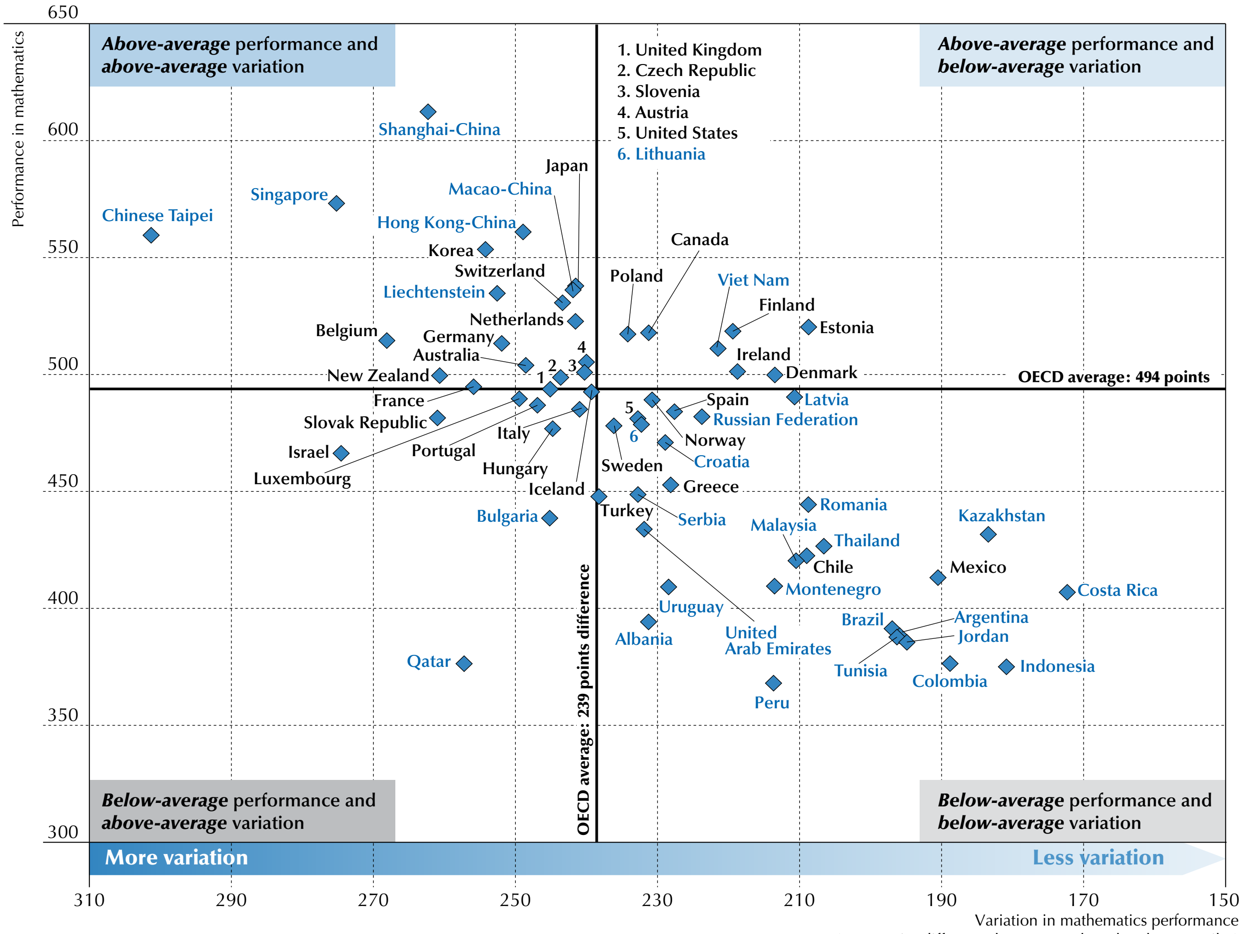
Gender differences in mathematics performance

Figure I.2.25 presents a summary of boys' and girls' performance in the PISA mathematics assessment (Table I.2.3a). On average across OECD countries, boys outperform girls in mathematics by 11 score points. Despite the stereotype that boys are better than girls at mathematics, boys show an advantage in only 38 out of the 65 countries and economies that participated in PISA 2012, and in only six countries is the gender gap larger than the equivalent of half a school year.

As shown in Figure I.2.25, the largest difference in scores between boys and girls – in favour of boys – is seen in the partner country Colombia, and the OECD countries Luxembourg and Chile, a difference of around 25 points. In the partner countries Costa Rica, Liechtenstein and the OECD country Austria, this difference is between 22 and 24 points.

■ Figure I.2.24 ■

Relationship between performance in mathematics and variation in performance



Source: OECD, PISA 2012 Database, Table I.2.3a.
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In Korea, Japan and the partner economy Hong Kong-China, all of which are among the 10 top-performing countries, as well as in Italy, Spain, Ireland and New Zealand, and in the partner countries Peru, Brazil and Tunisia, this difference is between 15 and 20 points. In Luxembourg, a larger proportion of boys than girls attains the three highest proficiency levels, and far fewer boys than girls are found in the three lowest proficiency levels, leading to a marked overall gender difference in favour of boys (Tables I.2.2a and I.2.3a).

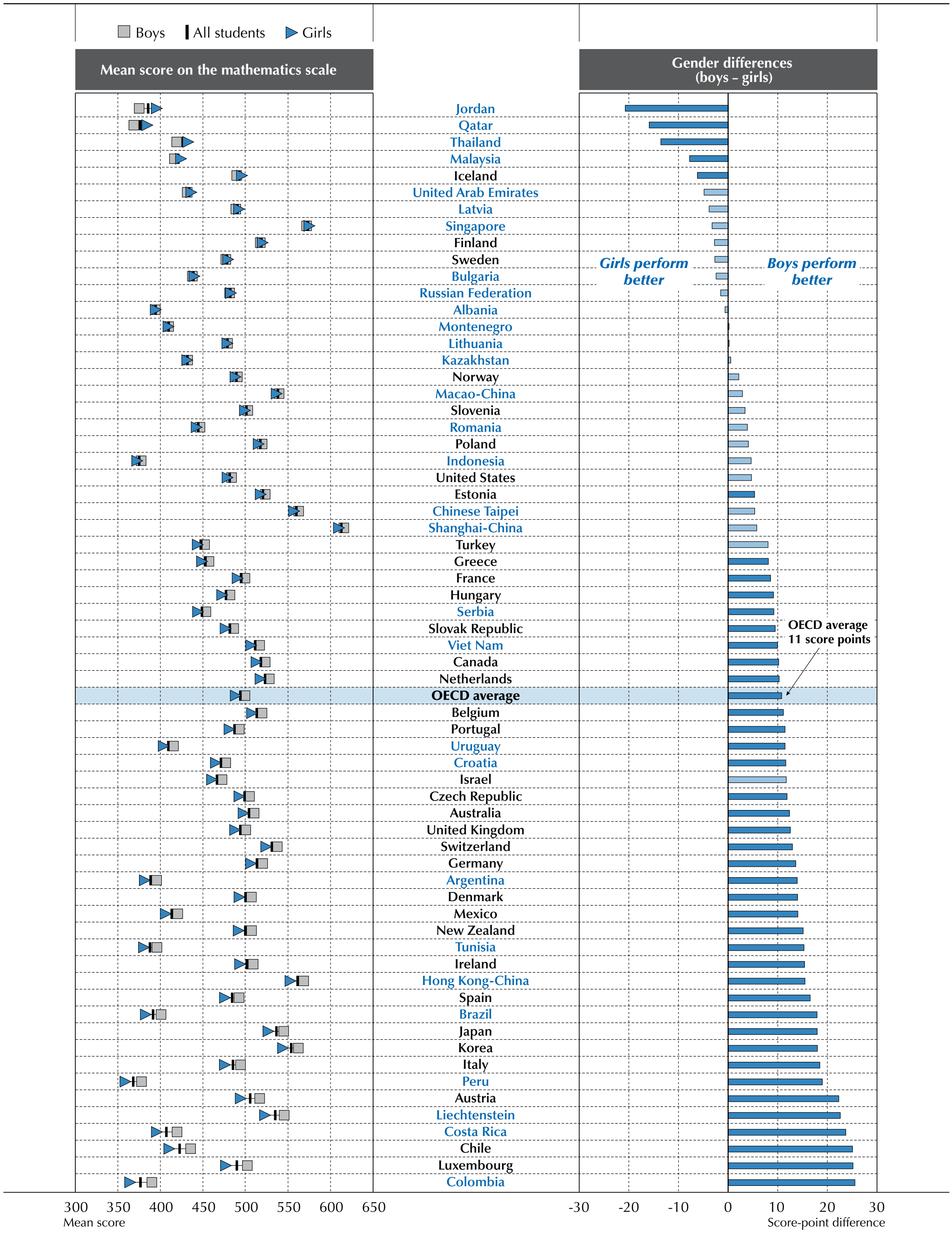
In contrast, in only five countries do girls outperform boys in mathematics. The largest difference is seen in the partner country Jordan, where girls score around 21 points higher than boys. Girls also outperform boys in the partner countries Qatar, Thailand, Malaysia and in the OECD country Iceland (Figure I.2.25 and Table I.2.3a). In all of these countries more boys score at or below Level 1 than girls. The difference is particularly large in the partner country Jordan, where around 43% of boys score at or below Level 1, compared to around 30% of girls. In Iceland, while girls and boys are well-represented at all proficiency levels, far more boys than girls score below proficiency Level 1 (Table I.2.2a).

Figure I.2.26 shows the average proportions of boys and girls in OECD countries within each of the defined mathematics proficiency levels. Larger proportions of boys than girls score at Level 5 or 6 (top performers) and at Level 4. Conversely, the proportion of girls is larger than the proportion of boys at all other proficiency levels, from Level 3.

In almost all participating countries and economies, a larger proportion of boys than girls are top performers in mathematics (Level 5 or 6). In high-performing countries and economies, where a relatively large share of students performs at these levels, the difference in the proportion of boys and girls scoring at these levels is generally larger.

■ Figure I.2.25 ■

Gender differences in mathematics performance

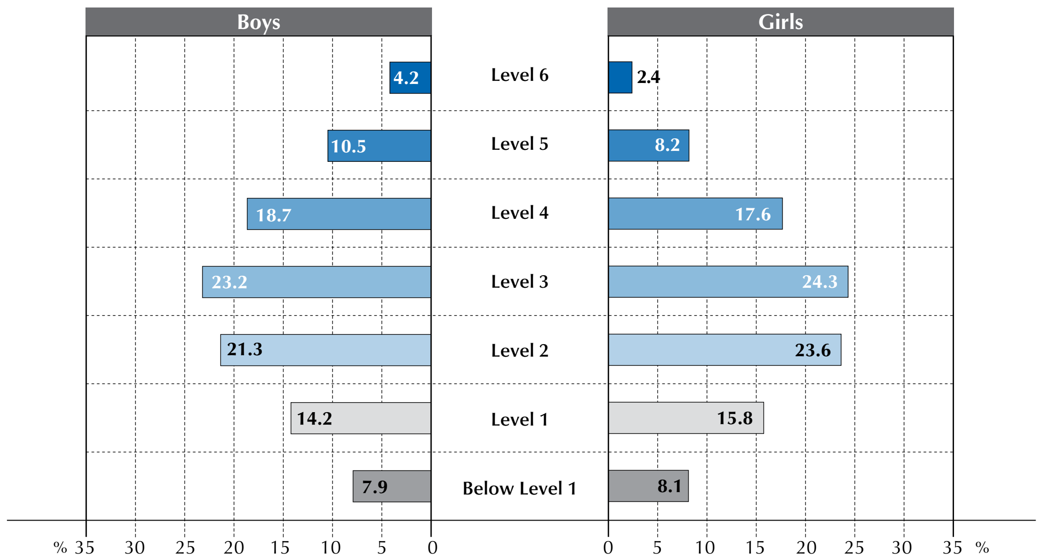


Note: Statistically significant gender differences are marked in a darker tone (see Annex A3).
Countries and economies are ranked in ascending order of the gender score-point difference (boys – girls).
Source: OECD, PISA 2012 Database, Table I.2.3a.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.26 ■

Proficiency in mathematics among boys and girls

OECD average percentages of boys and girls at each level of mathematics proficiency



Source: OECD, PISA 2012 Database, Table I.2.2a.
StatLink http://dx.doi.org/10.1787/888932935572

For example, in the high-performing OECD countries Korea and Japan, and the partner economy Hong Kong-China, the share of boys who are top performers is around 9 percentage points larger than that of girls. In Israel, Austria, Italy, New Zealand and Luxembourg, which are situated in the middle of the performance distribution, the share of boys who attain at the highest proficiency levels is considerably larger than the share of girls who do, by a difference of 7.7 to 5.8 percentage points. This difference is also larger than 5 percentage points in Belgium, Chinese Taipei, the Slovak Republic, Spain, Canada, Liechtenstein, Switzerland and Germany (Table I.2.2a).

While the proportion of girls is larger than the share of boys at the lower proficiency levels, there is considerable variation among countries and economies. In around a third of participating countries and economies, a higher proportion of boys than girls do not achieve the baseline level of proficiency. In Finland, Iceland and the partner countries Thailand, Jordan, Malaysia, the United Arab Emirates, Lithuania, Latvia and Singapore, a larger proportion of boys than girls perform below Level 2, the baseline proficiency level, and some of these countries, like Finland and the partner country Singapore, belong to the 15 top-performing countries and economies. Yet in many of the 15 lowest-performing countries and economies, including the OECD countries Chile and Mexico and the partner countries Costa Rica, Colombia, Brazil, Tunisia, Argentina and Peru, more girls than boys do not attain that level of proficiency. But in Luxembourg, which scores around the OECD average, and Liechtenstein, which scores well above the OECD average, the share of girls who score at or below Level 1 is considerably larger than that of boys by a difference of 8.6 and 6.1 percentage points, respectively (Table I.2.2a).

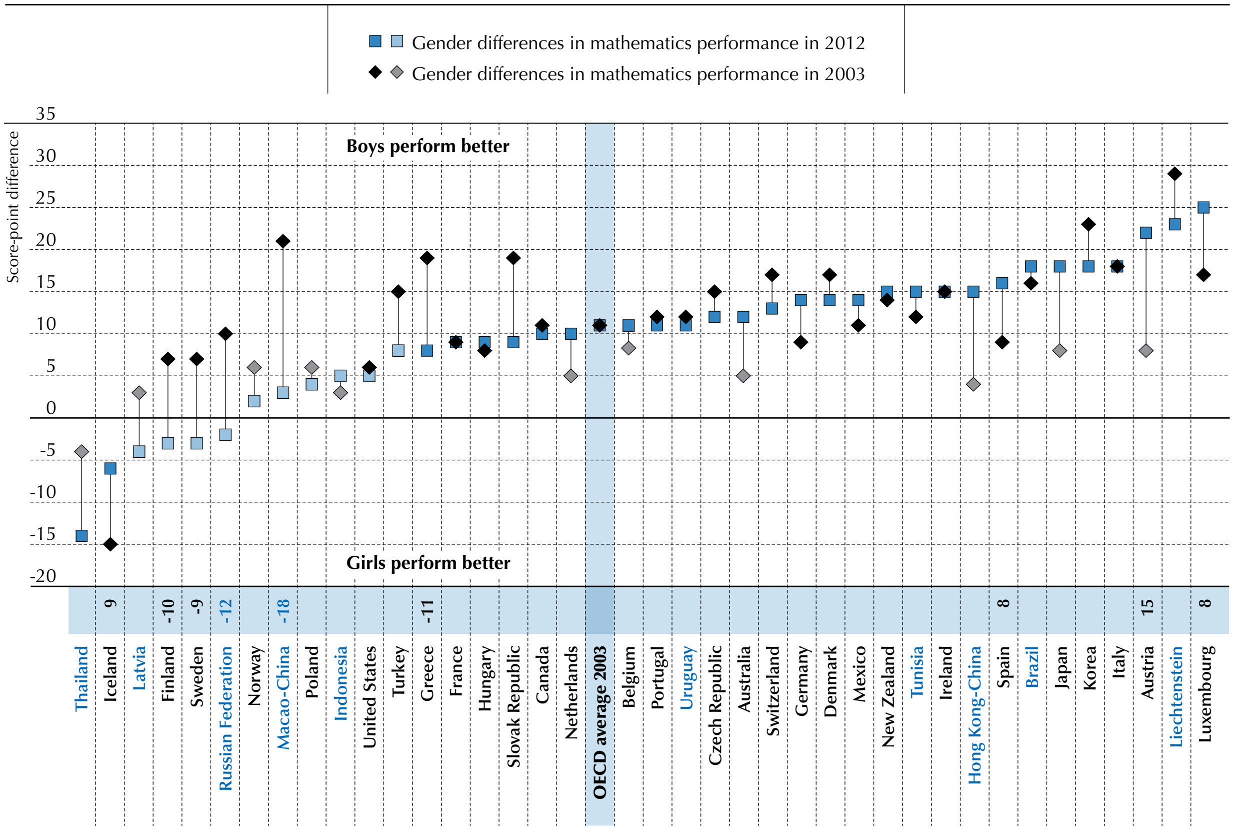
Trends in gender differences in mathematics performance

Among the countries and economies that showed a gender gap in mathematics performance in favour of boys in 2003, by 2012 the gender gap narrowed by nine score points or more in Finland, Greece, Macao-China, the Russian Federation and Sweden. Thus, in Greece, while boys outperformed girls in mathematics by 19 points in 2003, by 2012 this difference had shrunk to eight score points. In Finland, Macao-China, the Russian Federation, Sweden, Turkey and the United States, there was no longer a gender gap in mathematics performance favouring boys in 2012 compared to 2003. In Austria, Luxembourg and Spain, the gender gap favouring boys widened between 2003 and 2012. For example, in Austria in 2003, there was no observed gender gap in mathematics performance; but by 2012 there was a 22 score-point difference in performance in favour of boys. Iceland was one of the few countries where girls outperformed boys in mathematics in 2003; in 2012, girls still outperformed boys, but the gender gap had narrowed (Figure I.2.27 and Table I.2.3c).

Countries seeking to reduce girls' disadvantage in mathematics could examine the experiences of Korea, Latvia, Macao-China, the Russian Federation and Thailand. In Macao-China and the Russian Federation, for example, girls' mathematics performance improved by around 20 score points while boys' performance did not change, resulting in a narrowing of the gender gap in mathematics performance to the extent that the gender gap observed in 2003 lost statistical significance by 2012. In Thailand, boys' performance did not change between PISA 2003 and PISA 2012, but girls' performance improved by 14 score points.

■ Figure I.2.27 ■

Change between 2003 and 2012 in gender differences in mathematics performance



Notes: Gender differences in PISA 2003 and PISA 2012 that are statistically significant are marked in a darker tone (see Annex A3).
Statistically significant changes in the score-point difference between boys and girls in mathematics performance between PISA 2003 and PISA 2012 are shown next to the country/economy name.
OECD average 2003 compares only OECD countries with comparable mathematics scores since 2003.
Countries and economies are ranked in ascending order of gender differences (boys-girls) in 2012.
Source: OECD, PISA 2012 Database, Table I.2.3c.
StatLink http://dx.doi.org/10.1787/888932935572

These trends are also reflected in the changes in the proportion of boys and girls who can be considered top performers in PISA (those who score at or above proficiency Level 5) or who are considered low performers in PISA (because they score below proficiency Level 2). Consistent with the fact that the gender gap in mathematics has narrowed or now favours girls in certain countries and economies, in Latvia, Portugal, the Russian Federation and Thailand the share of girls who perform below proficiency Level 2 shrunk between 2003 and 2012 with no concurrent change in the share of low-performing boys. In Macao-China and the Russian Federation during the period, the share of top-performing girls increased with no such increase among boys. In addition, Italy, Poland, Portugal and the Russian Federation show a reduction in the share of girls who perform below Level 2 and an increase in the share of girls who perform at Level 5 or 6 (Table I.2.2b).

Box I.2.4. **Improving in PISA: Brazil**

With an economy that traditionally relied on the extraction of natural resources and suffered stagnating growth and spells of hyperinflation until the early 1990s, Brazil is today rapidly expanding its industrial and service sector. Its population of more than 190 million, which is spread across 27 states in geographic areas as vast and diverse as Rio de Janeiro and the Amazon River basin, recognises the critical role education plays in the country's economic development.

Like only a handful of other countries, Brazil's performance in mathematics, reading and science has improved notably over the past decade. Its mean score in the PISA mathematics assessment has improved by an average of 4.1 point per year – from 356 points in 2003 to 391 points in 2012. Since 2000, reading scores have improved by an average of 1.2 score points per year; and, since 2006, science scores have risen by an average of 2.3 score points per year. Lowest-achieving students (defined as the 10% of students who score the lowest) have improved their performance by 65 score points – the equivalent of more than a year and a half of schooling. Despite these considerable improvements, around two out of three Brazilian students still perform below Level 2 in mathematics (in 2003, three in four students did).

Not only have most Brazilian students remarkably improved their performance, Brazil has expanded enrolment in primary and secondary schools. While in 1995, 90% of students were enrolled in primary schools at age seven, only half of them continued to finish eighth grade. In 2003, 35% of 15-year-olds were not enrolled in school in grade 7 or above; by 2012 this percentage had shrunk to 22%. Enrolment rates for 15-year-olds thus increased, from 65% in 2003 to 78% in 2012. Many of the students who are now included in the school system come from rural communities or socio-economically disadvantaged families, so the population of students who participated in the PISA 2012 assessment is very different from that of 2003.

PISA compares the performance of 15-year-old students who are enrolled in schools; but for those countries where this population has changed dramatically in a short period of time, trend data for students with similar background characteristics provide another way of examining how students' performance is changing beyond changes in enrolment. Figure I.2.c compares the performance of students with similar socio-economic status across all years. The score attained by a socio-economically advantaged/average/disadvantaged student increased by 21/25/27 points, respectively, between 2003 and 2012.

The figure also simulates alternate scenarios, assuming that the students who are now enrolled in schools – but probably weren't in 2003 – score in the bottom half of the performance distribution, the bottom quarter of the performance distribution, or the bottom of the distribution and also come from the bottom half, bottom quarter, and bottom of the socio-economic distribution. Given that they assume that the newly enrolled students have lower scores than students who would have been enrolled in 2003, these simulations indicate the upper bounds of Brazil's improvement in performance.

For example, under the assumption that the newly enrolled students perform in the bottom quarter of mathematics performance, Brazil's improvement in mathematics, had enrolment rates retained their 2003 levels, would have been 56 score points. Similarly, if the assumption is that newly enrolled students come from the bottom quarter of the socio-economic distribution, Brazil's improvement in mathematics between 2003 and 2012 would have been 44 score points had enrolment rates not increased since 2003. Still, it is the observed enrolment rates and the observed performance in 2003 and 2012 that truly reflect the student population, its performance and the education challenges facing Brazil.

Brazil's increases in coverage are remarkable. However, although practically all students aged 7-14 start school at the beginning of the year, few continue until the end. They leave because the curriculum isn't engaging, or because they want or need to work, or because of the prevalence of grade repetition. The pervasiveness of grade repetition in Brazil has been linked to high dropout rates, high levels of student disengagement, and the more than 12 years it takes students, on average, to complete eight grades of primary school. (PISA results suggest that repetition rates remain high in Brazil: in 2003, 33% of students reported having repeated at least one grade in primary or secondary education; in 2012, 36% of students reported so.)

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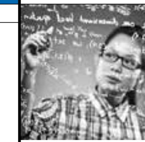

■ Figure I.2.c ■

Observed and expected trends in mathematics performance for Brazil (2003-12)

	2003		2012		Change between 2003 and 2012 (2012 - 2003)	
Total number of 15-year-olds	3 618 332		3 574 928		-43 404	
Total 15-year-olds enrolled in grades 7 or higher	2 359 854		2 786 064		+426 210	
Enrolment rates for 15-year-old students	65%		78%		+19%	
	Mean	S.E.	Mean	S.E.	Mean	S.E.
Mathematics performance	356	(4.8)	391	(2.1)	+35.4	(5.6)
Comparing the performance students with similar socio-economic backgrounds:						
Advantaged student in 2003	383	(5.2)	404	(2.3)	+20.5	(6.0)
Average student in 2003	357	(4.0)	382	(1.6)	+24.9	(4.7)
Disadvantaged student in 2003	342	(3.9)	369	(1.7)	+27.3	(4.7)
Average performance excluding newly enrolled students assuming that newly enrolled students are at:						
Bottom half of performance	356	(4.8)	406	(2.2)	+49.7	(5.6)
Bottom quarter of performance	356	(4.8)	412	(2.0)	+56.4	(5.6)
Bottom of the distribution	356	(4.8)	415	(1.8)	+58.6	(5.5)
Average performance excluding newly enrolled students assuming that newly enrolled students come from:						
Bottom half of ESCS	356	(4.8)	397	(2.2)	+40.5	(5.7)
Bottom quarter of ESCS	356	(4.8)	399	(2.3)	+43.5	(5.7)
Bottom of ESCS	356	(4.8)	400	(2.3)	+44.1	(5.7)

Notes: Enrolment rates are those reported as the coverage index 3 in Annex A3 in *Learning for Tomorrow's World: First Results from PISA 2003* (OECD, 2004) and in Annex A2 of this volume. An advantaged/disadvantaged student is one who has a *PISA index of economic, social and cultural status* (ESCS) that places him/her at the top/lower end of the fourth/first quartile of ESCS in 2003. Average students are those with an ESCS equal to the average in 2003. Average performance in PISA 2012 that excludes newly enrolled students assuming that they come from the bottom half/quarter of performance and ESCS is calculated by randomly deleting 19% of the sample only among students scoring bottom half/quarter in the performance and ESCS distribution, respectively. Average performance in PISA 2012 that excludes the bottom of the performance or ESCS distribution excludes the bottom 19% of the sample in the performance and ESCS distribution, respectively.

Despite the fact that primary and secondary education is managed and largely funded at the municipal and state levels, the central government has been a key actor in driving and shaping education reform. Over the past 15 years it has actively promoted reforms to increase funding, improve teacher quality, set national curriculum standards, improve high school completion rates, develop and put in place accountability measures, and set student achievement and learning targets for schools, municipalities and states.

After Brazil's economy stabilised, in the mid-1990s, the Cardoso administration increased federal spending on primary education through FUNDEF (*Fundo de Manutenção e Desenvolvimento do Ensino Fundamental*) and simultaneously distributed the funding more equitably, replacing a population-density formula that allocated the majority of funds to large cities and linking part of the funding to school enrolments. This was only possible after developing a student and school census to gather and consolidate information about schools and students. FUNDEF also raised teachers' salaries, increased the number of teachers, increased the length of teacher-preparation programmes, and contributed to higher enrolments in rural areas. A conditional cash-transfer programme for families who send their 7-14 year-old children to school (*Bolsa Escola*) lifted many families out of subsistence-level poverty encouraging their interest that their children receive an education.

In 2006, the Lula administration expanded FUNDEF to cover early childhood and after-school learning and increased overall funding for education, renaming the programme FUNDEB, as it now covered basic education more broadly. The administration also expanded the conditional cash transfers to cover students aged 15-17, thereby encouraging enrolment in upper secondary education, where enrolment is lowest. This expansion means that 6.1% of Brazil's GDP is now spent on education and the country aims to devote 10% of its GDP to education by 2020. Funding for this important increase in education expenditure will come from the recently approved allocation of 75% of public revenues from oil to education.

Improving the quality of teachers has also been at the centre of Brazil's reform initiatives. A core element of FUNDEF was increasing teacher salaries, which rose 13% on average after FUNDEF, and more than 60% in the poorer, northeast region of the country. At the same time, the 1996 Law of Directive and Bases of National Education (LDB)

...

mandated that, by 2006, all new teachers have a university qualification, and that initial and in-service teacher training programmes be free of charge. These regulations came at a time when coverage was expanding significantly, leading to an increase in the number of teachers in the system. In 2000, for example, there were 430 467 secondary school teachers, and 88% of whom had a tertiary degree; in 2012 there were 497 797 teachers, 95% of whom had tertiary qualifications (INEP, 2000 and 2012). Subsequent reforms in the late 2000s sought to create standards for teachers' career paths based on qualifications, not solely on tenure. The planned implementation of a new examination system for teacher certification, covering both content and pedagogy, has been delayed. Although universities are free to determine their curriculum for teacher-training programmes, the establishment of an examination system to certify teachers sends a strong signal of what content and pedagogical orientation should be developed.

To encourage more students to enrol – and stay – in school, upper secondary education has become mandatory (this policy is being phased in so that enrolment will be obligatory for students aged 4 to 17 by 2016), and a new grade level has been added at the start of primary school. Giving students more opportunities to learn in school has also meant shifting to a full school day, as underscored in the 2011-2020 National Plan for Education. Most school days are just four hours long; and even though FUNDEB provided incentives for full-day schools, they were not sufficient to prompt the investments in infrastructure required for schools that accommodate two or three shifts in a day to become full-day schools. Although enrolment in full-day schools increased 24% between 2010 and 2012, overall coverage in full-day schools remains low: only 2 million out of a total of almost 30 million students attended such schools in 2012 (INEP, 2013).

The reforms of the mid-1990s included provisions to improve the education information system and increase school accountability. It transformed the National Institute for Educational Studies and Research into an independent organisation responsible for the national assessment and evaluation of education. It turned a national assessment system into the Evaluation System for Basic Education (SAEB/*Prova Brazil*) for grades 4, 8 and 11 and the National Secondary Education Examination in Grade 11, which provides qualifications for further studies or entry into the labour market. SAEB changed over time to become a national census-based assessment for students in grades 4 and 8 and its results were combined with repetition and dropout rates in 2005 to create an index of schools quality, the Basic Education Development Index (IDEB). This gave schools, municipalities and states an incentive to reduce retention and dropout rates and a benchmark against which to which monitor their progress. The IDEB is set individually for each school and is scaled so that its levels are aligned with those of PISA. Results are widely published, and schools that show significant progress are granted more autonomy while schools that remain low performers are given additional assistance. Support for schools is also offered through the *Fundescola* programme. IDEB provides targets for each school; it is up to the schools, municipalities and states to develop strategic improvement plans. In line with Brazil's progress in PISA, national performance as measured by the SAEB has also improved between 1999 and 2009 (Bruns, Evans and Luque, 2011).

Perhaps a result of these reforms, not only are more Brazilian students attending school and performing at higher levels, they are also attending better-staffed schools (the *index of teacher shortage* dropped from 0.47 in 2003 to 0.19 in 2012, and the number of students per teacher in a school fell from 34 to 28 in the same period), and schools with better material resources (the *index of quality of educational resources* increased from -1.17 to -0.54). They are also attending schools with better learning environments, as shown by improved disciplinary climates and student-teacher relations. Students in 2012 also reported spending one-and-a-half hours less per week on homework than their counterparts in 2003 did.

Sources:

Bruns, B., D. Evans and J. Luque (2011), *Achieving World-Class Education in Brazil*, The World Bank, Washington, D.C.

INEP (Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira) (2000), *Sinopse Estatística da Educação Básica 2000*, INEP, Brasilia.

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STUDENT PERFORMANCE IN DIFFERENT AREAS OF MATHEMATICS

This section focuses on student performance on the process subscales of *formulating*, *employing* and *interpreting*; and on the content subscales of *change and relationships*, *space and shape*, *quantity* and *uncertainty and data*.

In general, the correlation between scores on the subscales and overall mathematics scores is high: students tend to perform as well on the mathematics subscales as they do in mathematics overall. However, there is some variation at the country level in the relationship between subscale performance and overall mathematics performance, which perhaps reflects differences in emphasis in the curriculum.

Process subscales

The three process categories in the mathematics framework relate to three parts of the mathematical modelling cycle, a key feature of the way PISA assesses mathematics.

As discussed earlier in this chapter, each item in the PISA 2012 mathematics survey was assigned to one of the process categories, even if solving an item often involves more than one of these processes. About a quarter of the items was designed primarily to elicit indicators of the *formulating situations mathematically* process; about half of them required mainly the *employing mathematical concepts, facts, procedures, and reasoning* process; and the remaining quarter emphasised the *interpreting, applying and evaluating mathematical outcomes* process.

Student performance on the mathematics subscale formulating situations mathematically

In order for individuals to use their mathematical knowledge and skills to solve a problem, they often first need to translate the problem into a form that is amenable to mathematical treatment. The framework refers to this process as one of *formulating situations mathematically*.

In the PISA assessment, students may need to recognise or introduce simplifying assumptions that would help make the given mathematics item amenable to analysis. They have to identify which aspects of the problem are relevant to the solution and which might safely be ignored. They must recognise words, images, relationships or other features of the problem that can be given a mathematical form; and they need to express the relevant information in an appropriate way, for example in the form of a numeric calculation or as an algebraic expression. This process is sometimes referred to as *translating* the problem as expressed, usually in real-world terms, into a *mathematical* problem. For example, in a problem about some form of motion (such as travel on public transport, or riding a bicycle), the student may need to recognise a reference to "speed" and understand that this is referring to the relationship between the distance travelled over a given time period, and perhaps invoke the formula $speed = {}^{distance}/_{time}$ as an essential step in giving the problem a clearly mathematical form.

Items listed in Figure I.2.9 that have been classified in this category are REVOLVING DOOR Question 2 and Question 3, and CLIMBING MOUNT FUJI Question 1 and Question 2.

Across OECD countries, the average score attained on the *formulating* subscale is 492 points. A substantially lower score on the *formulating* subscale compared to average scores in the other processes or in mathematics overall might indicate that some students might find the *formulating* process more difficult. This would be expected when students have less experience with this process, for example, when most students in school work on mathematics problems that have already been "translated" into mathematical form. Top-performing countries and economies on this subscale are Shanghai-China, Singapore, Chinese Taipei, Hong Kong-China, Korea, Japan, Macao-China, Switzerland, Liechtenstein and the Netherlands (Figure I.2.28 and Table I.2.7).

While across OECD countries, the average *formulating* score (492) is slightly lower than the average overall score for mathematics (494), this is not the case in the ten highest-performing countries on the overall mathematics scale. For nine of those countries and economies, the average national score on the *formulating* subscale is higher than the average overall score in mathematics. This is the case in Shanghai-China, Singapore, Hong Kong-China, Korea, Macao-China, Switzerland and the Netherlands, where the mean score in *formulating* is between 4 and 12 points higher than the overall mathematics average, and is particularly evident in Chinese Taipei and Japan, where it is 19 and 18 points higher, respectively, than the overall mathematics average. This implies that in these countries, students find the formulation process to be a relatively easy aspect of mathematics. The only exception among this highest-performing group is Liechtenstein, where the mean *formulating* score is similar to the country's mean overall mathematics score (Figure I.2.37).

■ Figure I.2.28 ■

Comparing countries' and economies' performance on the mathematics subscale *formulating*

(light blue)	Statistically significantly **above** the OECD average
(dark blue)	Not statistically significantly different from the OECD average
(medium blue)	Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
624	Shanghai-China	
582	Singapore	Chinese Taipei
578	Chinese Taipei	Singapore, Hong Kong-China
568	Hong Kong-China	Chinese Taipei, Korea
562	Korea	Hong Kong-China, Japan
554	Japan	Korea
545	Macao-China	Switzerland
538	Switzerland	Macao-China, Liechtenstein
535	Liechtenstein	Switzerland, Netherlands
527	Netherlands	Liechtenstein, Finland
519	Finland	Netherlands, Estonia, Canada, Poland, Belgium
517	Estonia	Finland, Canada, Poland, Belgium, Germany
516	Canada	Finland, Estonia, Poland, Belgium, Germany
516	Poland	Finland, Estonia, Canada, Belgium, Germany
512	Belgium	Finland, Estonia, Canada, Poland, Germany
511	Germany	Estonia, Canada, Poland, Belgium, Denmark
502	Denmark	Germany, Iceland, Austria, Australia, Viet Nam, New Zealand, Czech Republic
500	Iceland	Denmark, Austria, Australia, Viet Nam, New Zealand, Czech Republic
499	Austria	Denmark, Iceland, Australia, Viet Nam, New Zealand, Czech Republic, Ireland
498	Australia	Denmark, Iceland, Austria, Viet Nam, New Zealand, Czech Republic, Ireland
497	Viet Nam	Denmark, Iceland, Austria, Australia, New Zealand, Czech Republic, Ireland, Slovenia, Norway, United Kingdom, Latvia
496	New Zealand	Denmark, Iceland, Austria, Australia, Viet Nam, Czech Republic, Ireland, Slovenia, Norway, United Kingdom
495	Czech Republic	Denmark, Iceland, Austria, Australia, Viet Nam, New Zealand, Ireland, Slovenia, Norway, United Kingdom, Latvia
492	Ireland	Austria, Australia, Viet Nam, New Zealand, Czech Republic, Slovenia, Norway, United Kingdom, Latvia
492	Slovenia	Viet Nam, New Zealand, Czech Republic, Ireland, Norway, United Kingdom, Latvia
489	Norway	Viet Nam, New Zealand, Czech Republic, Ireland, Slovenia, United Kingdom, Latvia, France, Russian Federation, Slovak Republic
489	United Kingdom	Viet Nam, New Zealand, Czech Republic, Ireland, Slovenia, Norway, Latvia, France, Luxembourg, Russian Federation, Slovak Republic, Portugal
488	Latvia	Viet Nam, Czech Republic, Ireland, Slovenia, Norway, United Kingdom, France, Luxembourg, Russian Federation, Slovak Republic, Portugal
483	France	Norway, United Kingdom, Latvia, Luxembourg, Russian Federation, Slovak Republic, Sweden, Portugal, Lithuania, Spain, United States
482	Luxembourg	United Kingdom, Latvia, France, Russian Federation, Slovak Republic, Sweden, Portugal, Lithuania, United States
481	Russian Federation	Norway, United Kingdom, Latvia, France, Luxembourg, Slovak Republic, Sweden, Portugal, Lithuania, Spain, United States, Italy
480	Slovak Republic	Norway, United Kingdom, Latvia, France, Luxembourg, Russian Federation, Sweden, Portugal, Lithuania, Spain, United States, Italy
479	Sweden	France, Luxembourg, Russian Federation, Slovak Republic, Portugal, Lithuania, Spain, United States, Italy
479	Portugal	United Kingdom, Latvia, France, Luxembourg, Russian Federation, Slovak Republic, Sweden, Lithuania, Spain, United States, Italy, Hungary
477	Lithuania	France, Luxembourg, Russian Federation, Slovak Republic, Sweden, Portugal, Spain, United States, Italy, Hungary
477	Spain	France, Russian Federation, Slovak Republic, Sweden, Portugal, Lithuania, United States, Italy, Hungary
475	United States	France, Luxembourg, Russian Federation, Slovak Republic, Sweden, Portugal, Lithuania, Spain, Italy, Hungary, Israel
475	Italy	Russian Federation, Slovak Republic, Sweden, Portugal, Lithuania, Spain, United States, Hungary
469	Hungary	Portugal, Lithuania, Spain, United States, Italy, Israel
465	Israel	United States, Hungary, Croatia
453	Croatia	Israel, Turkey, Greece, Serbia, Romania, Kazakhstan
449	Turkey	Croatia, Greece, Serbia, Romania, Kazakhstan, Bulgaria
448	Greece	Croatia, Turkey, Serbia, Romania, Kazakhstan
447	Serbia	Croatia, Turkey, Greece, Romania, Kazakhstan, Bulgaria
445	Romania	Croatia, Turkey, Greece, Serbia, Kazakhstan, Bulgaria
442	Kazakhstan	Croatia, Turkey, Greece, Serbia, Romania, Bulgaria, Cyprus[1, 2]
437	Bulgaria	Turkey, Serbia, Romania, Kazakhstan, Cyprus[1, 2]
437	Cyprus[1, 2]	Kazakhstan, Bulgaria
426	United Arab Emirates	Chile
420	Chile	United Arab Emirates, Thailand
416	Thailand	Chile, Mexico, Uruguay, Malaysia
409	Mexico	Thailand, Uruguay, Malaysia
406	Uruguay	Thailand, Mexico, Malaysia, Montenegro, Costa Rica
406	Malaysia	Thailand, Mexico, Uruguay, Montenegro, Costa Rica, Albania
404	Montenegro	Uruguay, Malaysia, Costa Rica
399	Costa Rica	Uruguay, Malaysia, Montenegro, Albania, Jordan
398	Albania	Malaysia, Costa Rica
390	Jordan	Costa Rica, Argentina
383	Argentina	Jordan, Qatar, Brazil, Colombia, Tunisia
378	Qatar	Argentina, Brazil, Colombia, Tunisia
376	Brazil	Argentina, Qatar, Colombia, Tunisia, Peru, Indonesia
375	Colombia	Argentina, Qatar, Brazil, Tunisia, Peru, Indonesia
373	Tunisia	Argentina, Qatar, Brazil, Colombia, Peru, Indonesia
370	Peru	Brazil, Colombia, Tunisia, Indonesia
368	Indonesia	Brazil, Colombia, Tunisia, Peru

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Source: OECD, PISA 2012 Database.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.29 ■

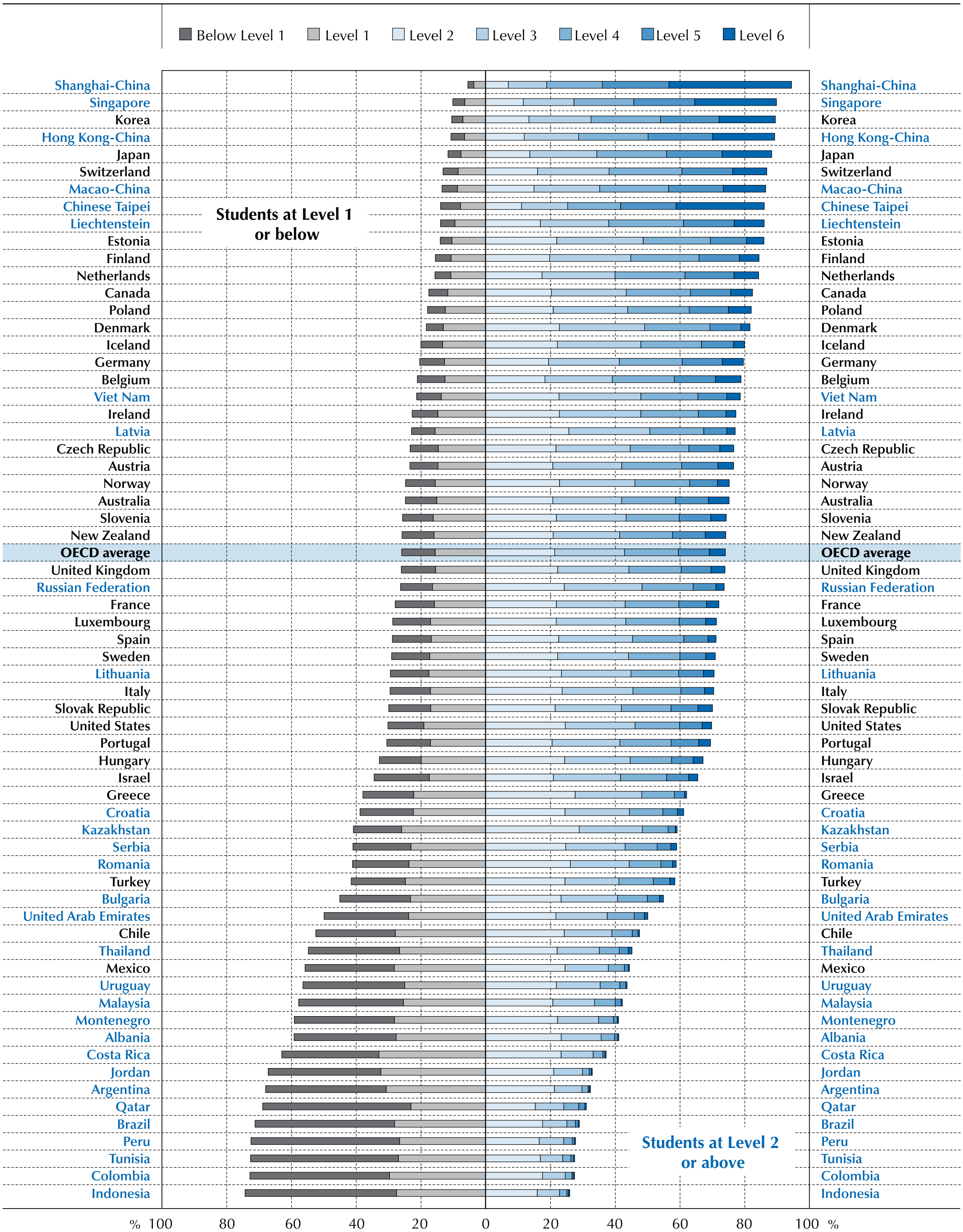
Summary descriptions of the six proficiency levels for the mathematical subscale *formulating*

Level	Percentage of students able to perform tasks at each level or above (OECD average)	What students can do
6	5.0%	Students at or above Level 6 can apply a wide variety of mathematical content knowledge to transform and represent contextual information or data, geometric patterns or objects into a mathematical form amenable to investigation. At this level, students can devise and follow a multi-step strategy involving significant modelling steps and extended calculation to formulate and solve complex real-world problems in a range of settings, for example involving material and cost calculations in a variety of contexts, or to find the area of an irregular region on a map; identify what information is relevant (and what is not) from contextual information about travel times, distances and speed to formulate appropriate relationships among them; apply reasoning across several linked variables to devise an appropriate way to present data in order to facilitate pertinent comparisons; and devise algebraic formulations that represent a given contextual situation.
5	14.5%	At this level, students can use their understanding in a range of mathematical areas to transform information or data from a problem context into mathematical form. They can transform information from different representations involving several variables, into a form suitable for mathematical treatment. They can formulate and modify algebraic expressions of relationships among variables; use proportional reasoning effectively to devise computations; gather information from different sources to formulate and solve problems involving geometric objects, features and properties, or analyse geometric patterns or relationships and express them in standard mathematical terms; transform a given model according to changed contextual circumstances; formulate a sequential calculation process based on text descriptions; and activate statistical concepts, such as randomness, or sample, and apply probability to formulate a model.
4	31.1%	At Level 4, students can link information and data from related representations (for example, a table and a map, or a spread sheet and a graphing tool) and apply a sequence of reasoning steps in order to formulate the mathematical expression needed to carry out a calculation or otherwise to solve a contextual problem. At this level, students can formulate a linear equation from a text description of a process, for example in a sales context, and formulate and apply cost comparisons to compare prices of sale items; identify which of given graphical representations corresponds to a given description of a physical process; specify a sequential calculation process in mathematical terms; identify geometrical features of a situation and use their geometric knowledge and reasoning to analyse a problem, for example to estimate areas or to link a contextual geometric situation involving similarity to the corresponding proportional reasoning; combine multiple decision rules needed to understand or implement a calculation where different constraints apply; and formulate algebraic expressions when the contextual information is reasonably straight-forward, for example to connect distance and speed information in time calculations.
3	52.7%	At this level, students can identify and extract information and data from text, tables, graphs, maps or other representations, and make use of them to express a relationship mathematically, including interpreting or adapting simple algebraic expressions related to an applied context. Students at this level can transform a textual description of a simple functional relationship into a mathematical form, for example with unit costs or payment rates; form a strategy involving two or more steps to link problem elements or to explore mathematical characteristics of the elements; apply reasoning with geometric concepts and skills to analyse patterns or identify properties of shapes or a specified map location, or to identify information needed to carry out some pertinent calculations, including calculations involving the use of simple proportional models and reasoning, where the relevant data and information is immediately accessible; and understand and link probabilistic statements to formulate probability calculations in contexts, such as in a manufacturing process or a medical test.
2	74.0%	At this level, students can understand written instructions and information about simple processes and tasks in order to express them in a mathematical form. They can use data presented in text or in a table (for example, giving information about the cost of some product or service) to formulate a computation required, such as to identify the length of a time period, or to present a cost comparison, or calculate an average; analyse a simple pattern, for example by formulating a counting rule or identifying and extending a numeric sequence; work effectively with different two- and three-dimensional standard representations of objects or situations, for example devising a strategy to match one representation with another compare different scenarios, or identify random experiment outcomes mathematically using standard conventions.
1	89.7%	At this level students can recognise or modify and use an explicit simple model of a contextual situation. Students can choose between several such models to match the situation. For example, they can choose between an additive and a multiplicative model in a shopping context; choose among given two-dimensional objects to represent a familiar three-dimensional object; and select one of several given graphs to represent growth of a population.

■ Figure I.2.30 ■

Proficiency in the mathematics subscale *formulating*

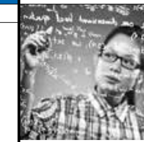
Percentage of students at each level of mathematics proficiency



Countries and economies are ranked in descending order of the percentage of students at Levels 2, 3, 4, 5 and 6.

Source: OECD, PISA 2012 Database, Table I.2.5.

StatLink http://dx.doi.org/10.1787/888932935572



In Croatia, Brazil, Tunisia, Malaysia, Viet Nam, Thailand and the OECD countries France and Italy, there is a difference of at least 10 points between student performance on the *formulating* subscale and overall mathematics performance. In all these countries, the scores in *formulating* are lower than the overall mathematics scores. All these countries show an average overall score in mathematics below the OECD average, except France, which is at the OECD average, and Viet Nam, which is above the OECD average.

Descriptions of the six levels of proficiency on the subscale *formulating situations mathematically* are given in Figure I.2.29 and the distribution of students among these six proficiency levels is shown in Figure I.2.30.

Student performance on the mathematics subscale employing mathematical concepts, facts, procedures, and reasoning

To employ mathematical concepts, facts, procedures and reasoning for the PISA assessment, students need to recognise which elements of their "mathematics tool kit" are relevant to the problem as it has been presented, or as they have formulated it, and apply that knowledge in a systematic and organised way to work towards a solution. For example, in a problem about travel on public transport or riding a bicycle, once the basic relationships underlying the problem have been understood and expressed in a suitable mathematical form, the student may need to carry out a calculation, substitute values into a formula, solve an equation, or apply their knowledge of the conventions of graphing to extract data or present information mathematically.

Items listed in Figure I.2.9 that have been classified in this category are REVOLVING DOOR Question 1, WHICH CAR? Question 2 and Question 3, CHARTS Question 5, GARAGE Question 2, CLIMBING MOUNT FUJI Question 3, and HELEN THE CYCLIST Question 1, Question 2 and Question 3.

Across OECD countries, the average score attained on the *employing* subscale is 493 points – 0.6 score point below the average score in overall mathematics proficiency. This small difference reflects both the centrality of using mathematical concepts, facts, procedures and reasoning in school mathematics classes and the fact that about half of the items in the PISA 2012 mathematics assessment are categorised as predominantly requiring the use of *employing* processes. Top-performing countries and economies on this subscale are Shanghai-China, Singapore, Hong Kong-China, Korea, Chinese Taipei, Liechtenstein, Macao-China, Japan, Switzerland and Estonia (Figure I.2.31 and Table I.2.10).

The great majority of participating countries and economies have an average *employing* score that is within about five score points of their average score on the overall mathematics proficiency scale. Only Chinese Taipei has an average score on the *employing* subscale that is more than 10 points lower than its average score in mathematics (an 11-point difference), indicating that more students have difficulty using this process. By contrast, Viet Nam's average score on the *employing* subscale is 12 points higher than its average score on the mathematics proficiency scale, suggesting that students in that country find this aspect of problem solving relatively easy (Figure I.2.37).

Descriptions of the six levels of proficiency on the subscale *employing mathematical concepts, facts, procedures, and reasoning* are given in Figure I.2.32 and the distribution of students among these six proficiency levels is shown in Figure I.2.33.

Student performance on the mathematics subscale interpreting, applying and evaluating mathematical outcomes

In interpreting mathematical outcomes, students need to make links between the outcomes and the situation from which they arose. For example, in a problem requiring a careful interpretation of some graphical data, students would have to make connections among the objects or relationships depicted in the graph, and the answer to the question might involve interpreting those objects or relationships. In a problem about travel on public transport or riding a bicycle, once the basic relationships underlying the problem have been understood and expressed in a suitable mathematical form, the required mathematical processing has been carried out, and results generated, the student may need to evaluate the results in relation to the original problem, or may need to show how the mathematical information obtained relates to the contextual elements of the problem.

Items listed in Figure I.2.9 that have been classified in this category are CHARTS Question 1 and Question 2, WHICH CAR? Question 1, and GARAGE Question 1.

■ Figure I.2.31 ■

Comparing countries' and economies' performance on the mathematics subscale *employing*

(light)	Statistically significantly **above** the OECD average
(medium)	Not statistically significantly different from the OECD average
(dark)	Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
613	Shanghai-China	
574	Singapore	
558	Hong Kong-China	Korea
553	Korea	Hong Kong-China, Chinese Taipei
549	Chinese Taipei	Korea
536	Liechtenstein	Macao-China, Japan, Switzerland
536	Macao-China	Liechtenstein, Japan
530	Japan	Liechtenstein, Macao-China, Switzerland, Estonia, Viet Nam
529	Switzerland	Liechtenstein, Japan, Estonia, Viet Nam
524	Estonia	Japan, Switzerland, Viet Nam, Poland, Netherlands
523	Viet Nam	Japan, Switzerland, Estonia, Poland, Netherlands, Canada, Germany, Belgium, Finland
519	Poland	Estonia, Viet Nam, Netherlands, Canada, Germany, Belgium, Finland
518	Netherlands	Estonia, Viet Nam, Poland, Canada, Germany, Belgium, Finland
517	Canada	Viet Nam, Poland, Netherlands, Germany, Belgium, Finland
516	Germany	Viet Nam, Poland, Netherlands, Canada, Belgium, Finland, Austria
516	Belgium	Viet Nam, Poland, Netherlands, Canada, Germany, Finland, Austria
516	Finland	Viet Nam, Poland, Netherlands, Canada, Germany, Belgium, Austria
510	Austria	Germany, Belgium, Finland, Slovenia, Czech Republic
505	Slovenia	Austria, Czech Republic, Ireland
504	Czech Republic	Austria, Slovenia, Ireland, Australia, France
502	Ireland	Slovenia, Czech Republic, Australia, France, Latvia
500	Australia	Czech Republic, Ireland, France, Latvia, New Zealand
496	France	Czech Republic, Ireland Australia, Latvia, New Zealand, Denmark, Luxembourg, United Kingdom, Portugal
495	Latvia	Ireland, Australia, France, New Zealand, Denmark, Luxembourg, United Kingdom, Iceland, Portugal
495	New Zealand	Australia, France, Latvia, Denmark, Luxembourg, United Kingdom, Iceland, Portugal
495	Denmark	France, Latvia, New Zealand, Luxembourg, United Kingdom, Iceland, Portugal
493	Luxembourg	France, Latvia, New Zealand, Denmark, United Kingdom, Iceland, Portugal, Russian Federation
492	United Kingdom	France, Latvia, New Zealand, Denmark, Luxembourg, Iceland, Portugal, Russian Federation, Norway, Italy, Slovak Republic
490	Iceland	Latvia, New Zealand, Denmark, Luxembourg, United Kingdom, Portugal, Russian Federation, Norway, Italy, Slovak Republic
489	Portugal	France, Latvia, New Zealand, Denmark, Luxembourg, United Kingdom, Iceland, Russian Federation, Norway, Italy, Slovak Republic, Lithuania, Spain Hungary, United States
487	Russian Federation	Luxembourg, United Kingdom, Iceland, Portugal, Norway, Italy, Slovak Republic, Lithuania, Spain, Hungary, United States, Croatia
486	Norway	United Kingdom, Iceland, Portugal, Russian Federation, Italy, Slovak Republic, Lithuania, Spain, Hungary, United States, Croatia
485	Italy	United Kingdom, Iceland, Portugal, Russian Federation, Norway, Slovak Republic, Lithuania, Spain, Hungary, United States, Croatia
485	Slovak Republic	United Kingdom, Iceland, Portugal, Russian Federation, Norway, Italy, Lithuania, Spain, Hungary, United States, Croatia
482	Lithuania	Portugal, Russian Federation, Norway, Italy, Slovak Republic, Spain, Hungary, United States, Croatia
481	Spain	Portugal, Russian Federation, Norway, Italy, Slovak Republic, Lithuania, Hungary, United States, Croatia
481	Hungary	Portugal, Russian Federation, Norway, Italy, Slovak Republic, Lithuania, Spain, United States, Croatia, Sweden
480	United States	Portugal, Russian Federation, Norway, Italy, Slovak Republic, Lithuania, Spain, Hungary, Croatia, Sweden, Israel
478	Croatia	Russian Federation, Norway, Italy, Slovak Republic, Lithuania, Spain, Hungary, United States, Sweden, Israel
474	Sweden	Hungary, United States, Croatia, Israel
469	Israel	United States, Croatia, Sweden
451	Serbia	Greece, Turkey, Romania
449	Greece	Serbia, Turkey, Romania, Cyprus[1, 2], Bulgaria
448	Turkey	Serbia, Greece, Romania, Cyprus[1, 2], United Arab Emirates, Bulgaria
446	Romania	Serbia, Greece, Turkey, Cyprus[1, 2], United Arab Emirates, Bulgaria
443	Cyprus[1, 2]	Greece, Turkey, Romania, United Arab Emirates, Bulgaria
440	United Arab Emirates	Turkey, Romania, Cyprus[1, 2], Bulgaria, Kazakhstan
439	Bulgaria	Greece, Turkey, Romania, Cyprus[1, 2], United Arab Emirates, Kazakhstan
433	Kazakhstan	United Arab Emirates, Bulgaria, Thailand
426	Thailand	Kazakhstan, Malaysia
423	Malaysia	Thailand, Chile
416	Chile	Malaysia, Mexico, Uruguay
413	Mexico	Chile, Uruguay
409	Montenegro	Uruguay
408	Uruguay	Chile, Mexico, Montenegro, Costa Rica
401	Costa Rica	Uruguay, Albania, Tunisia
397	Albania	Costa Rica, Tunisia
390	Tunisia	Costa Rica, Albania, Brazil, Argentina, Jordan
388	Brazil	Tunisia, Argentina, Jordan
387	Argentina	Tunisia, Brazil, Jordan
383	Jordan	Tunisia, Brazil, Argentina
373	Qatar	Indonesia, Peru, Colombia
369	Indonesia	Qatar, Peru, Colombia
368	Peru	Qatar, Indonesia, Colombia
367	Colombia	Qatar, Indonesia, Peru

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2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Source: OECD, PISA 2012 Database.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.32 ■

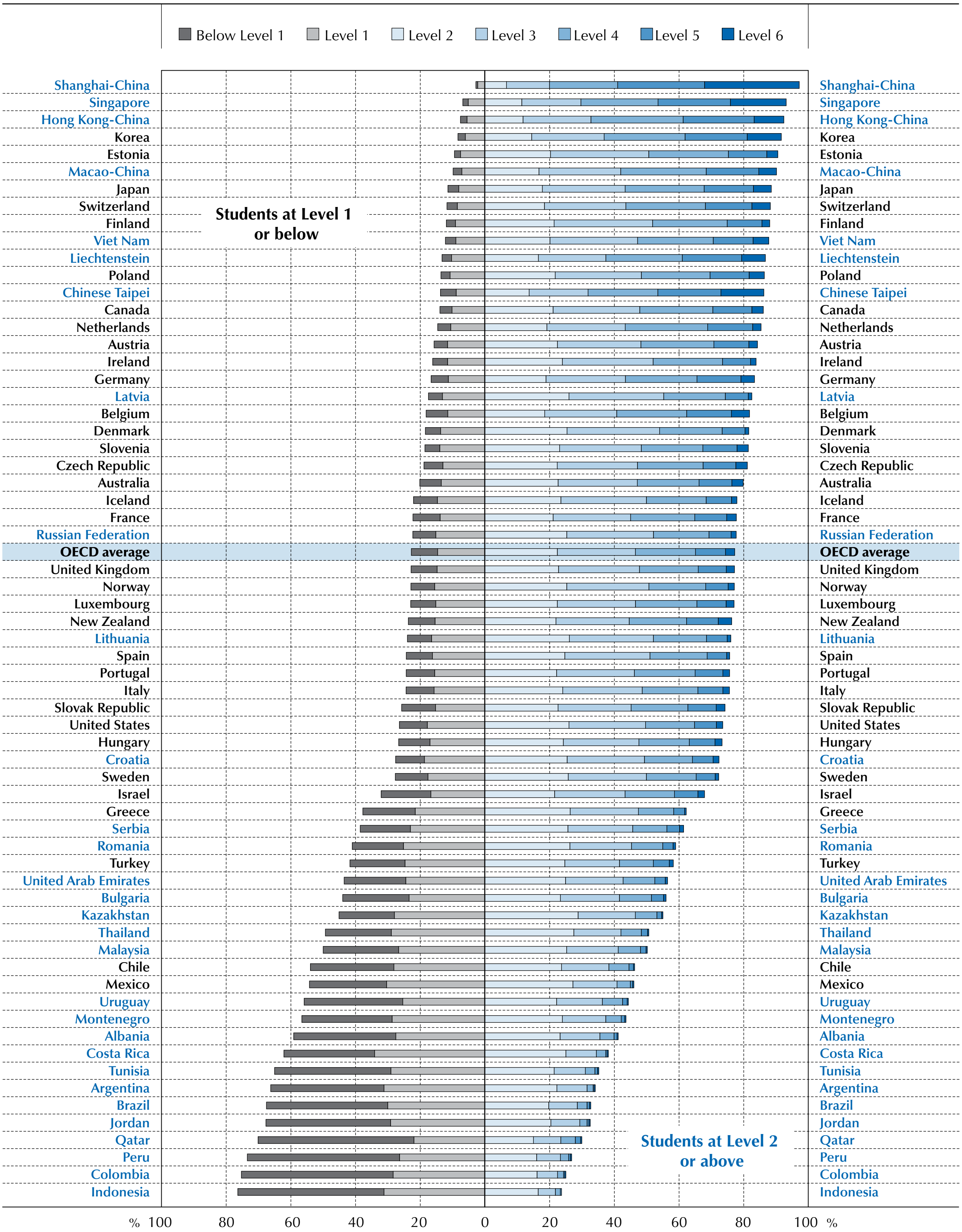
Summary descriptions of the six proficiency levels for the mathematical subscale *employing*

Level	Percentage of students able to perform tasks at each level or above (OECD average)	What students can do
6	2.8%	Students at or above Level 6 can use a strong repertoire of knowledge and procedural skills in a wide range of mathematical areas. They can form and follow a multi-step strategy to solve a problem involving several stages; apply reasoning in a connected way across several problem elements; set up and solve an algebraic equation with more than one variable; generate relevant data and information to explore problems, for example using a spread sheet to sort and analyse data; and justify their results mathematically and explain their conclusions and support them with well-formed mathematical arguments. At Level 6 students' work is consistently precise and accurate.
5	12.1%	Students at Level 5 can use a range of knowledge and skills to solve problems. They can sensibly link information in graphical and diagrammatic form to textual information. They can apply spatial and numeric reasoning skills to express and work with simple models in reasonably well-defined situations and where the constraints are clear. They usually work systematically, for example to explore combinatorial outcomes, and can sustain accuracy in their reasoning across a small number of steps and processes. They are generally able to work competently with expressions, can work with formulae and use proportional reasoning, and are able to work with and transform data presented in a variety of forms.
4	30.7%	At Level 4, students can identify relevant data and information from contextual material and use it to perform such tasks as calculating distances, using proportional reasoning to apply a scale factor, converting different units to a common scale, or relating different graph scales to each other. They can work flexibly with distance-time-speed relationships, and can carry out a sequence of arithmetic calculations. They can use algebraic formulations, and follow a straightforward strategy and describe it.
3	54.8%	Students at Level 3 frequently have sound spatial reasoning skills enabling them, for example, to use the symmetry properties of a figure, recognise patterns presented in graphical form, or use angle facts to solve a geometric problem. Students at this level can connect two different mathematical representations, such as data in a table and in a graph, or an algebraic expression with its graphical representation, enabling them, for example, to understand the effect of changing data in one representation on the other. They can handle percentages, fractions and decimal numbers and work with proportional relationships.
2	77.3%	Students at Level 2 can apply small reasoning steps to make direct use of given information to solve a problem, for example, to implement a simple calculation model, identify a calculation error, analyse a distance-time relationship, or analyse a simple spatial pattern. At this level students show an understanding of place value in decimal numbers and can use that understanding to compare numbers presented in a familiar context; correctly substitute values into a simple formula; recognise which of a set of given graphs correctly represents a set of percentages and apply reasoning skills to understand and explore different kinds of graphical representations of data; and can understand simple probability concepts.
1	91.9%	Students at Level 1 can identify simple data relating to a real-world context, such as that presented in a structured table or in an advertisement where the text and data labels match directly; perform practical tasks, such as decomposing money amounts into lower denominations; use direct reasoning from textual information that points to an obvious strategy to solve a given problem, particularly where the mathematical procedural knowledge required would be limited to, for example, arithmetic operations with whole numbers, or ordering and comparing whole numbers; understand graphing techniques and conventions; and use symmetry properties to explore characteristics of a figure, such as comparin g side lengths and angles.

■ Figure I.2.33 ■

Proficiency in the mathematics subscale *employing*

Percentage of students at each level of mathematics proficiency



Countries and economies are ranked in descending order of the percentage of students at Levels 2, 3, 4, 5 and 6.
Source: OECD, PISA 2012 Database, Table I.2.8.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.34 ■

Comparing countries' and economies' performance on the mathematics subscale *interpreting*

Statistically significantly **above** the OECD average
Not statistically significantly different from the OECD average
Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
579	Shanghai-China	
555	Singapore	Hong Kong-China, Chinese Taipei
551	Hong Kong-China	Singapore, Chinese Taipei
549	Chinese Taipei	Singapore, Hong Kong-China, Liechtenstein, Korea
540	Liechtenstein	Chinese Taipei, Korea, Japan
540	Korea	Chinese Taipei, Liechtenstein, Japan
531	Japan	Liechtenstein, Korea, Macao-China, Switzerland, Finland, Netherlands
530	Macao-China	Japan, Switzerland, Finland, Netherlands
529	Switzerland	Japan, Macao-China, Finland, Netherlands, Canada
528	Finland	Japan, Macao-China, Switzerland, Netherlands
526	Netherlands	Japan, Macao-China, Switzerland, Finland, Canada, Germany
521	Canada	Switzerland, Netherlands, Germany, Poland
517	Germany	Netherlands, Canada, Poland, Australia, Belgium, Estonia, New Zealand, France, Austria
515	Poland	Canada, Germany, Australia, Belgium, Estonia, New Zealand, France, Austria, Denmark, Ireland
514	Australia	Germany, Poland, Belgium, Estonia, New Zealand, France, Austria
513	Belgium	Germany, Poland, Australia, Estonia, New Zealand, France, Austria, Denmark, Ireland
513	Estonia	Germany, Poland, Australia, Belgium, New Zealand, France, Austria, Denmark, Ireland
511	New Zealand	Germany, Poland, Australia, Belgium, Estonia, France, Austria, Denmark, Ireland
511	France	Germany, Poland, Australia, Belgium, Estonia, New Zealand, Austria, Denmark, Ireland
509	Austria	Germany, Poland, Australia, Belgium, Estonia, New Zealand, France, Denmark, Ireland, United Kingdom
508	Denmark	Poland, Belgium, Estonia, New Zealand, France, Austria, Ireland, United Kingdom
507	Ireland	Poland, Belgium, Estonia, New Zealand, France, Austria, Denmark, United Kingdom, Viet Nam
501	United Kingdom	Austria, Denmark, Ireland, Norway, Italy, Slovenia, Viet Nam, Spain, Luxembourg, Czech Republic
499	Norway	United Kingdom, Italy, Slovenia, Viet Nam, Spain, Luxembourg, Czech Republic, Iceland, Portugal, United States
498	Italy	United Kingdom, Norway, Slovenia, Viet Nam, Spain, Luxembourg, Czech Republic, Portugal
498	Slovenia	United Kingdom, Norway, Italy, Viet Nam, Spain, Luxembourg, Czech Republic, Portugal
497	Viet Nam	Ireland, United Kingdom, Norway, Italy, Slovenia, Spain, Luxembourg, Czech Republic, Iceland, Portugal, United States, Latvia
495	Spain	United Kingdom, Norway, Italy, Slovenia, Viet Nam, Luxembourg, Czech Republic, Iceland, Portugal, United States
495	Luxembourg	United Kingdom, Norway, Italy, Slovenia, Viet Nam, Spain, Czech Republic, Iceland, Portugal, United States
494	Czech Republic	United Kingdom, Norway, Italy, Slovenia, Viet Nam, Spain, Luxembourg, Iceland, Portugal, United States, Latvia
492	Iceland	Norway, Viet Nam, Spain, Luxembourg, Czech Republic, Portugal, United States, Latvia
490	Portugal	Norway, Italy, Slovenia, Viet Nam, Spain, Luxembourg, Czech Republic, Iceland, United States, Latvia, Sweden
489	United States	Norway, Viet Nam, Spain, Luxembourg, Czech Republic, Iceland, Portugal, Latvia, Sweden
486	Latvia	Viet Nam, Czech Republic, Iceland, Portugal, United States, Sweden
485	Sweden	Portugal, United States, Latvia, Croatia
477	Croatia	Sweden, Hungary, Slovak Republic, Russian Federation, Lithuania
477	Hungary	Croatia, Slovak Republic, Russian Federation, Lithuania
473	Slovak Republic	Croatia, Hungary, Russian Federation, Lithuania, Greece, Israel
471	Russian Federation	Croatia, Hungary, Slovak Republic, Lithuania, Greece, Israel
471	Lithuania	Croatia, Hungary, Slovak Republic, Russian Federation, Greece, Israel
467	Greece	Slovak Republic, Russian Federation, Lithuania, Israel
462	Israel	Slovak Republic, Russian Federation, Lithuania, Greece
446	Turkey	Serbia, Bulgaria, Romania
445	Serbia	Turkey, Bulgaria, Romania
441	Bulgaria	Turkey, Serbia, Romania, Cyprus[1, 2], Chile, Thailand
438	Romania	Turkey, Serbia, Bulgaria, Cyprus[1, 2], Chile, Thailand
436	Cyprus[1, 2]	Bulgaria, Romania, Chile, Thailand
433	Chile	Bulgaria, Romania, Cyprus[1, 2], Thailand, United Arab Emirates
432	Thailand	Bulgaria, Romania, Cyprus[1, 2], Chile, United Arab Emirates
428	United Arab Emirates	Chile, Thailand
420	Kazakhstan	Malaysia, Costa Rica
418	Malaysia	Kazakhstan, Costa Rica, Montenegro, Mexico
418	Costa Rica	Kazakhstan, Malaysia, Montenegro, Mexico
413	Montenegro	Malaysia, Costa Rica, Mexico, Uruguay
413	Mexico	Malaysia, Costa Rica, Montenegro, Uruguay
409	Uruguay	Montenegro, Mexico
401	Brazil	
390	Argentina	Colombia, Tunisia, Jordan, Indonesia
387	Colombia	Argentina, Tunisia, Jordan, Indonesia
385	Tunisia	Argentina, Colombia, Jordan, Indonesia, Albania
383	Jordan	Argentina, Colombia, Tunisia, Indonesia, Albania
379	Indonesia	Argentina, Colombia, Tunisia, Jordan, Albania, Qatar, Peru
379	Albania	Tunisia, Jordan, Indonesia, Qatar
375	Qatar	Indonesia, Albania, Peru
368	Peru	Indonesia, Qatar

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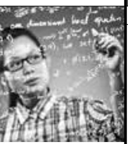
Source: OECD, PISA 2012 Database.

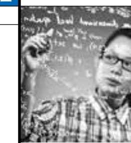
StatLink http://dx.doi.org/10.1787/888932935572



■ Figure I.2.35 ■

Summary descriptions of the six proficiency levels for the mathematical subscale *interpreting*

Level	Percentage of students able to perform tasks at each level or above (OECD average)	What students can do
6	4.2%	At Level 6, students can link multiple complex mathematical representations in an analytic way to identify and extract data and information that enables contextual questions to be answered, and can present their interpretations and conclusions in written form. For example, students may interpret two time-series graphs in relation to different contextual conditions; or link a relationship expressed both in a graph and in numeric form (such as in a price calculator) or in a spread sheet and graph, to present an argument or conclusion about contextual conditions. Students at this level can apply mathematical reasoning to data or information presented in order to generate a chain of linked steps to support a conclusion (for example, analysing a map using scale information; analysing a complex algebraic formula in relation to the variables represented; translating data into a new time-frame; performing a three-way currency conversion; or using a data-generation tool to find the information needed to answer a question). Students at this level can gather analysis, data and their interpretation across several different problem elements or across different questions about a context, showing a depth of insight and a capacity for sustained reasoning.
5	14.5%	At Level 5, students can combine several processes in order to formulate conclusions based on an interpretation of mathematical information with respect to context, such as formulating or modifying a model, solving an equation or carrying out computations, and using several reasoning steps to make the links to the identified context elements. At this level, students can make links between context and mathematics involving spatial or geometric concepts and complex statistical and algebraic concepts. They can easily interpret and evaluate a set of plausible mathematical representations, such as graphs, to identify which one highest reflects the contextual elements under analysis. Students at this level have begun to develop the ability to communicate conclusions and interpretations in written form.
4	33.0%	At Level 4, students can apply appropriate reasoning steps, possibly multiple steps, to extract information from a complex mathematical situation and interpret complicated mathematical objects, including algebraic expressions. They can interpret complex graphical representations to identify data or information that answers a question; perform a calculation or data manipulation (for example, in a spread sheet) to generate additional data needed to decide whether a constraint (such as a measurement condition or a size comparison) is met; interpret simple statistical or probabilistic statements in such contexts as public transport, or health and medical test interpretation, to link the meaning of the statements to the underlying contextual issues; conceptualise a change needed to a calculation procedure in response to a changed constraint; and analyse two data samples, for example relating to a manufacturing process, to make comparisons and draw and express conclusions.
3	55.9%	Students at Level 3 begin to be able to use reasoning, including spatial reasoning, to support their interpretations of mathematical information in order to make inferences about features of the context. They combine reasoning steps systematically to make various connections between mathematical and contextual material or when required to focus on different aspects of a context, for example where a graph shows two data series or a table contains data on two variables that must be actively related to each other to support a conclusion. They can test and explore alternative scenarios, using reasoning to interpret the possible effects of changing some of the variables under observation. They can use appropriate calculation steps to assist their analysis of data and support the formation of conclusions and interpretations, including calculations involving proportions and proportional reasoning, and in situations where systematic analysis across several related cases is needed. At this level, students can interpret and analyse relatively unfamiliar data presentations to support their conclusions.
2	77.0%	At Level 2, students can link contextual elements of the problem to mathematics, for example by performing appropriate calculations or reading tables. Students at this level can make comparisons repeatedly across several similar cases: for example, they can interpret a bar graph to identify and extract data to apply in a comparative condition where some insight is required. They can apply basic spatial skills to make connections between a situation presented visually and its mathematical elements; identify and carry out necessary calculations to support such comparisons as costs across several contexts; and can interpret a simple algebraic expression as it relates to a given context.
1	91.2%	At Level 1, students can interpret data or information expressed in a direct way in order to answer questions about the context described. They can interpret given data to answer questions about simple quantitative relational ideas (such as "larger", "shorter time", "in between") in a familiar context, for example by evaluating measurements of an object against given criterion values, by comparing average journey times for two methods of transport, or by comparing specified characteristics of a small number of similar objects. Similarly, they can make simple interpretations of data in a timetable or schedule to identify times or events. Students at this level may show rudimentary understanding of such concepts as randomness and data interpretation, for example by identifying the plausibility of a statement about chance outcomes of a lottery, by understanding numeric and relational information in a well-labelled graph, and by understanding basic contextual implications of links between related graphs.



Across OECD countries, the average score attained on the *interpreting* subscale is 497 points, 3 score points above the average score of 494 points on the overall mathematics proficiency scale. A substantially higher average score on the *interpreting* subscale might indicate that students find interpreting mathematical information a relatively less difficult aspect of the problem-solving process, perhaps because the task of evaluating mathematical results is commonly treated as part of that process in school mathematics classes. Top-performing countries and economies on this subscale are Shanghai-China, Singapore, Hong Kong-China, Chinese Taipei, Liechtenstein, Korea, Japan, Macao-China, Switzerland and Finland (Figure I.2.34 and Table I.2.13).

While across OECD countries the average score on the *interpreting* subscale is slightly higher than the average score on the mathematics proficiency scale, this is not the case in eight of the ten highest-performing countries and economies on the overall mathematics scale. In those countries and economies, the average score in *interpreting* is lower than the average score in overall mathematics proficiency, with a difference ranging from less than 10 points in Switzerland, Japan, Macao-China and Hong Kong-China, to between 10 and 20 points in Chinese Taipei, Korea and Singapore, to 34 points in Shanghai-China. In the high-performing OECD country, the Netherlands, and the partner country Liechtenstein, the opposite pattern is observed (Figure I.2.37).

In fact, performance on the *interpreting* subscale does not appear to be related to overall mathematics performance. In eight countries, students score at least ten points higher on the *interpreting* subscale than they do in mathematics overall, while in eight other countries the *interpreting* score is at least 10 points lower than the overall score. This latter group of countries includes the four highest-performing countries (Chinese Taipei, Korea, Singapore and Shanghai-China), one high-performing country (Viet Nam), and three countries that perform below the OECD average (Albania, Kazakhstan and the Russian Federation).

Descriptions of the six levels of proficiency on the subscale *interpreting, applying and evaluating mathematical outcomes* are given in Figure I.2.35 and the distribution of students among these six proficiency levels is shown in Figure I.2.36.

The relative strengths and weaknesses of countries in mathematics process subscales

Figure I.2.37 shows the country mean for the overall mathematics scale and the difference between each process subscale and the overall mathematics scale. As the figure makes clear, the levels of performance on the process subscales are somewhat aligned with each other and with the overall mean mathematics performance. However, it is also clear that countries' and economies' strengths in the three processes vary considerably.

Across all participating countries and economies, the average difference between the highest and lowest performance in mathematics processes is around 14 points. Within that variability, 16 countries/economies show the highest mean score in *formulating*; 21 countries/economies perform best in *employing;* and 28 countries/economies have the highest mean score in *interpreting*.

Shanghai-China shows the largest difference (46 points) between its highest (*formulating*) and lowest (*interpreting*) performance in processes, followed by Chinese Taipei, which has a difference 30 points between its highest (*formulating*) and lowest (*employing*) performance in processes. France shows a large difference (27 points) between its highest (*interpreting*) and lowest (*formulating*) performance in processes, the largest among OECD countries, and Singapore shows the same difference as France but its strongest performance is in *formulating* while its weakest is in *interpreting*. Viet Nam has a difference of 26 points between its strongest (*employing*) and weakest (*interpreting*) process subscales, and both Brazil and Croatia shows a difference of 25 points between their strongest and weakest process subscales. Peru, Turkey, Uruguay and Belgium show a negligible difference (2 to 3 score points) between their highest and lowest performance in processes (Figure I.2.37).

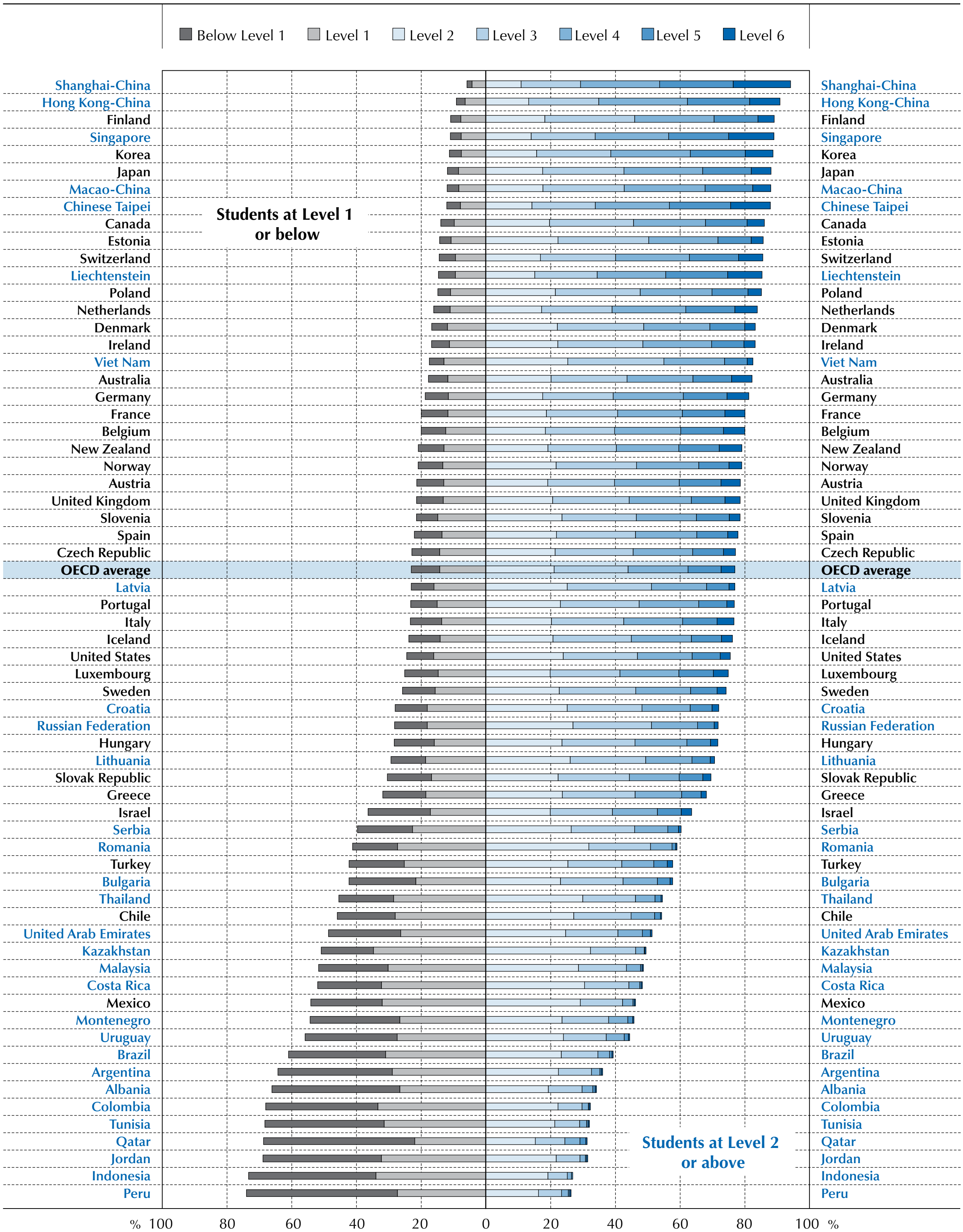
The OECD average difference between the highest and lowest performance in processes is around 5 points. Switzerland, Iceland, Japan, Korea, the Netherlands and Turkey have the highest mean score in *formulating*, and four of these countries are the best-performing OECD countries. Austria, Belgium, the Czech Republic, Estonia, Hungary, Israel, Mexico, Poland, the Slovak Republic and Slovenia perform best in *employing*; and the remaining 18 OECD countries have the highest mean scores in *interpreting*.

Ten partner countries and economies – Shanghai-China, Chinese Taipei, Singapore, Kazakhstan, Albania, Hong Kong-China, Macao-China, Jordan, Qatar and Peru – have the highest mean scores in *formulating*; ten other partner countries and economies – Brazil, Colombia, Costa Rica, Thailand, Indonesia, Montenegro, Argentina, Liechtenstein, Bulgaria and Uruguay – perform best in *interpreting*; and the remaining eleven partner countries and economies have the highest mean scores in *employing*.

■ Figure I.2.36 ■

Proficiency in the mathematics subscale *interpreting*

Percentage of students at each level of mathematics proficiency



Countries and economies are ranked in descending order of the percentage of students at Levels 2, 3, 4, 5 and 6.
Source: OECD, PISA 2012 Database, Table I.2.11.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.37 ■

Comparing countries and economies on the different mathematics process subscales

- Country's/economy's performance on the subscale is between 0 to 3 score points **higher** than on the overall mathematics scale
- Country's/economy's performance on the subscale is between 3 to 10 score points **higher** than on the overall mathematics scale
- Country's/economy's performance on the subscale is 10 or more score points **higher** than on the overall mathematics scale

- Country's/economy's performance on the subscale is between 0 to 3 score points **lower** than on the overall mathematics scale
- Country's/economy's performance on the subscale is between 3 to 10 score points **lower** than on the overall mathematics scale
- Country's/economy's performance on the subscale is 10 or more score points **lower** than on the overall mathematics scale

	Mathematics score	Performance difference between the overall mathematics scale and each process subscale		
		Formulating	Employing	Interpreting
Shanghai-China	613	12	0	-34
Singapore	573	8	1	-18
Hong Kong-China	561	7	-3	-10
Chinese Taipei	560	19	-11	-11
Korea	554	8	-1	-14
Macao-China	538	7	-2	-9
Japan	536	18	-6	-5
Liechtenstein	535	0	1	5
Switzerland	531	7	-2	-2
Netherlands	523	4	-4	3
Estonia	521	-3	4	-8
Finland	519	0	-3	9
Canada	518	-2	-2	3
Poland	518	-2	1	-3
Belgium	515	-2	1	-2
Germany	514	-3	2	3
Viet Nam	511	-14	12	-15
Austria	506	-6	4	3
Australia	504	-6	-4	10
Ireland	501	-9	1	5
Slovenia	501	-9	4	-3
Denmark	500	2	-5	8
New Zealand	500	-4	-5	11
Czech Republic	499	-4	5	-5
France	495	-12	1	16
OECD average	**494**	**-2**	**-1**	**3**
United Kingdom	494	-5	-2	7
Iceland	493	7	-3	0
Latvia	491	-3	5	-4
Luxembourg	490	-8	3	5
Norway	489	0	-3	9
Portugal	487	-8	2	3
Italy	485	-10	0	13
Spain	484	-8	-3	11
Russian Federation	482	-1	5	-11
Slovak Republic	482	-1	4	-8
United States	481	-6	-1	8
Lithuania	479	-1	3	-8
Sweden	478	1	-4	7
Hungary	477	-8	4	0
Croatia	471	-19	6	6
Israel	466	-2	2	-5
Greece	453	-5	-4	14
Serbia	449	-2	2	-3
Turkey	448	1	0	-2
Romania	445	0	1	-6
Cyprus[1, 2]	440	-3	3	-4
Bulgaria	439	-2	0	2
United Arab Emirates	434	-8	6	-6
Kazakhstan	432	10	1	-12
Thailand	427	-11	-1	5
Chile	423	-3	-6	10
Malaysia	421	-15	2	-3
Mexico	413	-4	0	0
Montenegro	410	-6	0	4
Uruguay	409	-3	-2	0
Costa Rica	407	-8	-6	11
Albania	394	4	3	-16
Brazil	391	-16	-4	10
Argentina	388	-5	-1	1
Tunisia	388	-15	2	-3
Jordan	386	4	-2	-3
Colombia	376	-2	-9	11
Qatar	376	1	-3	-1
Indonesia	375	-7	-6	4
Peru	368	2	0	0

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Source: OECD, PISA 2012 Database, Tables I.2.3a, I.2.7, I.2.10 and I.2.13.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.38 [Part 1/3] ■

Where countries and economies rank on the different mathematics process subscales

- Statistically significantly **above** the OECD average
- Not statistically significantly different from the OECD average
- Statistically significantly **below** the OECD average

	Formulating subscale				
		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Shanghai-China	624			1	1
Singapore	582			2	3
Chinese Taipei	578			2	3
Hong Kong-China	568			4	5
Korea	562	1	2	4	6
Japan	554	1	2	5	6
Macao-China	545			7	8
Switzerland	538	3	3	8	9
Liechtenstein	535			8	10
Netherlands	527	4	5	9	10
Finland	519	5	8	11	14
Estonia	517	5	9	11	15
Canada	516	5	9	11	15
Poland	516	5	10	11	16
Belgium	512	7	10	13	16
Germany	511	7	11	13	17
Denmark	502	11	14	16	20
Iceland	500	11	15	17	21
Austria	499	11	16	17	23
Australia	498	12	16	18	23
Viet Nam	497			17	27
New Zealand	496	12	18	18	25
Czech Republic	495	12	19	18	27
Ireland	492	15	20	21	27
Slovenia	492	16	20	22	27
Norway	489	16	21	22	29
United Kingdom	489	15	22	22	31
Latvia	488			23	30
France	483	20	25	27	34
Luxembourg	482	21	24	29	33
Russian Federation	481			27	37
Slovak Republic	480	20	28	28	38
Sweden	479	21	27	29	37
Portugal	479	20	28	28	38
Lithuania	477			30	38
Spain	477	23	28	32	38
United States	475	22	29	30	39
Italy	475	24	29	33	39
Hungary	469	27	30	37	40
Israel	465	28	30	38	41
Croatia	453			41	45
Turkey	449	31	32	41	46
Greece	448	31	32	41	45
Serbia	447			41	46
Romania	445			41	47
Kazakhstan	442			43	48
Bulgaria	437			45	48
Cyprus[1, 2]	437			46	48
United Arab Emirates	426			49	50
Chile	420	33	33	49	51
Thailand	416			50	52
Mexico	409	34	34	51	53
Uruguay	406			52	56
Malaysia	406			52	56
Montenegro	404			53	56
Costa Rica	399			54	57
Albania	398			56	57
Jordan	390			58	59
Argentina	383			58	61
Qatar	378			59	62
Brazil	376			60	64
Colombia	375			59	64
Tunisia	373			60	65
Peru	370			62	65
Indonesia	368			62	65

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
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Source: OECD, PISA 2012 Database.

StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.38 [Part 2/3] ■

Where countries and economies rank on the different mathematics process subscales

Statistically significantly **above** the OECD average
Not statistically significantly different from the OECD average
Statistically significantly **below** the OECD average

	Employing subscale				
		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Shanghai-China	613			1	1
Singapore	574			2	2
Hong Kong-China	558			3	4
Korea	553	1	1	3	5
Chinese Taipei	549			4	5
Liechtenstein	536			6	8
Macao-China	536			6	7
Japan	530	2	4	6	10
Switzerland	529	2	4	7	10
Estonia	524	3	5	9	12
Viet Nam	523			8	17
Poland	519	4	10	10	17
Netherlands	518	4	10	10	17
Canada	517	5	10	12	17
Germany	516	5	11	12	18
Belgium	516	5	10	12	17
Finland	516	6	10	12	17
Austria	510	9	12	16	19
Slovenia	505	12	14	19	21
Czech Republic	504	11	15	18	22
Ireland	502	12	16	19	23
Australia	500	13	16	20	23
France	496	15	20	22	28
Latvia	495			22	29
New Zealand	495	15	20	22	28
Denmark	495	16	21	23	29
Luxembourg	493	17	21	25	29
United Kingdom	492	16	23	23	32
Iceland	490	19	23	27	32
Portugal	489	17	26	24	36
Russian Federation	487			28	37
Norway	486	20	26	28	36
Italy	485	22	27	30	36
Slovak Republic	485	21	28	28	38
Lithuania	482			32	39
Spain	481	24	28	33	39
Hungary	481	23	29	32	40
United States	480	24	29	33	40
Croatia	478			35	41
Sweden	474	28	30	38	41
Israel	469	29	30	39	41
Serbia	451			42	45
Greece	449	31	32	42	45
Turkey	448	31	32	42	47
Romania	446			42	48
Cyprus[1, 2]	443			44	47
United Arab Emirates	440			45	48
Bulgaria	439			45	49
Kazakhstan	433			48	50
Thailand	426			49	51
Malaysia	423			50	52
Chile	416	33	34	51	53
Mexico	413	33	34	52	54
Montenegro	409			54	55
Uruguay	408			53	56
Costa Rica	401			55	57
Albania	397			56	58
Tunisia	390			57	61
Brazil	388			58	61
Argentina	387			58	61
Jordan	383			59	61
Qatar	373			62	63
Indonesia	369			62	65
Peru	368			62	65
Colombia	367			63	65

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Source: OECD, PISA 2012 Database.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.38 [Part 3/3] ■

Where countries and economies rank on the different mathematics process subscales

- Statistically significantly **above** the OECD average
- Not statistically significantly different from the OECD average
- Statistically significantly **below** the OECD average

	Interpreting subscale				
		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Shanghai-China	579			1	1
Singapore	555			2	3
Hong Kong-China	551			2	4
Chinese Taipei	549			3	5
Liechtenstein	540			4	7
Korea	540	1	2	4	7
Japan	531	2	5	6	11
Macao-China	530			7	10
Switzerland	529	2	5	7	11
Finland	528	2	5	7	11
Netherlands	526	2	6	7	12
Canada	521	5	7	11	13
Germany	517	6	12	12	18
Poland	515	6	14	12	20
Australia	514	7	12	13	18
Belgium	513	7	14	13	20
Estonia	513	8	14	13	20
New Zealand	511	8	16	14	22
France	511	9	16	14	22
Austria	509	9	17	15	23
Denmark	508	11	17	17	23
Ireland	507	12	17	18	23
United Kingdom	501	15	22	21	29
Norway	499	16	23	22	30
Italy	498	17	22	23	29
Slovenia	498	17	21	23	28
Viet Nam	497			22	33
Spain	495	18	25	25	32
Luxembourg	495	20	24	26	31
Czech Republic	494	18	26	24	33
Iceland	492	21	26	28	33
Portugal	490	20	27	26	35
United States	489	21	27	28	35
Latvia	486			31	35
Sweden	485	25	27	33	36
Croatia	477			35	39
Hungary	477	28	29	35	39
Slovak Republic	473	28	30	36	41
Russian Federation	471			37	41
Lithuania	471			37	41
Greece	467	29	31	39	42
Israel	462	30	31	40	42
Turkey	446	32	32	43	46
Serbia	445			43	45
Bulgaria	441			43	47
Romania	438			44	48
Cyprus[1, 2]	436			45	48
Chile	433	33	33	46	50
Thailand	432			46	50
United Arab Emirates	428			48	50
Kazakhstan	420			51	53
Malaysia	418			51	55
Costa Rica	418			51	54
Montenegro	413			53	56
Mexico	413	34	34	53	56
Uruguay	409			54	56
Brazil	401			57	57
Argentina	390			58	61
Colombia	387			58	61
Tunisia	385			58	62
Jordan	383			59	63
Indonesia	379			60	65
Albania	379			61	64
Qatar	375			63	64
Peru	368			64	65

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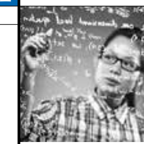
Source: OECD, PISA 2012 Database.
StatLink http://dx.doi.org/10.1787/888932935572



Gender differences in performance on the process subscales

Figures I.2.39a, b and c show the extent of gender-related differences in performance on the three mathematical processes. In most countries, boys and girls show similar performance on the processes subscales as on the mathematics proficiency scale. Boys also outnumber girls in the top three proficiency levels of the subscales, while girls outnumber boys in the lower levels of the subscales (Tables I.2.6, I.2.9 and I.2.12).

On average across OECD countries, boys outperform girls on the *formulating* subscale by around 16 points. The largest differences in favour of boys are observed in Luxembourg (33 points), Austria (32 points), Chile (29 points), Italy (24 points), New Zealand (23 points) and Korea (22 points). Ireland, Switzerland and Mexico show a gender difference of 20 points. The difference was less than 10 points in the United States (8 points). Among partner countries and economies, boys outperform girls by 33 points in Costa Rica, and by between 20 and 30 points in Colombia, Liechtenstein, Brazil, Tunisia, Peru, Hong Kong-China, and Uruguay. Several partner countries and economies show gender differences of less than 10 points, including Macao-China (9 points), Shanghai-China (8 points), Kazakhstan (7 points) and Montenegro (6 points). Only one country shows performance differences in favour of girls – Qatar (9 points).

On average among OECD countries, boys outperform girls on the *employing* subscale by 9 points. In only one OECD country, Iceland, do girls outperform boys – by 7 points. Among partner countries and economies, girls outperform boys on the *employing* subscale in 6 countries and economies, notably in Jordan (25 points), Thailand (17 points), Qatar (15 points), Malaysia (9 points), Latvia (6 points) and Singapore (6 points). Boys outperform girls by more than 20 points in the partner countries Colombia (28 points) and Costa Rica (23 points).

On average across OECD countries, boys outperform girls on the *interpreting* subscale by 9 points. The largest differences in favour of boys are recorded in Chile (22 points), Spain (21 points) and Luxembourg (20 points). Among partner countries and economies, large differences in favour of boys are recorded in Liechtenstein (27 points), Costa Rica (21 points) and Colombia (21 points). In Iceland and Finland, girls outperform boys by 11 points, and four partner countries show differences in favour of girls, with measurable differences in Jordan (25 points), Qatar (23 points), Thailand (15 points) and Malaysia (11 points).

Content subscales

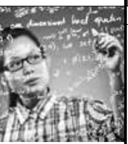
The four content categories in the PISA 2012 assessment – *change and relationships*, *space and shape*, *quantity* and *uncertainty and data* – aim to capture broad groups of mathematical phenomena that involve different kinds of mathematical thinking and expertise, and that relate to broad parts of the mathematics curriculum found in all countries and economies.

PISA outcomes presented according to this categorisation may reflect differences in curriculum priorities and in course content available to 15-year-olds. For example, in previous PISA assessment, a different profile of outcomes related to the *uncertainty and data* category compared to the other areas was observed and could be attributed to the fact that the teaching of probability and statistics is not uniform among countries/economies or even within them. Similarly, it might be expected that students who have studied predominantly basic computation and quantitative skills (related most strongly to the *quantity* category) might have different outcomes from those whose courses emphasised algebra and the study of mathematical functions and relations (which link most strongly to the *change and relationships* category); and that students in school systems that emphasise geometry can be expected to perform better on the items related to the *space and shape* category.

***Student performance on the mathematics subscale* change and relationships**

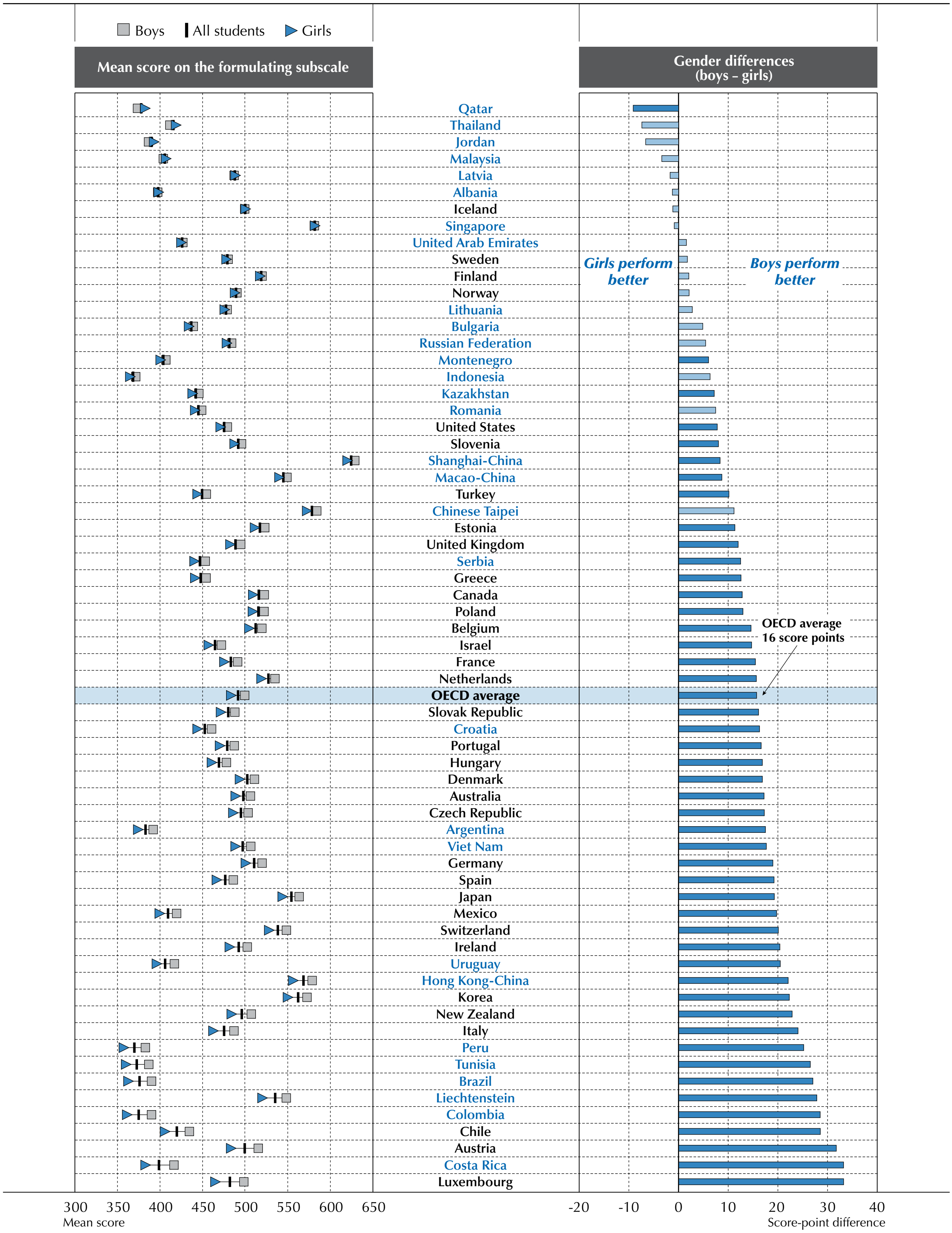
PISA items in this category emphasise the relationships among objects, and the mathematical processes associated with changes in those relationships. Items listed in Figure I.2.9 that have been classified in this category are HELEN THE CYCLIST Question 1, Question 2 and Question 3, and CLIMBING MOUNT FUJI Question 2. The questions in HELEN THE CYCLIST relate to the relationships among the variables speed, distance and time in relation to travel by bicycle. CLIMBING MOUNT FUJI also involves thinking about the relationships among the variables distance, speed and time in relation to a walking trip.

The OECD average score on the *change and relationships* subscale is 493 points. The ten top-performing countries, with a mean score of at least 530 points on this subscale, are Shanghai-China, Singapore, Hong Kong-China, Chinese Taipei, Korea, Macao-China, Japan, Liechtenstein, Estonia and Switzerland (Figure I.2.40 and Table I.2.16). The average score among OECD countries on this subscale is one point lower than the average score on the overall mathematics proficiency scale (Figure I.2.52).



■ Figure I.2.39a ■

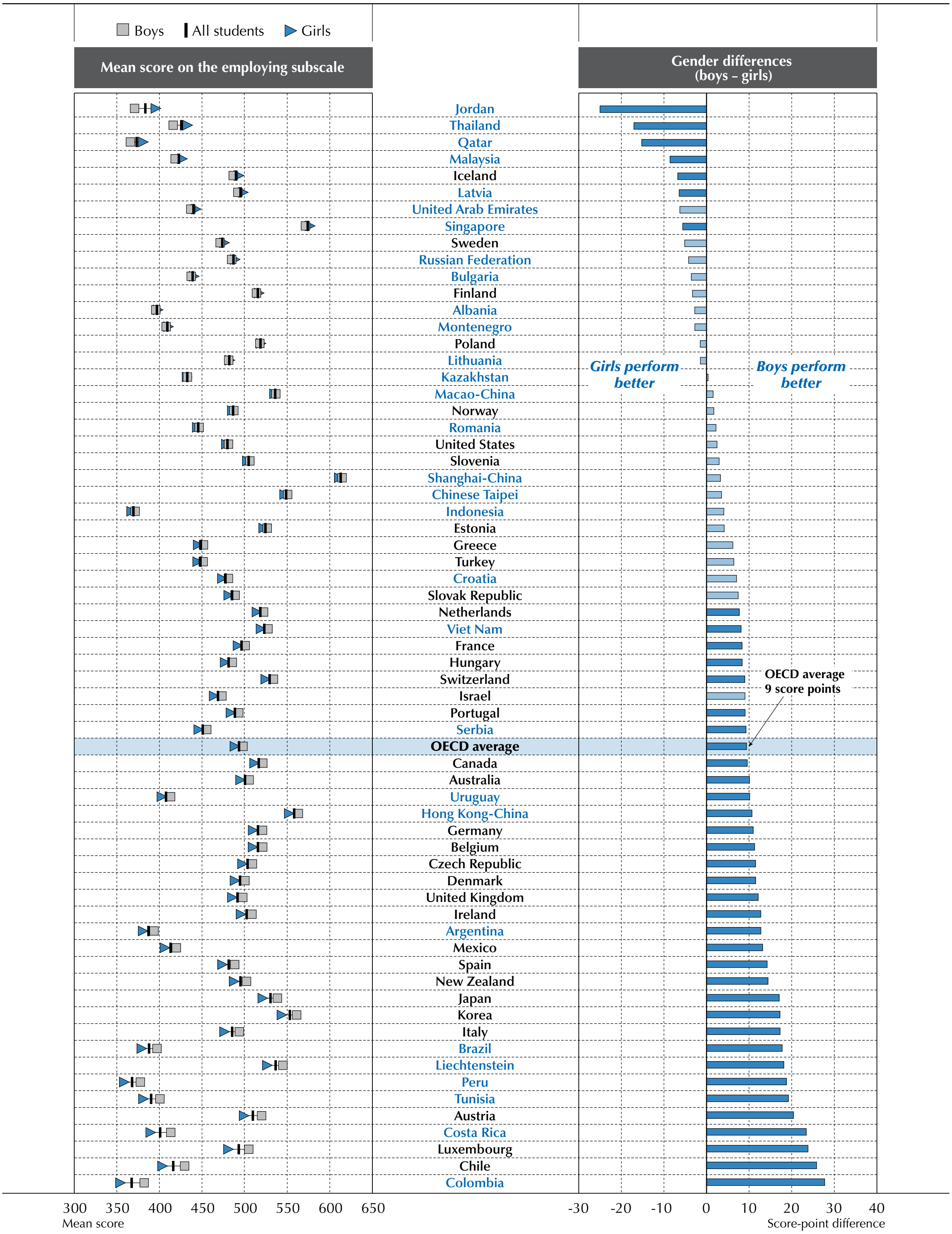
Gender differences in performance on the *formulating* subscale



Note: Statistically significant gender differences are marked in a darker tone (see Annex A3).
Countries and economies are ranked in ascending order of the gender score-point difference (boys – girls).
Source: OECD, PISA 2012 Database, Table I.2.7.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.39b ■

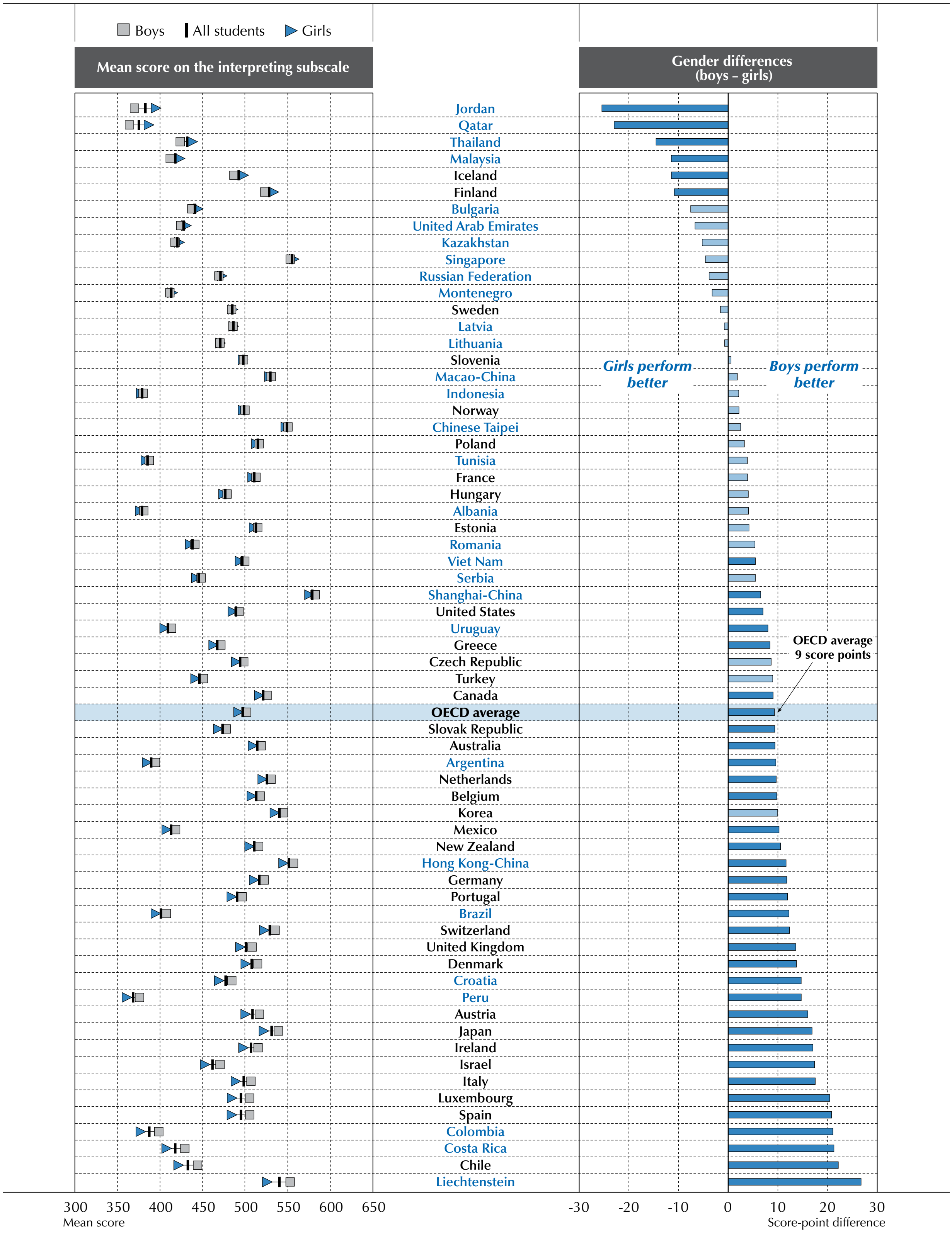
Gender differences in performance on the *employing* subscale



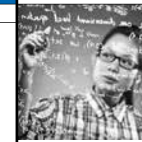
Note: Statistically significant gender differences are marked in a darker tone (see Annex A3).
Countries and economies are ranked in ascending order of the gender score-point difference (boys – girls).
Source: OECD, PISA 2012 Database, Table I.2.10.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.39c ■

Gender differences in performance on the *interpreting* subscale



Note: Statistically significant gender differences are marked in a darker tone (see Annex A3).
Countries and economies are ranked in ascending order of the gender score-point difference (boys – girls).
Source: OECD, PISA 2012 Database, Table I.2.13.
StatLink http://dx.doi.org/10.1787/888932935572



■ Figure I.2.40 ■

Comparing countries' and economies' performance on the mathematics subscale *change and relationships*

Statistically significantly **above** the OECD average
Not statistically significantly different from the OECD average
Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
624	Shanghai-China	
580	Singapore	
564	Hong Kong-China	Chinese Taipei, Korea
561	Chinese Taipei	Hong Kong-China, Korea
559	Korea	Hong Kong-China, Chinese Taipei
542	Macao-China	Japan, Liechtenstein
542	Japan	Macao-China, Liechtenstein
542	Liechtenstein	Macao-China, Japan
530	Estonia	Switzerland, Canada
530	Switzerland	Estonia, Canada
525	Canada	Estonia, Switzerland, Finland, Netherlands
520	Finland	Canada, Netherlands, Germany, Belgium, Viet Nam
518	Netherlands	Canada, Finland, Germany, Belgium, Viet Nam, Poland
516	Germany	Finland, Netherlands Belgium, Viet Nam, Poland, Australia, Austria
513	Belgium	Finland, Netherlands, Germany, Viet Nam, Poland, Australia, Austria
509	Viet Nam	Finland, Netherlands, Germany, Belgium, Poland, Australia, Austria, Ireland, New Zealand, Czech Republic, Slovenia
509	Poland	Netherlands, Germany, Belgium, Viet Nam, Australia, Austria, Ireland, New Zealand, Czech Republic
509	Australia	Germany, Belgium, Viet Nam, Poland, Austria
506	Austria	Germany, Belgium, Viet Nam, Poland, Australia, Ireland, New Zealand, Czech Republic
501	Ireland	Viet Nam, Poland, Austria, New Zealand, Czech Republic, Slovenia, France, Latvia, United Kingdom, Denmark
501	New Zealand	Viet Nam, Poland, Austria, Ireland, Czech Republic, Slovenia, France, Latvia, United Kingdom, Denmark
499	Czech Republic	Viet Nam, Poland, Austria, Ireland, New Zealand, Slovenia, France, Latvia, United Kingdom, Denmark, Russian Federation
499	Slovenia	Viet Nam, Ireland, New Zealand, Czech Republic, France, Latvia, United Kingdom, Denmark
497	France	Ireland, New Zealand, Czech Republic, Slovenia, Latvia, United Kingdom, Denmark, Russian Federation, United States
496	Latvia	Ireland, New Zealand, Czech Republic, Slovenia, France, United Kingdom, Denmark, Russian Federation, United States, Portugal
496	United Kingdom	Ireland, New Zealand, Czech Republic, Slovenia, France, Latvia, Denmark, Russian Federation, United States, Portugal
494	Denmark	Ireland, New Zealand, Czech Republic, Slovenia, France, Latvia, United Kingdom, Russian Federation, United States, Portugal
491	Russian Federation	Czech Republic, France, Latvia, United Kingdom, Denmark, United States, Luxembourg, Iceland, Portugal
488	United States	France, Latvia, United Kingdom, Denmark, Russian Federation, Luxembourg, Iceland, Portugal, Spain, Hungary, Lithuania
488	Luxembourg	Russian Federation, United States, Iceland, Portugal, Hungary
487	Iceland	Russian Federation, United States, Luxembourg, Portugal, Spain, Hungary
486	Portugal	Latvia, United Kingdom, Denmark, Russian Federation, United States, Luxembourg, Iceland, Spain, Hungary, Lithuania, Norway
482	Spain	United States, Iceland, Portugal, Hungary, Lithuania, Norway, Italy, Slovak Republic
481	Hungary	United States, Luxembourg, Iceland, Portugal, Spain, Lithuania, Norway, Italy, Slovak Republic
479	Lithuania	United States, Portugal, Spain, Hungary, Norway, Italy, Slovak Republic
478	Norway	Portugal, Spain, Hungary, Lithuania, Italy, Slovak Republic, Croatia
477	Italy	Spain, Hungary, Lithuania, Norway, Slovak Republic, Croatia
474	Slovak Republic	Spain, Hungary, Lithuania, Norway, Italy, Sweden, Croatia, Israel
469	Sweden	Slovak Republic, Croatia, Israel
468	Croatia	Norway, Italy, Slovak Republic, Sweden, Israel
462	Israel	Slovak Republic, Sweden, Croatia, Turkey
448	Turkey	Israel, Greece, Romania, United Arab Emirates, Serbia, Cyprus[1, 2]
446	Greece	Turkey, Romania, United Arab Emirates, Serbia, Cyprus[1, 2]
446	Romania	Turkey, Greece, United Arab Emirates, Serbia, Cyprus[1, 2], Bulgaria
442	United Arab Emirates	Turkey, Greece, Romania, Serbia, Cyprus[1, 2], Bulgaria
442	Serbia	Turkey, Greece, Romania, United Arab Emirates, Cyprus[1, 2], Bulgaria, Kazakhstan
440	Cyprus[1, 2]	Turkey, Greece, Romania, United Arab Emirates, Serbia, Bulgaria
434	Bulgaria	Romania, United Arab Emirates, Serbia, Cyprus[1, 2], Kazakhstan
433	Kazakhstan	Serbia, Bulgaria
414	Thailand	Chile
411	Chile	Thailand, Mexico, Costa Rica, Malaysia
405	Mexico	Chile, Costa Rica, Uruguay, Malaysia
402	Costa Rica	Chile, Mexico, Uruguay, Malaysia, Montenegro
401	Uruguay	Mexico, Costa Rica, Malaysia, Montenegro
401	Malaysia	Chile, Mexico, Costa Rica, Uruguay, Montenegro
399	Montenegro	Costa Rica, Uruguay, Malaysia
388	Albania	Jordan, Tunisia, Argentina
387	Jordan	Albania, Tunisia, Argentina
379	Tunisia	Albania, Jordan, Argentina, Brazil, Indonesia
379	Argentina	Albania, Jordan, Tunisia, Brazil, Indonesia
372	Brazil	Tunisia, Argentina, Indonesia
364	Indonesia	Brazil, Qatar, Colombia
363	Qatar	Colombia
357	Colombia	Qatar, Peru
349	Peru	Colombia

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
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Fourteen countries and economies score more than three points higher on this subscale than on the overall mathematics scale. Eleven of these countries and economies score more than five points above the overall mathematics scale. They include Shanghai-China, which scores 11 points higher (the largest difference) on the *change and relationships* subscale than on the overall mathematics scale, followed by Estonia, the Russian Federation, the United Arab Emirates, Liechtenstein, Canada, Singapore, the United States, Japan, Latvia and Korea. Seven of these countries and economies score well above the OECD average on the overall mathematics proficiency scale.

At the other end of the spectrum, 28 countries show average scores on the *change and relationships* subscale that are more than three points lower than the average score on the overall mathematics proficiency scale. Among these countries, Brazil, Colombia, Malaysia and Peru score between 19 and 20 points lower on the subscale than on the overall mathematics proficiency scale; Qatar, Thailand, Norway, Chile, Montenegro and Indonesia score between 10 and 14 points lower; and 14 other countries and economies also score lower on the subscale than on the overall proficiency scale, by a difference of at least 5 points (Figure I.2.52).

Figure I.2.41 describes the six levels of proficiency on the mathematics subscale *change and relationships* and the distribution of students among these six proficiency levels is shown in Figure I.2.42.

■ Figure I.2.41 ■

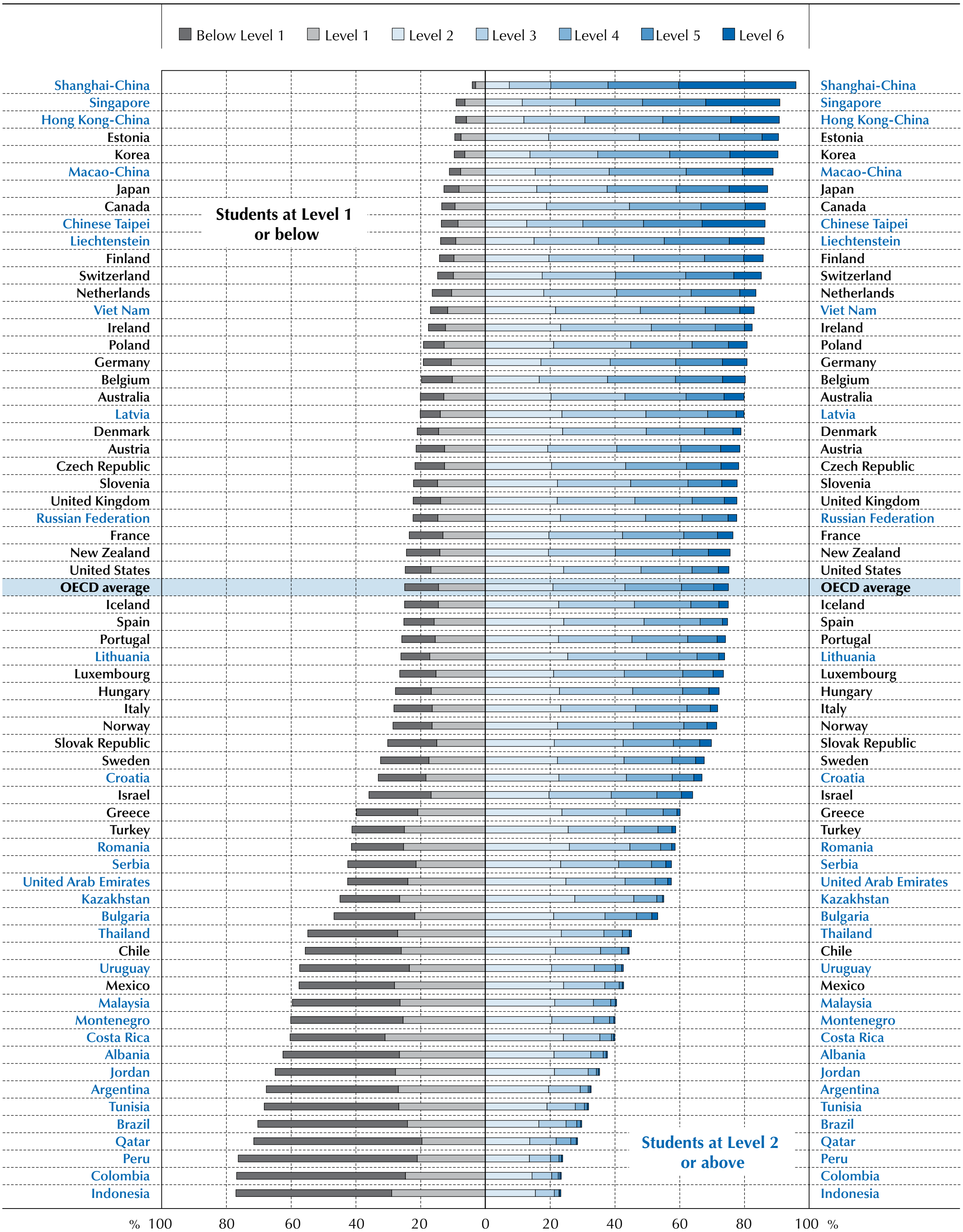
Summary descriptions of the six proficiency levels for the mathematical subscale *change and relationships*

Level	Percentage of students able to perform tasks at each level or above (OECD average)	What students can do
6	4.5%	At Level 6, students use significant insight, abstract reasoning and argumentation skills, and technical knowledge and conventions to solve problems involving relationships among variables and to generalise mathematical solutions to complex real-world problems. They can create and use an algebraic model of a functional relationship incorporating multiple quantities. They apply deep geometrical insight to work with complex patterns; and they can use complex proportional reasoning, and complex calculations with percentages to explore quantitative relationships and change.
5	14.5%	At Level 5, students can solve problems by using algebraic and other formal mathematical models, including in scientific contexts. They can use complex and multi-step problem-solving skills, and can reflect on and communicate reasoning and arguments, for example in evaluating and using a formula to predict the quantitative effect of change in one variable on another. They can use complex proportional reasoning, for example to work with rates, and they can work competently with formulae and with expressions including inequalities.
4	31.9%	Students at Level 4 can understand and work with multiple representations, including algebraic models of real-world situations. They can reason about simple functional relationships between variables, going beyond individual data points to identifying simple underlying patterns. They can use some flexibility in interpretation and reasoning about functional relationships (for example, in exploring distance-time-speed relationships) and can modify a functional model or graph to fit a specified change to the situation; and they can communicate the resulting explanations and arguments.
3	54.2%	At Level 3, students can solve problems that involve working with information from two related representations (text, graph, table, formulae), requiring some interpretation, and use reasoning in familiar contexts. They show some ability to communicate their arguments. Students at this level can make a straightforward modification to a given functional model to fit a new situation; and they use a range of calculation procedures to solve problems, including ordering data, time difference calculations, substitution of values into a formula, or linear interpolation.
2	75.1%	Students at Level 2 can locate relevant information about a relationship from data provided in a table or graph and make direct comparisons, for example, to match given graphs to a specified change process. They can reason about the basic meaning of simple relationships expressed in text or numeric form by linking text with a single representation of a relationship (graph, table, simple formula), and can correctly substitute numbers into simple formulae, sometimes expressed in words. At this level, student can use interpretation and reasoning skills in a straightforward context involving linked quantities.
1	89.6%	Students at Level 1 can evaluate single given statements about a relationship expressed clearly and directly in a formula, or in a graph. Their ability to reason about relationships, and to change in those relationships, is limited to simple expressions and to those located in familiar situations. They may apply simple calculations needed to solve problems related to clearly expressed relationships.

■ Figure I.2.42 ■

Proficiency in the mathematics subscale *change and relationships*

Percentage of students at each level of mathematics proficiency



Countries and economies are ranked in descending order of the percentage of students at Levels 2, 3, 4, 5 and 6.
Source: OECD, PISA 2012 Database, Table I.2.14.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.43 ■

Comparing countries' and economies' performance on the mathematics subscale *space and shape*

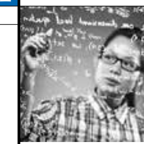
- Statistically significantly **above** the OECD average
- Not statistically significantly different from the OECD average
- Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
649	Shanghai-China	
592	Chinese Taipei	
580	Singapore	Korea
573	Korea	Singapore, Hong Kong-China
567	Hong Kong-China	Korea, Japan
558	Macao-China	Japan
558	Japan	Hong Kong-China, Macao-China
544	Switzerland	Liechtenstein
539	Liechtenstein	Switzerland
524	Poland	
513	Estonia	Canada, Belgium, Netherlands, Germany, Viet Nam, Finland
510	Canada	Estonia, Belgium, Netherlands, Germany, Viet Nam, Finland
509	Belgium	Estonia, Canada, Netherlands, Germany, Viet Nam, Finland
507	Netherlands	Estonia, Canada, Belgium, Germany, Viet Nam, Finland, Slovenia, Austria, Czech Republic
507	Germany	Estonia, Canada, Belgium, Netherlands, Viet Nam, Finland, Slovenia, Austria, Czech Republic
507	Viet Nam	Estonia, Canada, Belgium, Netherlands, Germany, Finland, Slovenia, Austria, Czech Republic, Latvia, Denmark, Australia, Russian Federation
507	Finland	Estonia, Canada, Belgium, Netherlands, Germany, Viet Nam, Slovenia, Austria
503	Slovenia	Netherlands, Germany, Viet Nam, Finland, Austria, Czech Republic, Latvia, Russian Federation
501	Austria	Netherlands, Germany, Viet Nam, Finland, Slovenia, Czech Republic, Latvia, Denmark, Australia, Russian Federation, Portugal
499	Czech Republic	Netherlands, Germany, Viet Nam, Slovenia, Austria, Latvia, Denmark, Australia, Russian Federation, Portugal, New Zealand, Slovak Republic
497	Latvia	Viet Nam, Slovenia, Austria, Czech Republic, Denmark, Australia, Russian Federation, Portugal, New Zealand, Slovak Republic, France
497	Denmark	Viet Nam, Austria, Czech Republic, Latvia, Australia, Russian Federation, Portugal, New Zealand, Slovak Republic
497	Australia	Viet Nam, Austria, Czech Republic, Latvia, Denmark, Russian Federation, Portugal, New Zealand, Slovak Republic
496	Russian Federation	Viet Nam, Slovenia, Austria, Czech Republic, Latvia, Denmark, Australia, Portugal, New Zealand, Slovak Republic, France, Iceland, Italy
491	Portugal	Austria, Czech Republic, Latvia, Denmark, Australia, Russian Federation, New Zealand, Slovak Republic, France, Iceland, Italy, Luxembourg
491	New Zealand	Czech Republic, Latvia, Denmark, Australia, Russian Federation, Portugal, Slovak Republic, France, Iceland, Italy, Luxembourg
490	Slovak Republic	Czech Republic, Latvia, Denmark, Australia, Russian Federation, Portugal, New Zealand, France, Iceland, Italy, Luxembourg, Norway
489	France	Latvia, Russian Federation, Portugal, New Zealand, Slovak Republic, Iceland, Italy, Luxembourg
489	Iceland	Russian Federation, Portugal, New Zealand, Slovak Republic, France, Italy, Luxembourg
487	Italy	Russian Federation, Portugal, New Zealand, Slovak Republic, France, Iceland Luxembourg, Norway
486	Luxembourg	Portugal, New Zealand, Slovak Republic, France, Iceland, Italy, Norway
480	Norway	Slovak Republic, Italy, Luxembourg, Ireland, Spain, United Kingdom, Hungary, Lithuania
478	Ireland	Norway, Spain, United Kingdom, Hungary, Lithuania
477	Spain	Norway, Ireland, United Kingdom, Hungary, Lithuania
475	United Kingdom	Norway, Ireland, Spain, Hungary, Lithuania, Sweden
474	Hungary	Norway, Ireland, Spain, United Kingdom, Lithuania, Sweden, United States
472	Lithuania	Norway, Ireland, Spain, United Kingdom, Hungary, Sweden, United States
469	Sweden	United Kingdom, Hungary, Lithuania, United States, Croatia
463	United States	Hungary, Lithuania, Sweden, Croatia
460	Croatia	Sweden, United States, Kazakhstan, Israel
450	Kazakhstan	Croatia, Israel, Romania, Serbia, Turkey, Bulgaria
449	Israel	Croatia, Kazakhstan, Romania, Serbia, Turkey, Bulgaria
447	Romania	Kazakhstan, Israel, Serbia, Turkey, Bulgaria
446	Serbia	Kazakhstan, Israel, Romania, Turkey, Bulgaria
443	Turkey	Kazakhstan, Israel, Romania, Serbia, Bulgaria, Greece, Cyprus[1, 2], Malaysia, Thailand
442	Bulgaria	Kazakhstan, Israel, Romania, Serbia, Turkey, Greece, Cyprus[1, 2], Malaysia, Thailand
436	Greece	Turkey, Bulgaria, Cyprus[1, 2], Malaysia, Thailand
436	Cyprus[1, 2]	Turkey, Bulgaria, Greece, Malaysia, Thailand
434	Malaysia	Turkey, Bulgaria, Greece, Cyprus[1, 2], Thailand
432	Thailand	Turkey, Bulgaria, Greece, Cyprus[1, 2], Malaysia, United Arab Emirates
425	United Arab Emirates	Thailand, Chile
419	Chile	United Arab Emirates, Albania, Uruguay, Mexico
418	Albania	Chile, Uruguay, Mexico, Montenegro
413	Uruguay	Chile, Albania, Mexico, Montenegro
413	Mexico	Chile, Albania, Uruguay, Montenegro
412	Montenegro	Albania, Uruguay, Mexico
397	Costa Rica	
385	Jordan	Argentina, Indonesia, Tunisia, Brazil, Qatar
385	Argentina	Jordan, Indonesia, Tunisia, Brazil, Qatar
383	Indonesia	Jordan, Argentina, Tunisia, Brazil, Qatar
382	Tunisia	Jordan, Argentina, Indonesia, Brazil, Qatar
381	Brazil	Jordan, Argentina, Indonesia, Tunisia, Qatar
380	Qatar	Jordan, Argentina, Indonesia, Tunisia, Brazil
370	Peru	Colombia
369	Colombia	Peru

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Source: OECD, PISA 2012 Database.

StatLink http://dx.doi.org/10.1787/888932935572



Student performance on the mathematics subscale space and shape

PISA items in this category emphasise spatial relationships among objects, and measurement and other geometric aspects of the spatial world. Items listed in Figure I.2.9 that have been classified in this category are GARAGE Question 1 and Question 2, and REVOLVING DOOR Question 1 and Question 2. The questions in GARAGE involve spatial reasoning (Question 1), and working with measurements and area calculations with a model of a real-world object. REVOLVING DOOR involves knowledge of angle relationships, spatial reasoning and some calculations with circle geometry.

Across OECD countries, the average score attained on the *space and shape* subscale is 490 points. Top-performing countries and economies on this subscale are Shanghai-China, Chinese Taipei, Singapore, Korea, Hong Kong-China, Macao-China, Japan, Switzerland, Liechtenstein and Poland (Figure I.2.43 and Table I.2.19). The average score among OECD countries on this subscale is four points lower than the average score on the overall mathematics proficiency scale (Figure I.2.52). However, this difference varies widely among countries.

■ Figure I.2.44 ■

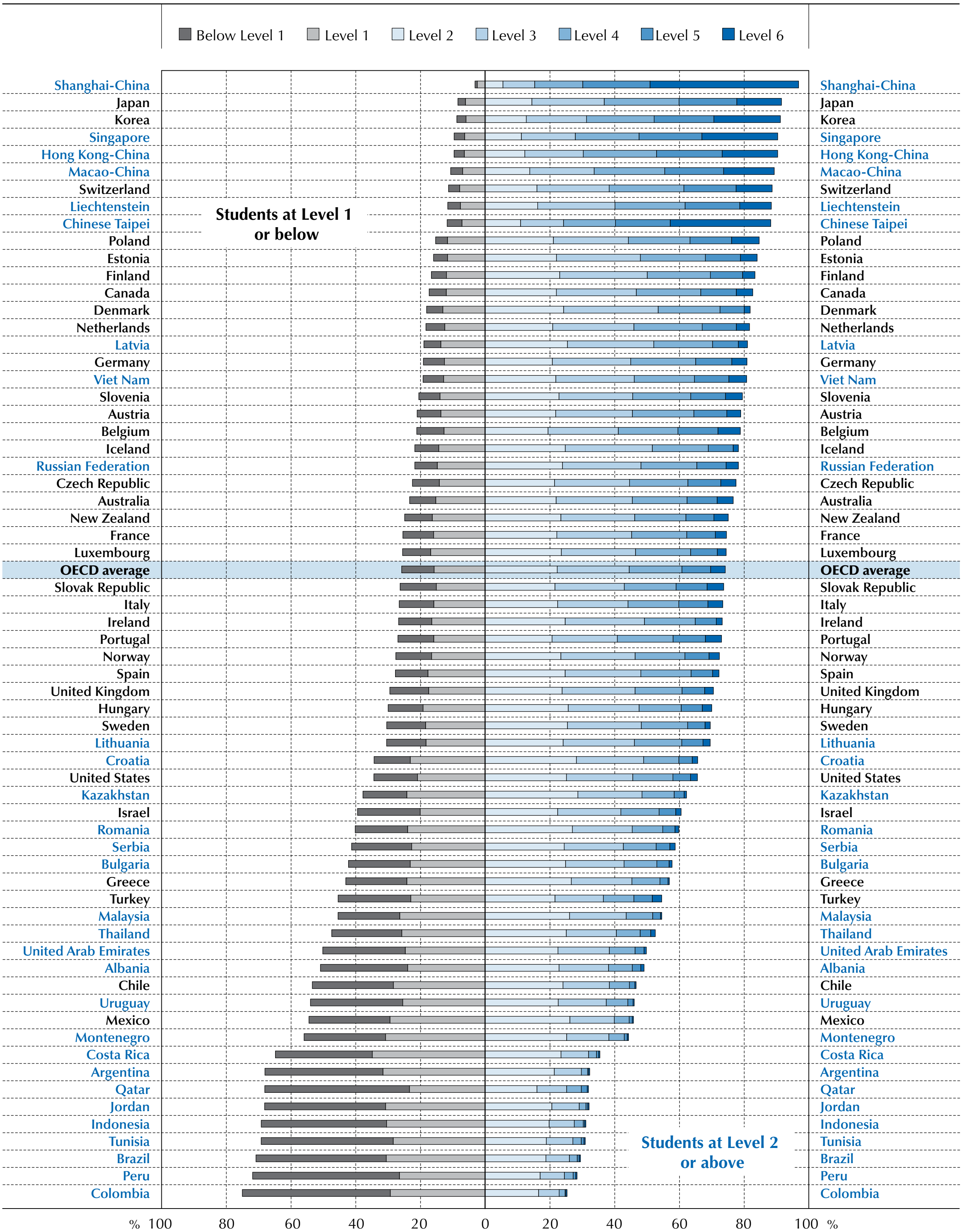
Summary descriptions of the six proficiency levels for the mathematical subscale *space and shape*

Level	Percentage of students able to perform tasks at each level or above (OECD average)	What students can do
6	4.5%	At Level 6, students can solve complex problems involving multiple representations or calculations; identify, extract, and link relevant information, for example by extracting relevant dimensions from a diagram or map and using scale to calculate an area or distance; use spatial reasoning, significant insight and reflection, for example, by interpreting text and related contextual material to formulate a useful geometric model and applying it while taking into account contextual constraints; recall and apply relevant procedural knowledge from their base of mathematical knowledge, such as in circle geometry, trigonometry, Pythagoras's rule, or area and volume formulae to solve problems; and can generalise results and findings, communicate solutions and provide justifications and argumentation.
5	13.4%	At Level 5, students can solve problems that require appropriate assumptions to be made, or that involve reasoning from assumptions provided while taking into account explicitly stated constraints, for example, in exploring and analysing the layout of a room and the furniture it contains. They solve problems using theorems or procedural knowledge, such as symmetry properties, or similar triangle properties or formulae including those for calculating area, perimeter or volume of familiar shapes. They use well-developed spatial reasoning, argument and insight to infer relevant conclusions and to interpret and link different representations, for example to identify a direction or location on a map from textual information.
4	29.7%	Students at Level 4 can solve problems by using basic mathematical knowledge, such as angle and side-length relationships in triangles, and by doing so in a way that involves multistep, visual and spatial reasoning, and argumentation in unfamiliar contexts. They can link and integrate different representations, for example to analyse the structure of a three-dimensional object based on two different perspectives of it; and can compare objects using geometric properties.
3	51.9%	At Level 3, students can solve problems that involve elementary visual and spatial reasoning in familiar contexts, such as calculating a distance or a direction from a map or a GPS device; link different representations of familiar objects or appreciate properties of objects under some simple specified transformation; and devise simple strategies and apply basic properties of triangles and circles. They can use appropriate supporting calculation techniques, such as scale conversions needed to analyse distances on a map.
2	74.2%	At Level 2, students can solve problems involving a single familiar geometric representation (for example, a diagram or other graphic) by comprehending and drawing conclusions in relation to clearly presented basic geometric properties and associated constraints. They can also evaluate and compare spatial characteristics of familiar objects in a situation where given constraints apply, such as comparing the height or circumference of two cylinders having the same surface area, or deciding whether a given shape can be dissected to produce another specified shape.
1	90.0%	Students at Level 1 can recognise and solve simple problems in a familiar context using pictures or drawings of familiar geometric objects and applying basic spatial skills, such as recognising elementary symmetry properties, comparing lengths or angle sizes, or using procedures, such as dissection of shapes.

■ Figure I.2.45 ■

Proficiency in the mathematics subscale *space and shape*

Percentage of students at each level of mathematics proficiency



Countries and economies are ranked in descending order of the percentage of students at Levels 2, 3, 4, 5 and 6.
Source: OECD, PISA 2012 Database, Table I.2.17.
StatLink http://dx.doi.org/10.1787/888932935572

Ten countries and economies score more than 10 points higher on the *space and shape* subscale than on their overall proficiency scale. These differences are quiet large in some countries, with Shanghai-China showing the largest difference (36 points), followed by Chinese Taipei (32 points), Albania (23 points), Japan (21 points), Macao-China (20 points), Korea (19 points), Kazakhstan (18 points), Malaysia (14 points), the Russian Federation (14 points) and Switzerland (13 points). Five of the best-performing countries and economies on the mathematics scale, Shanghai-China, Chinese Taipei, Korea, Macao-China and Japan, are included in this group.

Conversely, nine countries score at least 10 points lower on the *space and shape* subscale than on the overall proficiency scale. Ireland shows the largest difference (24 points), while in the eight other countries, differences range from 10 to 20 points: the United Kingdom (19 points), the United States (18 points), Israel (17 points), Greece (17 points), the Netherlands (16 points), Finland (12 points), Croatia (11 points) and Brazil (11 points) (Figure I.2.52).

Figure I.2.44 describes the six levels of proficiency on the mathematics subscale *space and shape* and the distribution of students among these six proficiency levels is shown in Figure I.2.45.

Student performance on the mathematics subscale quantity

PISA items in this category emphasise comparisons and calculations based on quantitative relationships and numeric properties of objects and phenomena. Items listed in Figure I.2.9 that have been classified in this category are WHICH CAR*?* Question 2 and Question 3, CLIMBING MOUNT FUJI Question 1 and Question 3, and REVOLVING DOOR Question 3. The questions in WHICH CAR*?* involve reasoning about quantities of given properties of different objects, and computation with percentages. CLIMBING MOUNT FUJI also involves calculations with given quantities. REVOLVING DOOR Question 3 involves reasoning and calculations using given quantitative information.

The average score on the *quantity* subscale is 495 points. The ten top-performing countries and economies on this subscale are Shanghai-China, Singapore, Hong Kong-China, Chinese Taipei, Liechtenstein, Korea, the Netherlands, Switzerland, Macao-China and Finland (Figure I.2.46 and Table I.2.22).

The average score among OECD countries on the *quantity* subscale is one point higher than the average score on the overall mathematics proficiency scale (Figure I.2.52). Twenty-two countries and economies have an average *quantity* score that is within about three score points of their average score on the overall mathematics proficiency scale.

Israel scores 13 points higher on the *quantity* subscale than on the overall mathematics scale, and seven other countries also score higher on this subscale than on the main scale by at least five points: Croatia (9 points), the Netherlands (9 points), Finland (8 points), Serbia (7 points), Spain (7 points), the Czech Republic (6 points) and Italy (5 points).

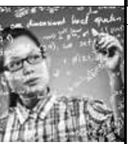
Shanghai-China scores 22 points lower on the *quantity* subscale than on the main proficiency scale, and Jordan scores 19 points lower. Japan (18 points), Chinese Taipei (16 points), Korea (16 points), Indonesia (13 points) and Malaysia (11 points) score at least 10 points lower on the subscale than on the main scale.

Figure I.2.47 describes the six levels of proficiency on the mathematics subscale *quantity* and the distribution of students among these six proficiency levels is shown in Figure I.2.48.

Student performance on the mathematics subscale uncertainty and data

PISA items in this category emphasise interpreting and working with data and with different data presentation forms, and problems involving probabilistic reasoning. Items listed in Figure I.2.9 that have been classified in this category are WHICH CAR*?* Question 1, and CHARTS Question 1, Question 2 and Question 3. The question in WHICH CAR*?* involves interpreting data in a two-way table to identify an object that satisfies various criteria. The questions in CHARTS involve interpreting a bar chart and understanding the relationships depicted in the chart.

Across OECD countries, the average score on the *uncertainty and data* subscale is 493 points. Top-performing countries and economies on this subscale are Shanghai-China, Singapore, Hong Kong-China, Chinese Taipei, Korea, the Netherlands, Japan, Liechtenstein, Macao-China and Switzerland (Figure I.2.49 and Table I.2.25). The average score among OECD countries on the *uncertainty and data* subscale is one point lower than the average score on the overall mathematics scale, but the difference between the two sets of scores varies widely among countries (Figure I.2.52).



■ Figure I.2.46 ■

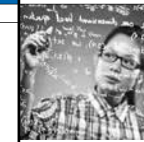
Comparing countries' and economies' performance on the mathematics subscale *quantity*

	Statistically significantly **above** the OECD average
	Not statistically significantly different from the OECD average
	Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
591	Shanghai-China	
569	Singapore	Hong Kong-China
566	Hong Kong-China	Singapore
543	Chinese Taipei	Liechtenstein, Korea
538	Liechtenstein	Chinese Taipei, Korea, Netherlands, Switzerland, Macao-China
537	Korea	Chinese Taipei, Liechtenstein, Netherlands, Switzerland, Macao-China
532	Netherlands	Liechtenstein, Korea, Switzerland, Macao-China, Finland, Estonia
531	Switzerland	Liechtenstein, Korea, Netherlands, Macao-China, Finland, Estonia
531	Macao-China	Liechtenstein, Korea, Netherlands, Switzerland, Finland
527	Finland	Netherlands, Switzerland, Macao-China, Estonia
525	Estonia	Netherlands, Switzerland, Finland, Belgium, Poland, Japan
519	Belgium	Estonia, Poland, Japan, Germany, Canada, Viet Nam
519	Poland	Estonia, Belgium, Japan, Germany, Canada, Austria, Viet Nam
518	Japan	Estonia, Belgium, Poland, Germany, Canada, Austria, Viet Nam
517	Germany	Belgium, Poland, Japan, Canada, Austria, Viet Nam
515	Canada	Belgium, Poland, Japan, Germany, Austria, Viet Nam
510	Austria	Poland, Japan, Germany, Canada, Viet Nam, Ireland, Czech Republic
509	Viet Nam	Belgium, Poland, Japan, Germany, Canada, Austria, Ireland, Czech Republic, Slovenia, Denmark, Australia, New Zealand
505	Ireland	Austria, Viet Nam, Czech Republic, Slovenia, Denmark, Australia, New Zealand
505	Czech Republic	Austria, Viet Nam, Ireland, Slovenia, Denmark, Australia, New Zealand
504	Slovenia	Viet Nam, Ireland, Czech Republic, Denmark, Australia
502	Denmark	Viet Nam, Ireland, Czech Republic, Slovenia, Australia, New Zealand, Iceland, France, United Kingdom
500	Australia	Viet Nam, Ireland, Czech Republic, Slovenia, Denmark, New Zealand, Iceland, France, United Kingdom
499	New Zealand	Viet Nam, Ireland, Czech Republic, Denmark, Australia, Iceland, France, Luxembourg, United Kingdom, Norway
496	Iceland	Denmark, Australia, New Zealand, France, Luxembourg, United Kingdom, Norway, Spain
496	France	Denmark, Australia, New Zealand, Iceland, Luxembourg, United Kingdom, Norway, Spain, Italy
495	Luxembourg	New Zealand, Iceland, France, United Kingdom, Norway, Spain, Italy
494	United Kingdom	Denmark, Australia, New Zealand, Iceland, France, Luxembourg, Norway, Spain, Italy, Latvia, Slovak Republic
492	Norway	New Zealand, Iceland, France, Luxembourg, United Kingdom, Spain, Italy, Latvia, Slovak Republic
491	Spain	Iceland, France, Luxembourg, United Kingdom, Norway, Italy, Latvia, Slovak Republic
491	Italy	France, Luxembourg, United Kingdom, Norway, Spain, Latvia, Slovak Republic
487	Latvia	United Kingdom, Norway, Spain, Italy, Slovak Republic, Lithuania, Sweden, Portugal, Croatia, Israel, United States
486	Slovak Republic	United Kingdom, Norway, Spain, Italy, Latvia, Lithuania, Sweden, Portugal, Croatia, Israel, Russian Federation, United States
483	Lithuania	Latvia, Slovak Republic, Sweden, Portugal, Croatia, Israel, Russian Federation, United States, Hungary
482	Sweden	Latvia, Slovak Republic, Lithuania, Portugal, Croatia, Israel, Russian Federation, United States, Hungary
481	Portugal	Latvia, Slovak Republic, Lithuania, Sweden, Croatia, Israel, Russian Federation, United States, Hungary
480	Croatia	Latvia, Slovak Republic, Lithuania, Sweden, Portugal, Israel, Russian Federation, United States, Hungary
480	Israel	Latvia, Slovak Republic, Lithuania, Sweden, Portugal, Croatia, Russian Federation, United States, Hungary
478	Russian Federation	Slovak Republic, Lithuania, Sweden, Portugal, Croatia, Israel, United States, Hungary
478	United States	Latvia, Slovak Republic, Lithuania, Sweden, Portugal, Croatia, Israel, Russian Federation, Hungary
476	Hungary	Lithuania, Sweden, Portugal, Croatia, Israel, Russian Federation, United States
456	Serbia	Greece
455	Greece	Serbia
443	Romania	Bulgaria, Turkey, Cyprus[1, 2]
443	Bulgaria	Romania, Turkey, Cyprus[1, 2]
442	Turkey	Romania, Bulgaria, Cyprus[1, 2], United Arab Emirates
439	Cyprus[1, 2]	Romania, Bulgaria, Turkey
431	United Arab Emirates	Turkey, Kazakhstan
428	Kazakhstan	United Arab Emirates, Chile, Thailand
421	Chile	Kazakhstan, Thailand
419	Thailand	Kazakhstan, Chile, Mexico, Uruguay, Malaysia
414	Mexico	Thailand, Uruguay, Malaysia, Costa Rica
411	Uruguay	Thailand, Mexico, Malaysia, Montenegro, Costa Rica
409	Malaysia	Thailand, Mexico, Uruguay, Montenegro, Costa Rica
409	Montenegro	Uruguay, Malaysia, Costa Rica
406	Costa Rica	Mexico, Uruguay, Malaysia, Montenegro
393	Brazil	Argentina, Albania
391	Argentina	Brazil, Albania
386	Albania	Brazil, Argentina, Tunisia
378	Tunisia	Albania, Colombia, Qatar, Jordan
375	Colombia	Tunisia, Qatar, Jordan, Peru
371	Qatar	Tunisia, Colombia, Jordan, Peru, Indonesia
367	Jordan	Tunisia, Colombia, Qatar, Peru, Indonesia
365	Peru	Colombia, Qatar, Jordan, Indonesia
362	Indonesia	Qatar, Jordan, Peru

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Source: OECD, PISA 2012 Database.
StatLink http://dx.doi.org/10.1787/888932935572



■ Figure I.2.47 ■

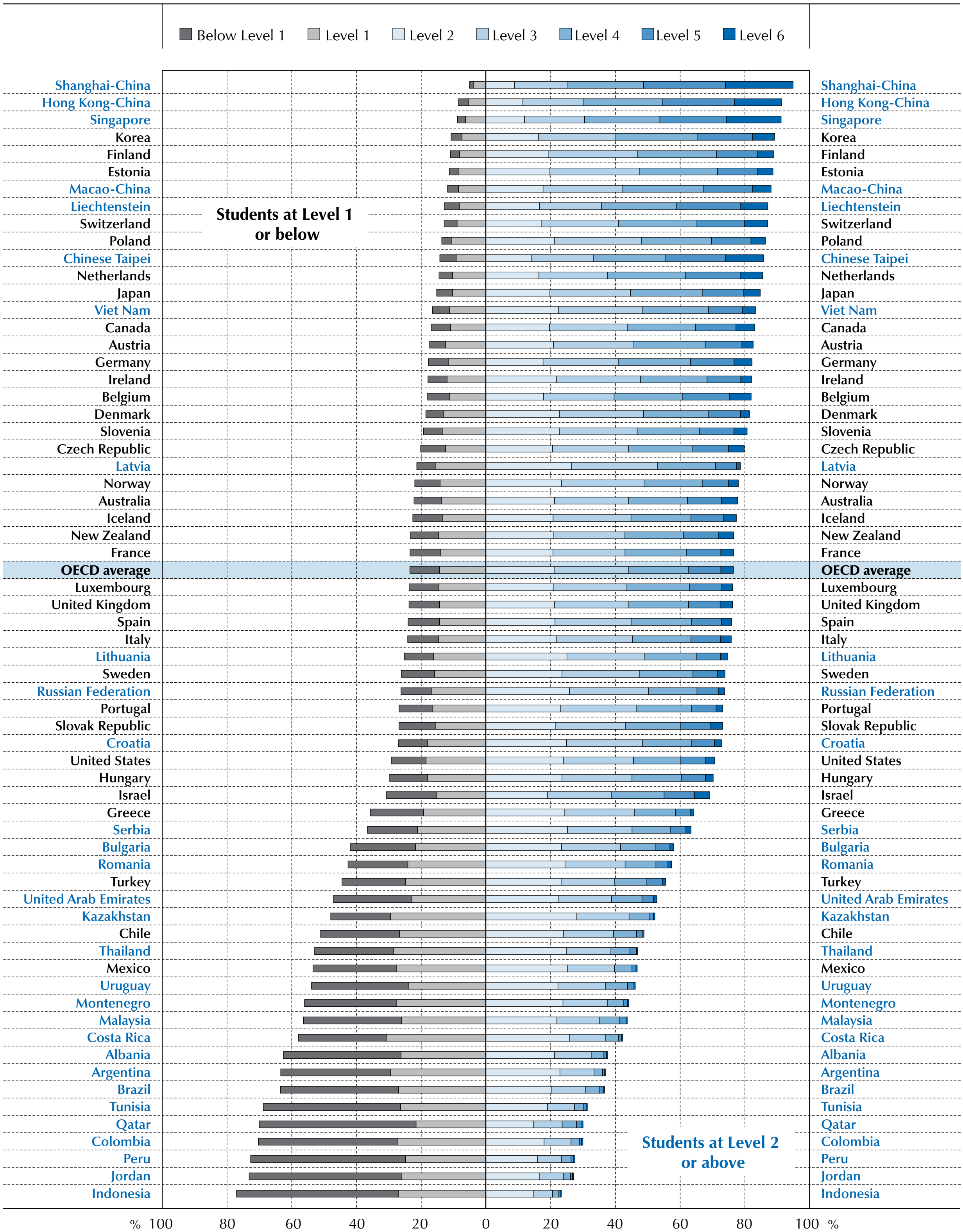
Summary descriptions of the six proficiency levels on the mathematical subscale *quantity*

Level	Percentage of students able to perform tasks at each level or above (OECD average)	What students can do
6	3.9%	At Level 6 and above, students conceptualise and work with models of complex quantitative processes and relationships; devise strategies for solving problems; formulate conclusions, arguments and precise explanations; interpret and understand complex information, and link multiple complex information sources; interpret graphical information and apply reasoning to identify, model and apply a numeric pattern. They can analyse and evaluate interpretive statements based on data provided; work with formal and symbolic expressions; plan and implement sequential calculations in complex and unfamiliar contexts, including working with large numbers, for example to perform a sequence of currency conversions, entering values correctly and rounding results. Students at this level work accurately with decimal fractions; they use advanced reasoning concerning proportions, geometric representations of quantities, combinatorics and integer number relationships; and they interpret and understand formal expressions of relationships among numbers, including in a scientific context.
5	14.0%	At Level 5, students can formulate comparison models and compare outcomes to determine highest price, and interpret complex information about real-world situations (including graphs, drawings and complex tables, for example two graphs using different scales). They can generate data for two variables and evaluate propositions about the relationship between them. Students can communicate reasoning and argument; recognise the significance of numbers to draw inferences; and provide a written argument evaluating a proposition based on data provided. They can make an estimation using knowledge about daily life; calculate relative and/or absolute change; calculate an average; calculate relative and/or absolute difference, including percentage difference, given raw difference data; and can convert units (for example calculations involving areas in different units).
4	32.5%	At Level 4, students can interpret complex instructions and situations; relate text-based numerical information to a graphic representation; identify and use quantitative information from multiple sources; deduce system rules from unfamiliar representations; formulate a simple numeric model; set up comparison models; and explain their results. They can carry out accurate and more complex or repeated calculations, such as adding 13 given times in hour/minute format; carry out time calculations using given data on distance and speed of a journey; perform simple division of large multiples in context; carry out calculations involving a sequence of steps; and accurately apply a given numeric algorithm involving a number of steps. Students at this level can perform calculations involving proportional reasoning, divisibility or percentages in simple models of complex situations.
3	55.4%	At Level 3, students can use basic problem-solving processes, including devising a simple strategy to test scenarios, understand and work with given constraints, use trial and error, and use simple reasoning in familiar contexts. At this level students can interpret a text description of a sequential calculation process, and correctly implement the process; identify and extract data presented directly in textual explanations of unfamiliar data; interpret text and diagrams describing a simple pattern; and perform calculations, including working with large numbers, calculations with speed and time, conversion of units (for example from an annual rate to a daily rate). They understand place value involving mixed 2- and 3-decimal values and including working with prices; can order a small series of (4) decimal values; calculate percentages of up to 3-digit numbers; and apply calculation rules given in natural language.
2	76.5%	At Level 2, students can interpret simple tables to identify and extract relevant quantitative information, and can interpret a simple quantitative model (such as a proportional relationship) and apply it using basic arithmetic calculations. They can identify the links between relevant textual information and tabular data to solve word problems; interpret and apply simple models involving quantitative relationships; identify the simple calculation required to solve a straight-forward problem; carry out simple calculations involving basic arithmetic operations; order 2- and 3-digit whole numbers and decimal numbers with one or two decimal places; and calculate percentages.
1	90.8%	At Level 1, students can solve basic problems in which relevant information is explicitly presented, and the situation is straightforward and very limited in scope. Students at this level can handle situations where the required computational activity is obvious and the mathematical task is basic, such as a one-step simple arithmetic operation, or to total the columns of a simple table and compare the results. They can read and interpret a simple table of numbers; extract data and perform simple calculations; use a calculator to generate relevant data; and extrapolate from the data generated, using reasoning and calculation with a simple linear model.

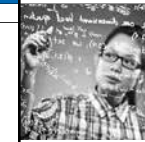
■ Figure I.2.48 ■

Proficiency in the mathematics subscale *quantity*

Percentage of students at each level of mathematics proficiency



Countries and economies are ranked in descending order of the percentage of students at Levels 2, 3, 4, 5 and 6.
Source: OECD, PISA 2012 Database, Table I.2.20.
StatLink http://dx.doi.org/10.1787/888932935572



■ Figure I.2.49 ■

Comparing countries' and economies' performance on the mathematics subscale *uncertainty and data*

- Statistically significantly **above** the OECD average
- Not statistically significantly different from the OECD average
- Statistically significantly **below** the OECD average

Mean score	Comparison country/economy	Countries/economies whose mean score is NOT statistically significantly different from that comparison country's/economy's score
592	Shanghai-China	
559	Singapore	Hong Kong-China
553	Hong Kong-China	Singapore, Chinese Taipei
549	Chinese Taipei	Hong Kong-China
538	Korea	Netherlands, Japan
532	Netherlands	Korea, Japan, Liechtenstein, Macao-China
528	Japan	Korea, Netherlands, Liechtenstein, Macao-China, Switzerland, Viet Nam
526	Liechtenstein	Netherlands, Japan, Macao-China, Switzerland, Viet Nam, Finland, Poland
525	Macao-China	Netherlands, Japan, Liechtenstein, Switzerland, Viet Nam
522	Switzerland	Japan, Liechtenstein, Macao-China, Viet Nam, Finland, Poland, Canada
519	Viet Nam	Japan, Liechtenstein, Macao-China, Switzerland, Finland, Poland, Canada, Estonia
519	Finland	Liechtenstein, Switzerland, Viet Nam, Poland, Canada
517	Poland	Liechtenstein, Switzerland, Viet Nam, Finland, Canada, Estonia, Germany, Ireland
516	Canada	Switzerland, Viet Nam, Finland, Poland
510	Estonia	Viet Nam, Poland, Germany, Ireland, Belgium, Australia, New Zealand, Denmark
509	Germany	Poland, Estonia, Ireland, Belgium, Australia, New Zealand, Denmark, United Kingdom
509	Ireland	Poland, Estonia, Germany, Belgium, Australia, New Zealand, Denmark, United Kingdom
508	Belgium	Estonia, Germany, Ireland, Australia, New Zealand, Denmark, United Kingdom
508	Australia	Estonia, Germany, Ireland, Belgium, New Zealand, Denmark, United Kingdom
506	New Zealand	Estonia, Germany, Ireland, Belgium, Australia, Denmark, United Kingdom, Austria
505	Denmark	Estonia, Germany, Ireland, Belgium, Australia, New Zealand, United Kingdom, Austria
502	United Kingdom	Germany, Ireland, Belgium, Australia, New Zealand, Denmark, Austria, Norway, Iceland
499	Austria	New Zealand, Denmark, United Kingdom, Norway, Slovenia, Iceland, France
497	Norway	United Kingdom, Austria, Slovenia, Iceland, France, United States
496	Slovenia	Austria, Norway, Iceland, France
496	Iceland	United Kingdom, Austria, Norway, Slovenia, France, United States
492	France	Austria, Norway, Slovenia, Iceland, Czech Republic, United States, Spain, Portugal
488	Czech Republic	France, United States, Spain, Portugal, Luxembourg, Sweden, Italy
488	United States	Norway, Iceland, France, Czech Republic, Spain, Portugal, Luxembourg, Sweden, Italy
487	Spain	France, Czech Republic, United States, Portugal, Luxembourg, Sweden, Italy
486	Portugal	France, Czech Republic, United States, Spain, Luxembourg, Sweden, Italy, Latvia
483	Luxembourg	Czech Republic, United States, Spain, Portugal, Sweden, Italy, Latvia
483	Sweden	Czech Republic, United States, Spain, Portugal, Luxembourg, Italy, Latvia, Hungary
482	Italy	Czech Republic, United States, Spain, Portugal, Luxembourg, Sweden, Latvia, Hungary
478	Latvia	Portugal, Luxembourg, Sweden, Italy, Hungary, Lithuania, Slovak Republic
476	Hungary	Sweden, Italy, Latvia, Lithuania, Slovak Republic, Croatia, Israel
474	Lithuania	Latvia, Hungary, Slovak Republic, Croatia, Israel
472	Slovak Republic	Latvia, Hungary, Lithuania, Croatia, Israel, Russian Federation
468	Croatia	Hungary, Lithuania, Slovak Republic, Israel, Russian Federation, Greece
465	Israel	Hungary, Lithuania, Slovak Republic, Croatia, Russian Federation, Greece
463	Russian Federation	Slovak Republic, Croatia, Israel, Greece
460	Greece	Croatia, Israel, Russian Federation
448	Serbia	Turkey, Cyprus[1, 2]
447	Turkey	Serbia, Cyprus[1, 2], Romania
442	Cyprus[1, 2]	Serbia, Turkey, Romania
437	Romania	Turkey, Cyprus[1, 2], Thailand, United Arab Emirates, Bulgaria, Chile
433	Thailand	Romania, United Arab Emirates, Bulgaria, Chile
432	United Arab Emirates	Romania, Thailand, Bulgaria, Chile
432	Bulgaria	Romania, Thailand, United Arab Emirates, Chile, Malaysia
430	Chile	Romania, Thailand, United Arab Emirates, Bulgaria
422	Malaysia	Bulgaria, Costa Rica
415	Montenegro	Costa Rica, Kazakhstan, Mexico
414	Costa Rica	Malaysia, Montenegro, Kazakhstan, Mexico, Uruguay
414	Kazakhstan	Montenegro, Costa Rica, Mexico, Uruguay
413	Mexico	Montenegro, Costa Rica, Kazakhstan
407	Uruguay	Costa Rica, Kazakhstan, Brazil, Tunisia
402	Brazil	Uruguay, Tunisia
399	Tunisia	Uruguay, Brazil, Jordan
394	Jordan	Tunisia, Argentina, Colombia, Albania, Indonesia
389	Argentina	Jordan, Colombia, Albania, Indonesia, Qatar
388	Colombia	Jordan, Argentina, Albania, Indonesia
386	Albania	Jordan, Argentina, Colombia, Indonesia, Qatar
384	Indonesia	Jordan, Argentina, Colombia, Albania, Qatar
382	Qatar	Argentina, Albania, Indonesia
373	Peru	

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
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Source: OECD, PISA 2012 Database.

StatLink http://dx.doi.org/10.1787/888932935572

Colombia (12 points), Tunisia (12 points) and Brazil (11 points) score more than 10 points higher on the subscale than on the mathematics proficiency scale. Twenty other countries scores between three and ten points lower on this subscale than on the overall proficiency scale.

Eleven countries and economies score 10 points or more lower on the *uncertainty and data* subscale than they do on the mathematics proficiency scale. Shanghai-China (21 points lower), the Russian Federation (19 points lower) and Kazakhstan (18 points lower) show the largest differences. Korea (16 points), Singapore (14 points), Macao-China (13 points), Latvia (12 points), Chinese Taipei (11 points), the Czech Republic (11 points), Estonia (10 points) and the Slovak Republic (10 points) complete this group.

Figure I.2.50 describes the six levels of proficiency in the mathematics subscale *uncertainty and data* and the distribution of students among these six proficiency levels is shown in Figure I.2.51.

■ Figure I.2.50 ■

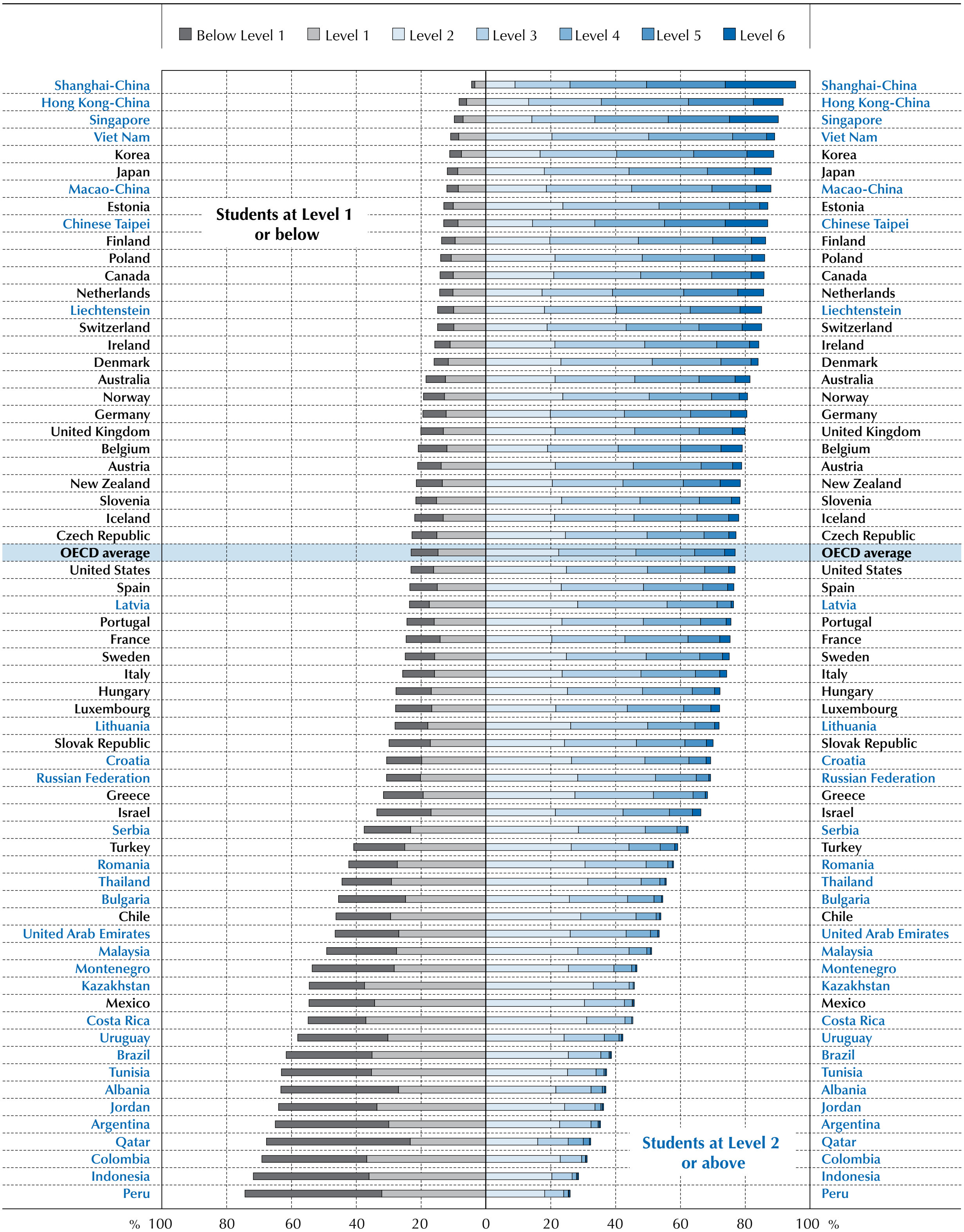
Summary descriptions of the six proficiency levels on the mathematical subscale *uncertainty and data*

Level	Percentage of students able to perform tasks at each level or above (OECD average)	What students can do
6	3.2%	At Level 6, students can interpret, evaluate and critically reflect on a range of complex statistical or probabilistic data, information and situations to analyse problems. Students at this level bring insight and sustained reasoning across several problem elements; they understand the connections between data and the situations they represent and are able to make use of those connections to explore problem situations fully. They bring appropriate calculation techniques to bear to explore data or to solve probability problems; and they can produce and communicate conclusions, reasoning and explanations.
5	12.5%	At Level 5, students can interpret and analyse a range of statistical or probabilistic data, information and situations to solve problems in complex contexts that require linking of different problem components. They can use proportional reasoning effectively to link sample data to the population they represent, can appropriately interpret data series over time, and are systematic in their use and exploration of data. Students at this level can use statistical and probabilistic concepts and knowledge to reflect, draw inferences and produce and communicate results.
4	30.6%	Students at Level 4 can activate and employ a range of data representations and statistical or probabilistic processes to interpret data, information and situations to solve problems. They can work effectively with constraints, such as statistical conditions that might apply in a sampling experiment, and they can interpret and actively translate between two related data representations (such as a graph and a data table). Students at this level can perform statistical and probabilistic reasoning to make contextual conclusions.
3	54.4%	At Level 3, students can interpret and work with data and statistical information from a single representation that may include multiple data sources, such as a graph representing several variables, or from two related data representations ,such as a simple data table and graph. They can work with and interpret descriptive statistical, probabilistic concepts and conventions in contexts such as coin tossing or lotteries, and draw conclusions from data, such as calculating or using simple measures of centre and spread. Students at this level can perform basic statistical and probabilistic reasoning in simple contexts.
2	76.9%	Students at Level 2 can identify, extract and comprehend statistical data presented in a simple and familiar form such as a simple table, a bar graph or pie chart. They can identify, understand and use basic descriptive statistical and probabilistic concepts in familiar contexts, such as tossing coins or rolling dice. At this level students can interpret data in simple representations, and apply suitable calculation procedures that connect given data to the problem context represented.
1	91.7%	At Level 1, students can identify and read information presented in a small table or simple well-labelled graph to locate and extract specific data values while ignoring distracting information, and recognise how these relate to the context. Students at this level can recognise and use basic concepts of randomness to identify misconceptions in familiar experimental contexts, such as lottery outcomes.

■ Figure I.2.51 ■

Proficiency in the mathematics subscale *uncertainty and data*

Percentage of students at each level of mathematics proficiency



Countries and economies are ranked in descending order of the percentage of students at Levels 2, 3, 4, 5 and 6.

Source: OECD, PISA 2012 Database, Table I.2.23.

StatLink http://dx.doi.org/10.1787/888932935572

The relative strengths and weaknesses of countries in different mathematics content areas

Figure I.2.52 shows the country means for the overall mathematics scale and the difference in performance between each content subscale and the overall mathematics scale. As the figure makes clear, the levels of performance on the content subscales are relatively well aligned with each other and with overall mean mathematics performance, as is the case with the process subscales. However, it is also clear that the relative strength of countries in relation to the four content categories varies considerably; in fact, there is even more variability than is the case with the process subscales. It is also evident that while *space and shape* is frequently the strongest area among some of the higher-performing countries, this is certainly not always the case; and similarly, while *change and relationships* is the weakest of the four areas in several of the lower-performing countries, this is by no means true for all countries and economies.

Among OECD countries, where the average score on the easiest subscale (*quantity*) and the most difficult subscale (*space and shape*), relative to overall mathematical performance, is about 6 points, Japan shows the largest difference between its strongest (*space and shape*) and weakest (*quantity*) content areas of 39 points; Turkey has the smallest difference between its strongest and weakest content areas, as it did between its strongest and weakest process areas, this time of about 7 points. Between these extremes there is a great spread, with an average difference between the strongest and weakest performance of about 17 points. Within that variation, six countries had the highest mean score for *change and relationships* (Estonia, Canada, Australia, Hungary, France and Turkey); six countries performed strongest in *space and shape* (Japan, Korea, Switzerland, the Slovak Republic, Poland and Portugal); 13 performed strongest in *quantity* (Israel, the Netherlands, Finland, Spain, the Czech Republic, Italy, Luxembourg, Austria, Belgium, Iceland, Germany, Slovenia and Mexico)*;* and the remaining nine had the highest mean scores in *uncertainty and data* (the United Kingdom, Chile, Norway, Greece, Ireland, the United States, New Zealand, Denmark, and Sweden).

Among partner countries and economies, Shanghai-China shows the largest difference (about 58 points) between its strongest content category (*space and shape*) and its weakest (*quantity*); while the smallest difference between the best and worst performance in the content subscales is around 11 points, seen in Uruguay, Bulgaria, Lithuania and Romania. Once again, between these extremes there is a great spread, with an average difference between the best and worst performance of about 22 points. Within that variation, three countries had the highest mean score for *change and relationships*; 11 countries performed best in *space and shape*; five had the highest mean score in *quantity;* and 12 performed best in *uncertainty and data*.

Figure I.2.53 shows the mean score on each of the four content scales for all countries, and indicates the range of ranks (highest and lowest) that might apply to each country, taking into account the statistical uncertainty in the estimates of ranks.

Gender differences in performance on the content subscales

Figures I.2.54a, b, c and d, show the performance differences between boys and girls on the content subscales. On average, a larger proportion of boys than girls attains the top two proficiency levels on all four of the content subscales (Tables I.2.15, I.2.18, I.2.21 and I.2.25).

On the *change and relationships* subscale, boys outperform girls by 11 points, on average across OECD countries. Differences of more than 20 points, in favour of boys, are seen in Chile (32 points), Colombia (29 points), Luxembourg (25 points), Austria (23 points), Japan (22 points), Korea, Liechtenstein and Costa Rica (21 points each). Twenty-four other countries and economies show significant differences in favour of boys.

Six partner countries and economies show girls outperforming boys on the *change and relationships* subscale: Jordan (29 points), Thailand (20 points), Qatar (18 points), Malaysia (15 points), Latvia (9 points), and Kazakhstan (8 points). By contrast, in no OECD country did girls outperform boys on the subscale.

On the *space and shape* subscale, boys outperform girls by 15 points, on average across OECD countries. Differences of more than 20 points, in favour of boys, are seen in 18 countries and economies, with the largest differences in Austria (37 points), Luxembourg (34 points), Colombia (34 points) and Chile (31 points). Twenty-seven other countries and economies show differences in favour of boys. In Iceland, girls outperform boys by a statistically significant 8 points. Statistically significant differences in favour of girls are observed in Albania (10 points), Qatar (15 points) and Jordan (15 points).

Boys outperform girls on the *quantity* subscale by an average of 11 points across OECD countries. Differences of more than 20 points in favour of boys are seen in Colombia (31 points), Costa Rica (29 points), Luxembourg (23 points), Chile (22 points), Peru (22 points) and Liechtenstein (22 points). Meanwhile, only in four countries do girls outperform boys: Qatar (19 points), Thailand (16 points), Sweden (7 points) and Singapore (6 points).

■ Figure I.2.52 ■

Comparing countries and economies on the different mathematics content subscales

- Country's/economy's performance on the subscale is between 0 to 3 score points **higher** than on the overall mathematics scale
- Country's/economy's performance on the subscale is between 3 to 10 score points **higher** than on the overall mathematics scale
- Country's/economy's performance on the subscale is 10 or more score points **higher** than on the overall mathematics scale

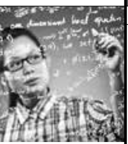
- Country's/economy's performance on the subscale is between 0 to 3 score points **lower** than on the overall mathematics scale
- Country's/economy's performance on the subscale is between 3 to 10 score points **lower** than on the overall mathematics scale
- Country's/economy's performance on the subscale is 10 or more score points **lower** than on the overall mathematics scale

	Mathematics score	Performance difference between the overall mathematics scale and each content subscale			
		Change and relationships	Space and shape	Quantity	Uncertainty and data
Shanghai-China	613	11	36	-22	-21
Singapore	573	7	6	-5	-14
Hong Kong-China	561	3	6	4	-8
Chinese Taipei	560	1	32	-16	-11
Korea	554	5	19	-16	-16
Macao-China	538	4	20	-8	-13
Japan	536	6	21	-18	-8
Liechtenstein	535	7	4	3	-9
Switzerland	531	-1	13	0	-9
Netherlands	523	-5	-16	9	9
Estonia	521	9	-8	4	-10
Finland	519	2	-12	8	0
Canada	518	7	-8	-3	-2
Poland	518	-8	7	1	-1
Belgium	515	-1	-6	4	-7
Germany	514	2	-6	4	-5
Viet Nam	511	-2	-4	-2	8
Austria	506	1	-5	5	-7
Australia	504	5	-8	-4	4
Ireland	501	0	-24	4	7
Slovenia	501	-2	2	3	-5
Denmark	500	-6	-3	2	5
New Zealand	500	1	-9	-1	6
Czech Republic	499	0	0	6	-11
France	495	2	-6	1	-3
OECD average	**494**	**-1**	**-4**	**1**	**-1**
United Kingdom	494	2	-19	0	8
Iceland	493	-6	-4	4	3
Latvia	491	6	6	-3	-12
Luxembourg	490	-2	-3	5	-7
Norway	489	-12	-10	3	7
Portugal	487	-1	4	-6	-1
Italy	485	-9	2	5	-3
Spain	484	-3	-7	7	2
Russian Federation	482	9	14	-4	-19
Slovak Republic	482	-7	8	5	-10
United States	481	7	-18	-4	7
Lithuania	479	0	-7	4	-5
Sweden	478	-9	-10	3	4
Hungary	477	4	-3	-2	-1
Croatia	471	-3	-11	9	-3
Israel	466	-4	-17	13	-1
Greece	453	-7	-17	2	7
Serbia	449	-7	-3	7	-1
Turkey	448	0	-5	-6	-1
Romania	445	1	3	-1	-8
Cyprus[1, 2]	440	0	-3	-1	3
Bulgaria	439	-4	3	4	-7
United Arab Emirates	434	8	-9	-3	-2
Kazakhstan	432	1	18	-4	-18
Thailand	427	-13	5	-8	6
Chile	423	-12	-4	-1	8
Malaysia	421	-19	14	-11	2
Mexico	413	-9	-1	0	0
Montenegro	410	-11	2	-1	5
Uruguay	409	-8	3	2	-2
Costa Rica	407	-5	-10	-1	7
Albania	394	-6	23	-8	-8
Brazil	391	-20	-11	1	11
Argentina	388	-10	-3	3	0
Tunisia	388	-9	-5	-10	12
Jordan	386	2	-1	-19	8
Colombia	376	-20	-8	-1	12
Qatar	376	-14	4	-6	5
Indonesia	375	-11	7	-13	9
Peru	368	-19	2	-3	5

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Source: OECD, PISA 2012 Database, Tables I.2.3a, I.2.16, I.2.19, I.2.22 and I.2.25.

StatLink http://dx.doi.org/10.1787/888932935572



■ Figure I.2.53 [Part 1/4] ■

Where countries and economies rank on the different mathematics content subscales

- Statistically significantly **above** the OECD average
- Not statistically significantly different from the OECD average
- Statistically significantly **below** the OECD average

	Change and relationships subscale				
		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Shanghai-China	624			1	1
Singapore	580			2	2
Hong Kong-China	564			3	5
Chinese Taipei	561			3	5
Korea	559	1	1	3	5
Macao-China	542			6	8
Japan	542	2	2	6	8
Liechtenstein	542			6	8
Estonia	530	3	4	9	10
Switzerland	530	3	5	9	11
Canada	525	4	6	10	12
Finland	520	5	8	11	14
Netherlands	518	5	9	11	16
Germany	516	6	10	12	17
Belgium	513	7	11	13	17
Viet Nam	509			13	21
Poland	509	7	13	13	20
Australia	509	9	12	15	19
Austria	506	9	14	15	21
Ireland	501	12	17	19	25
New Zealand	501	12	17	19	25
Czech Republic	499	12	19	19	27
Slovenia	499	13	17	20	25
France	497	13	19	21	28
Latvia	496			20	28
United Kingdom	496	13	20	20	28
Denmark	494	15	20	23	29
Russian Federation	491			24	32
United States	488	18	24	26	33
Luxembourg	488	20	23	28	32
Iceland	487	20	24	28	33
Portugal	486	19	26	27	36
Spain	482	23	26	32	36
Hungary	481	22	28	31	38
Lithuania	479			32	38
Norway	478	24	28	33	38
Italy	477	25	28	34	38
Slovak Republic	474	25	29	34	40
Sweden	469	28	30	38	41
Croatia	468			38	41
Israel	462	28	30	39	42
Turkey	448	31	32	42	47
Greece	446	31	32	42	46
Romania	446			42	47
United Arab Emirates	442			43	48
Serbia	442			42	48
Cyprus[1, 2]	440			45	48
Bulgaria	434			46	49
Kazakhstan	433			48	49
Thailand	414			50	51
Chile	411	33	34	50	52
Mexico	405	33	34	51	54
Costa Rica	402			52	56
Uruguay	401			52	56
Malaysia	401			52	56
Montenegro	399			54	56
Albania	388			57	58
Jordan	387			57	59
Tunisia	379			58	61
Argentina	379			58	61
Brazil	372			60	62
Indonesia	364			61	64
Qatar	363			62	63
Colombia	357			63	65
Peru	349			64	65

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
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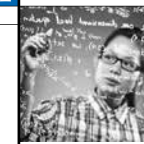

■ Figure I.2.53 [Part 2/4] ■

Where countries and economies rank on the different mathematics content subscales

- Statistically significantly **above** the OECD average
- Not statistically significantly different from the OECD average
- Statistically significantly **below** the OECD average

	Space and shape subscale				
		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Shanghai-China	649			1	1
Chinese Taipei	592			2	2
Singapore	580			3	4
Korea	573	1	1	3	5
Hong Kong-China	567			4	6
Macao-China	558			6	7
Japan	558	2	2	5	7
Switzerland	544	3	3	8	9
Liechtenstein	539			8	9
Poland	524	4	4	10	10
Estonia	513	5	8	11	14
Canada	510	5	9	11	16
Belgium	509	5	10	11	17
Netherlands	507	5	12	11	19
Germany	507	5	12	11	19
Viet Nam	507			11	21
Finland	507	6	11	12	18
Slovenia	503	9	12	16	20
Austria	501	9	15	16	24
Czech Republic	499	10	16	17	25
Latvia	497			18	26
Denmark	497	12	16	19	25
Australia	497	12	16	20	25
Russian Federation	496			18	28
Portugal	491	13	22	21	31
New Zealand	491	15	21	23	30
Slovak Republic	490	14	22	22	32
France	489	16	22	24	31
Iceland	489	16	21	25	30
Italy	487	16	22	25	31
Luxembourg	486	19	22	28	31
Norway	480	22	27	31	36
Ireland	478	23	27	32	36
Spain	477	23	27	32	36
United Kingdom	475	23	28	32	37
Hungary	474	24	28	32	38
Lithuania	472			33	38
Sweden	469	27	29	36	39
United States	463	28	29	37	40
Croatia	460			39	41
Kazakhstan	450			41	45
Israel	449	30	31	40	46
Romania	447			41	46
Serbia	446			41	46
Turkey	443	30	32	41	49
Bulgaria	442			42	49
Greece	436	31	32	46	50
Cyprus[1, 2]	436			46	49
Malaysia	434			46	50
Thailand	432			46	51
United Arab Emirates	425			50	52
Chile	419	33	33	51	54
Albania	418			52	55
Uruguay	413			53	56
Mexico	413	34	34	53	56
Montenegro	412			54	56
Costa Rica	397			57	57
Jordan	385			58	62
Argentina	385			58	62
Indonesia	383			58	63
Tunisia	382			58	63
Brazil	381			59	63
Qatar	380			60	63
Peru	370			64	65
Colombia	369			64	65

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Source: OECD, PISA 2012 Database.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.53 [Part 3/4] ■

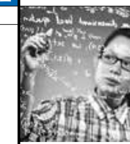
Where countries and economies rank on the different mathematics content subscales

- Statistically significantly **above** the OECD average
- Not statistically significantly different from the OECD average
- Statistically significantly **below** the OECD average

	Quantity subscale				
		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Shanghai-China	591			1	1
Singapore	569			2	3
Hong Kong-China	566			2	3
Chinese Taipei	543			4	5
Liechtenstein	538			4	7
Korea	537	1	3	4	8
Netherlands	532	1	4	5	10
Switzerland	531	1	4	6	10
Macao-China	531			7	9
Finland	527	3	5	8	11
Estonia	525	3	6	9	12
Belgium	519	6	10	12	16
Poland	519	5	10	11	17
Japan	518	5	11	11	17
Germany	517	6	11	12	17
Canada	515	7	11	13	17
Austria	510	9	13	15	19
Viet Nam	509			13	24
Ireland	505	11	15	17	22
Czech Republic	505	11	16	17	23
Slovenia	504	12	15	18	22
Denmark	502	12	17	18	24
Australia	500	14	19	21	26
New Zealand	499	14	20	21	27
Iceland	496	16	22	23	29
France	496	16	23	22	29
Luxembourg	495	18	22	25	29
United Kingdom	494	16	25	22	32
Norway	492	18	25	25	33
Spain	491	20	25	27	33
Italy	491	21	25	28	33
Latvia	487			29	36
Slovak Republic	486	22	28	29	37
Lithuania	483			32	39
Sweden	482	25	29	33	40
Portugal	481	25	30	32	41
Croatia	480			33	41
Israel	480	25	30	32	41
Russian Federation	478			35	41
United States	478	26	30	34	41
Hungary	476	27	30	36	41
Serbia	456			42	43
Greece	455	31	31	42	43
Romania	443			44	47
Bulgaria	443			44	47
Turkey	442	32	32	44	48
Cyprus[1, 2]	439			45	47
United Arab Emirates	431			47	49
Kazakhstan	428			48	50
Chile	421	33	33	49	51
Thailand	419			50	53
Mexico	414	34	34	51	54
Uruguay	411			52	56
Malaysia	409			52	56
Montenegro	409			53	56
Costa Rica	406			53	56
Brazil	393			57	58
Argentina	391			57	59
Albania	386			58	60
Tunisia	378			59	62
Colombia	375			60	62
Qatar	371			61	63
Jordan	367			62	65
Peru	365			62	65
Indonesia	362			63	65

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
2. Note by all the European Union Member States of the OECD and the European Union: The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.

Source: OECD, PISA 2012 Database.
StatLink http://dx.doi.org/10.1787/888932935572



■ Figure I.2.53 [Part 4/4] ■

Where countries and economies rank on the different mathematics content subscales

Statistically significantly **above** the OECD average
Not statistically significantly different from the OECD average
Statistically significantly **below** the OECD average

Uncertainty and data subscale

		Range of ranks			
		OECD countries		All countries/economies	
	Mean score	Upper rank	Lower rank	Upper rank	Lower rank
Shanghai-China	592			1	1
Singapore	559			2	2
Hong Kong-China	553			3	4
Chinese Taipei	549			3	4
Korea	538	1	2	5	7
Netherlands	532	1	3	5	8
Japan	528	2	4	6	10
Liechtenstein	526			6	11
Macao-China	525			7	10
Switzerland	522	3	6	7	13
Viet Nam	519			8	15
Finland	519	4	7	10	14
Poland	517	4	8	10	16
Canada	516	4	7	11	14
Estonia	510	7	12	14	19
Germany	509	7	14	14	21
Ireland	509	8	14	15	21
Belgium	508	8	14	15	21
Australia	508	9	14	16	21
New Zealand	506	9	15	16	22
Denmark	505	10	16	17	23
United Kingdom	502	11	17	18	24
Austria	499	14	19	21	26
Norway	497	15	20	22	27
Slovenia	496	16	20	23	27
Iceland	496	16	20	23	27
France	492	18	23	24	30
Czech Republic	488	20	25	27	32
United States	488	19	26	26	34
Spain	487	20	25	28	33
Portugal	486	20	27	27	35
Luxembourg	483	24	27	31	34
Sweden	483	23	28	29	35
Italy	482	23	27	30	35
Latvia	478			32	37
Hungary	476	27	29	34	39
Lithuania	474			35	39
Slovak Republic	472	28	30	35	40
Croatia	468			37	41
Israel	465	29	31	38	42
Russian Federation	463			39	42
Greece	460	30	31	40	42
Serbia	448			43	44
Turkey	447	32	32	43	45
Cyprus[1, 2]	442			44	46
Romania	437			45	49
Thailand	433			46	50
United Arab Emirates	432			46	50
Bulgaria	432			46	50
Chile	430	33	33	47	50
Malaysia	422			50	52
Montenegro	415			52	55
Costa Rica	414			52	55
Kazakhstan	414			52	55
Mexico	413	34	34	52	55
Uruguay	407			55	57
Brazil	402			56	58
Tunisia	399			56	59
Jordan	394			58	61
Argentina	389			59	63
Colombia	388			59	63
Albania	386			60	63
Indonesia	384			60	64
Qatar	382			63	64
Peru	373			65	65

1. Note by Turkey: The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
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Source: OECD, PISA 2012 Database.
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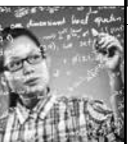
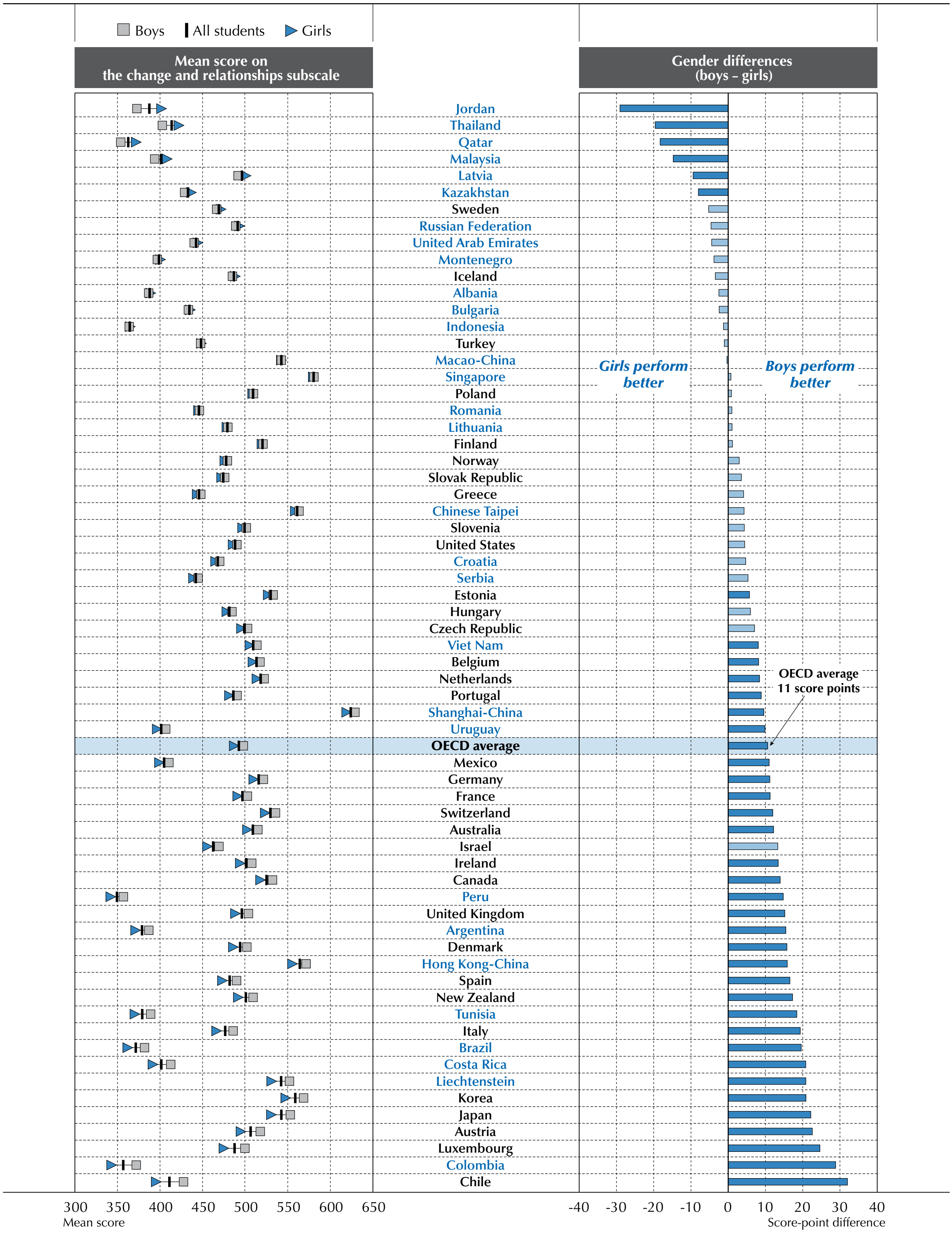


■ Figure I.2.54a ■

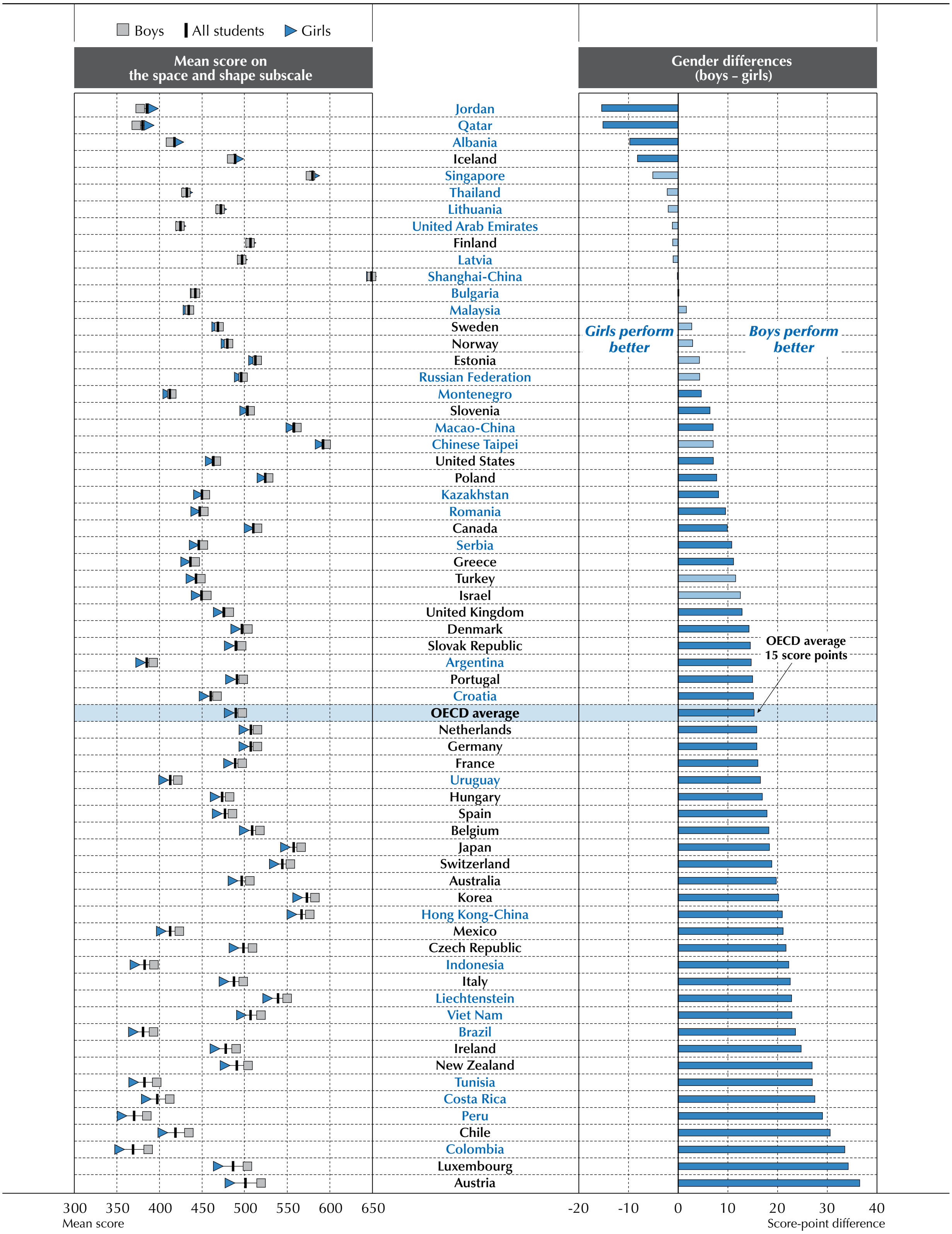
Gender differences in performance on the *change and relationships* subscale



Note: Statistically significant gender differences are marked in a darker tone (see Annex A3).
Countries and economies are ranked in ascending order of the gender score-point difference (boys – girls).
Source: OECD, PISA 2012 Database, Table I.2.16.
StatLink http://dx.doi.org/10.1787/888932935572

■ Figure I.2.54b ■

Gender differences in performance on the *space and shape* subscale



Note: Statistically significant gender differences are marked in a darker tone (see Annex A3).
Countries and economies are ranked in ascending order of the gender score-point difference (boys – girls).
Source: OECD, PISA 2012 Database, Table I.2.19.

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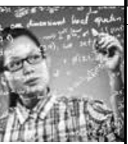
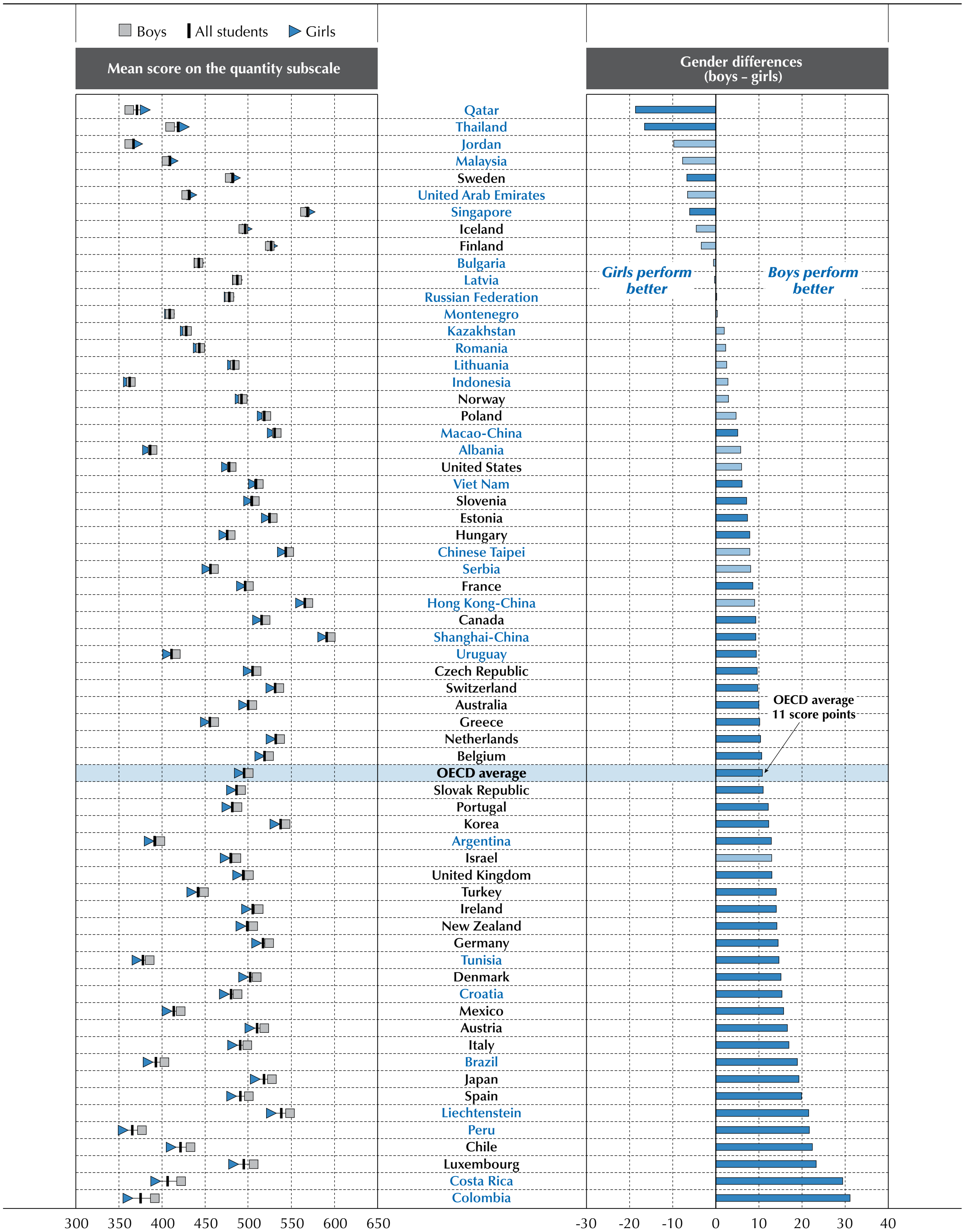


■ Figure I.2.54c ■

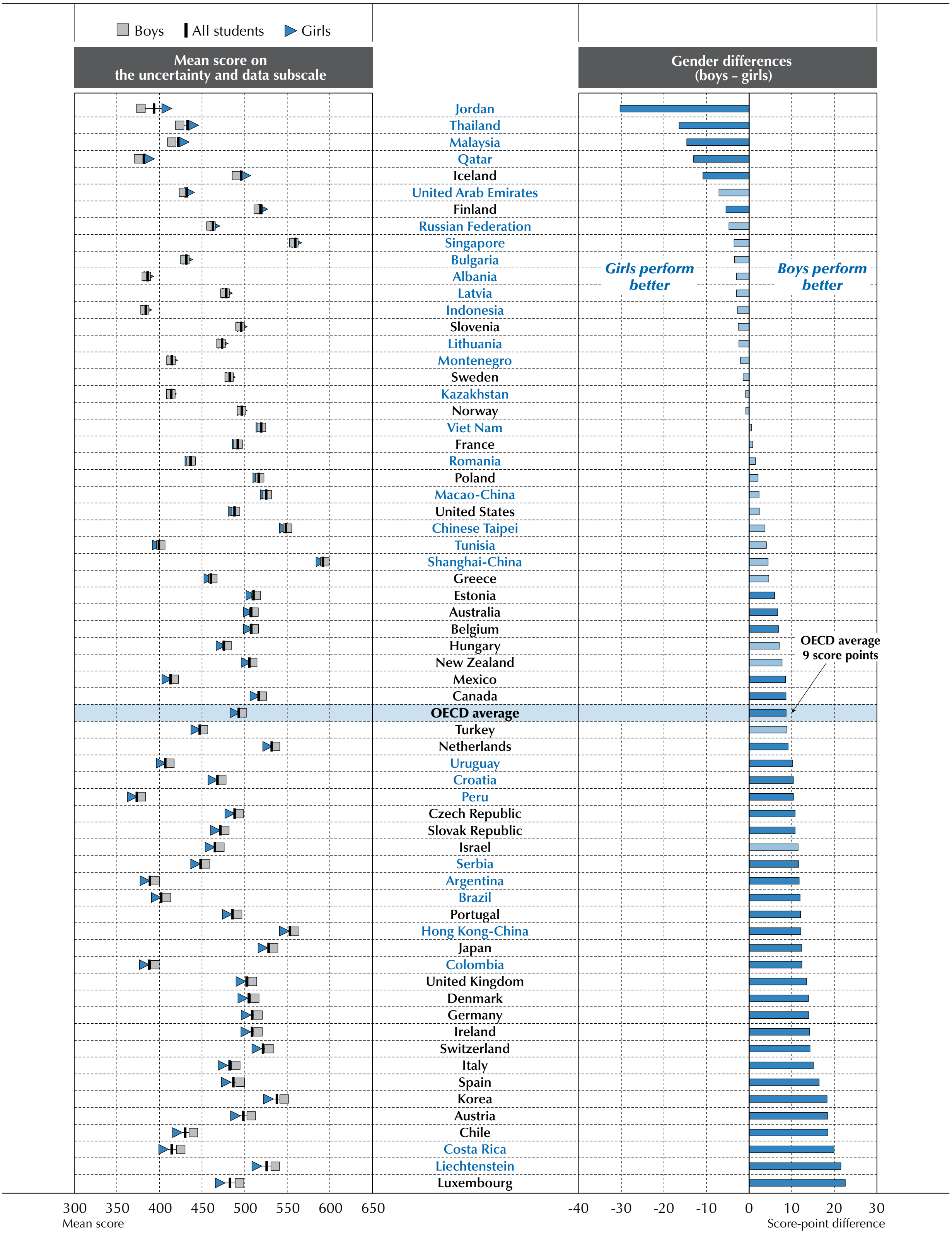
Gender differences in performance on the *quantity* subscale



Note: Statistically significant gender differences are marked in a darker tone (see Annex A3).
Countries and economies are ranked in ascending order of the gender score-point difference (boys – girls).
Source: OECD, PISA 2012 Database, Table I.2.22.
StatLink http://dx.doi.org/10.1787/888932935572

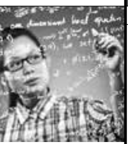
■ Figure I.2.54d ■

Gender differences in performance on the *uncertainty and data* subscale



Note: Statistically significant gender differences are marked in a darker tone (see Annex A3).
Countries and economies are ranked in ascending order of the gender score-point difference (boys – girls).
Source: OECD, PISA 2012 Database, Table I.2.25.

StatLink http://dx.doi.org/10.1787/888932935572



Across OECD countries, boys outperform girls on the *uncertainty and data* subscale by an average of 9 points – the smallest average difference of the four content subscales. The largest performance difference in favour of boys (23 points) is seen in Luxembourg. In Liechtenstein this difference is about 22 points, and in 31 other countries and economies boys outperform girls on this subscale by less than 20 points. Iceland and Finland are the only OECD countries where girls outperform boys on this subscale (11 and 5 points in favour for girls, respectively), but among partner countries and economies, four show substantial differences in favour of girls: Jordan (30 points), Thailand (16 points), Malaysia (15 points) and Qatar (13 points).

Box I.2.5. **Improving in PISA: Turkey**

When it first participated in PISA, in 2003, Turkey was among the lowest-performing OECD countries in mathematics, reading and science. Yet Turkey's performance in all three domains has improved markedly since then, at an average yearly rate of 3.2, 4.1 and 6.4 points per year. In 2003, for example, the average 15-year-old student in Turkey scored 423 points in mathematics. With an average annual increase of 3.2 points, the average score in mathematics in 2012 was 448 points – an improvement over 2003 scores that is the equivalent of more than half a year of schooling. Much of this improvement was concentrated among students with the greatest educational needs. The mathematics scores of Turkey's lowest-achieving students (the 10th percentile) improved from 300 to 338 points between 2003 and 2012, with no significant change among the highest-achieving students during the period. Consistent with this trend, the share of students who perform below proficiency Level 2 in mathematics shrank from 52% in 2003 to 42% in 2012. Between-school differences in average mathematics performance did not change between 2003 and 2012, but differences in performance among students within schools narrowed during that time, meaning that much of the improvement in mathematics performance observed between 2003 and 2012 is the result of low-performing students across all schools improving their performance (Table II.2.1b).

The observed improvement in mathematics was concentrated among socio-economically disadvantaged and low-achieving students. Between 2003 and 2012, both the average difference in performance between advantaged and disadvantaged students and the degree to which students' socio-economic status predicts their performance shrank. In 2003, advantaged students outperformed disadvantaged students by almost 100 score points; in 2012, the difference was around 60 score points. In 2003, 28% of the variation in students' scores (around the OECD average) was explained by students' socio-economic status; by 2012, 15% of the variation (below the OECD average) was explained by students' socio-economic status. While all students, on average, improved their scores no matter where their schools were located, students attending schools in towns (population of 3 000 to 100 000) improved their mathematics scores by 59 points between 2003 and 2012 – more than the increase observed among students in cities or large cities (population greater than 100 000; no change in performance detected).

Turkey has a highly centralised school system: education policy is set centrally at the Ministry of National Education and schools have comparatively little autonomy. Education policy is guided by a two-year Strategic Plan and a four-year Development Plan. The Basic Education Programme (BEP), launched in 1998, sought to expand primary education, improve the quality of education and overall student outcomes, narrow the gender gap in performance, align performance indicators with those of the European Union, develop school libraries, ensure that qualified teachers were employed, integrate information and communication technologies into the education system, and create local learning centres, based in schools, that are open to everyone (OECD, 2007). The Master Implementation Plan (2001-05), designed in collaboration with UNICEF, and the Secondary Project (2006-11), in collaboration with the World Bank, included multiple projects to improve both equity and quality in the education system. The Standards for Primary Education, piloted in 2010 and recently expanded to all primary institutions, defines quality standards for primary education, guides schools in achieving these standards, develops a system of school self-assessments, and guides local and central authorities in addressing inequalities among schools.

One of the major changes introduced with the BEP programme involved the compulsory education law. This change was first implemented in the 1997/98 school year, and in 2003 the first students graduated from the eight-year compulsory education system. Since the launch of this programme, the attendance rate among primary students increased from around 85% to nearly 100%, while the attendance rate in pre-primary programmes increased from 10% to 25%. In addition, the system was expanded to include 3.5 million more pupils, average class size was reduced to roughly 30 students, all students learn at least one foreign language, computer laboratories were established in every primary school, and overall physical conditions were improved in all 35 000 rural schools.

...

Resources devoted to the programme exceeded USD 11 billion. This programme did not directly affect school participation for most of the 15-year-olds assessed by PISA, who are mainly in secondary schools where enrolment rates are close to 60%. In 2012, compulsory education was increased from 8 to 12 years of schooling, and the school system was redefined into three levels (primary, lower secondary and upper secondary) of four years each.

Fifteen-year-old students in Turkey are the least likely among students in all OECD countries to have attended pre-primary education. Several initiatives are in place to change this, but none has yet had a direct impact on the students who participated in PISA 2012. Early childhood education and care is featured in the current Development Plan (2014-18) and other on-going programmes include the Mobile Classroom (for children aged 36-66 months from low-income families), the Summer Preschool (for children aged 60-66 months), the Turkey Country Programme, and the Pre-School Education Project.

New curricula were introduced in the 2006/07 school year, starting from the 6th grade. The secondary school mathematics and language curricula were also revised and a new science curriculum was applied in the 9th grade for the 2008/09 school year. In PISA 2012 students had already been taught the new curriculum for four years, although their primary school education was part of the former system. The standards of the new curricula were intended to meet PISA goals: "Increased importance has been placed on students' doing mathematics which means exploring mathematical ideas, solving problems, making connections among mathematical ideas, and applying them in real life situations" (Talim ve Terbiye Kurulu [TTKB] [Board of Education], 2008).

The curricular reform was designed not only to change the content of school education and encourage the introduction of innovative teaching methods, but above all to change the teaching philosophy and culture within schools. The new curricula and teaching materials emphasise "student-centred learning", giving students a more active role than before, when memorising information had been the predominant approach. They also reflect the assumption, on which PISA is based, that schools should equip students with the skills needed to ensure success at school and in life, in general.

In 2003, more than one in four students reported having arrived late for school at least once in the two weeks prior to the PISA test; by 2012, more than four in ten students reported having arrived late. By contrast, students' sense of belonging at school seems to have improved during the same period. Students in 2012 also spent one half an hour less per week in mathematics instruction than students in 2003 did, and almost an hour and a half less per week in after-school study.

Students in 2012 attended schools with better physical infrastructure and better educational resources than their counterparts in 2003 did. Throughout 2004 and 2005, private-sector investments funded 14 000 additional classrooms in the country. Taxes were reduced for private businesses that invested in education. This was particularly helpful in provinces where there was large internal migration (OECD, 2006).

Several policies had sought to change the culture and management of schools. Schools were obliged to propose a plan of work, including development targets and strategic plans for reaching them. More democratic governance, parental involvement and teamwork were suggested. In 2004, a project aimed at teaching students democratic skills was started in all primary and secondary schools, with many responsibilities assigned to student assemblies. In addition, more transparent and performance-oriented inspection tools were introduced.

Teachers were also the target of policy changes. New arrangements were implemented in 2008 to train teachers for upper secondary education through five-year graduate programmes. The arrangements also stipulated that graduates in other fields, such as science or literature, who wanted to teach would also have to attend a year-and-a-half of graduate training in education. The Teacher Formation Programmes of Education Faculties (2008) links pre-service training courses to the Ministry's curriculum and teacher-practice standards while giving more autonomy to faculties on the courses that should be taught. The New Teacher Programme, introduced in 2011, established stricter requirements for certain subjects.

Several projects implemented over the past decade have addressed equity issues. The Girls to Schools Now campaign, in collaboration with UNICEF, that started in 2003 aimed to ensure that all girls aged 6 to 14 attend primary school. Efforts to increase enrolment in school continue through programmes like the Address-Based Population Registry System, which creates a registry to identify non-schooled children, the Education with Transport programme, which benefits students who have no access to school, and the Complementary Transitional Training

...

Programme, which tries to ensure that 10-14 year-olds acquire a basic education even if they have never been enrolled in a school or if they had dropped out of school. The Project for Increasing Enrolment Rates Especially for Girls, in a pilot phase in the 16 provinces with the lowest enrolment rates among girls, addresses families' awareness about the links between education and the labour market. Since 2003, textbooks for all primary students have been supplied free of charge by the Ministry of National Education. The International Inspiration Project, begun in 2011, and the Strengthening Special Education Project, begun in 2010, are designed to promote disadvantaged students' performance.

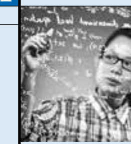
Sources:

OECD (2013d), *Education Policy Outlook: Turkey,* OECD Publishing.
http://www.oecd.org/edu/EDUCATION%20POLICY%20OUTLOOK%20TURKEY_EN.pdf

OECD (2007), *Reviews of National Policies for Education: Basic Education in Turkey*, OECD Publishing.
http://dx.doi.org/10.1787/9789264030206-en

OECD (2006), *Economic Survey of Turkey: 2006*, OECD Publishing.
http://dx.doi.org/10.1787/eco_surveys-tur-2006-en

Talim ve Terbiye Kurulu (TTKB) (2008), *Ilkögretim Matematik Dersi 6-8 Sınıflar Öğretim Programı ve Kılavuzu (Teaching Syllabus and Curriculum Guidebook for Elementary School Mathematics Course: Grades 6 to 8)*, Milli Eğitim Bakanlığı, Ankara.



EXAMPLES OF PISA MATHEMATICS UNITS

■ Figure I.2.55 ■

HELEN THE CYCLIST

Helen has just got a new bike. It has a speedometer which sits on the handlebar.

The speedometer can tell Helen the distance she travels and her average speed for a trip.

This unit is concerned with journeys by bicycle. Its storyline about an individual person places it into the *personal* context category. Slight changes in the context of the unit could place these questions into the *occupational* or *scientific* categories. These categories are designed to ensure breadth of appeal to students in the contexts used in the assessment and are a checklist to promote inclusion of all aspects of life. They are not reporting categories. The concern with relationships between distance, time and speed puts these questions in the *change and relationships* content category.

HELEN THE CYCLIST – *QUESTION 1*

On one trip, Helen rode 4 km in the first 10 minutes and then 2 km in the next 5 minutes.

Which one of the following statements is correct?

A. Helen's average speed was greater in the first 10 minutes than in the next 5 minutes.

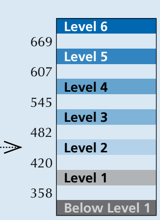
B. Helen's average speed was the same in the first 10 minutes and in the next 5 minutes.

C. Helen's average speed was less in the first 10 minutes than in the next 5 minutes.

D. It is not possible to tell anything about Helen's average speed from the information given.

Scoring

Description: *Compare average speeds given distances travelled and times taken*
Mathematical content area: *Change and relationships*
Context: *Personal*
Process: *Employ*
Question format: *Simple multiple choice*
Difficulty: *440.5*



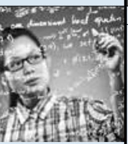
Full Credit

B. Helen's average speed was the same in the first 10 minutes and in the next 5 minutes.

No Credit

Other responses.

Missing.



Comment

Question 1, a simple multiple choice item, requires comparison of speed when travelling 4 km in 10 minutes versus 2 km in 5 minutes. It is been classified within the employing *process category because it requires the precise mathematical understanding that speed is a rate and that proportionality is the key. This question can be solved by recognising the doubles involved (2 km – 4 km; 5 km – 10 km), which is the very simplest notion of proportion. Consequently, with this Level 2 question, successful students demonstrate a very basic understanding of speed and of proportion calculations. If distance and time are in the same proportion, the speed is the same. Of course, students could correctly solve the problem in more complicated ways (e.g. calculating that both speeds are 24 km per hour) but this is not necessary. PISA results for this question do not incorporate information about the solution method used. The correct response option here is B (Helen's average speed was the same in the first 10 minutes and in the next 5 minutes).*

HELEN THE CYCLIST – *QUESTION 2*

Helen rode 6 km to her aunt's house. Her speedometer showed that she had averaged 18 km/h for the whole trip.

Which one of the following statements is correct?

A. It took Helen 20 minutes to get to her aunt's house.

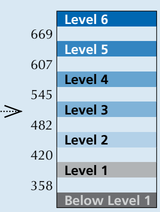
B. It took Helen 30 minutes to get to her aunt's house.

C. It took Helen 3 hours to get to her aunt's house.

D. It is not possible to tell how long it took Helen to get to her aunt's house.

Scoring

Description: *Calculate time travelled given average speed and distance travelled*
Mathematical content area: *Change and relationships*
Context: *Personal*
Process: *Employ*
Question format: *Simple multiple choice*
Difficulty: *510.6*



Full Credit

A. It took Helen 20 minutes to get to her aunt's house.

No Credit

Other responses.

Missing.

Comment

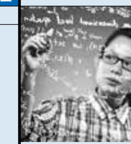
Question 2 is at Level 3. Again, it is classified in the employing *process category and can be solved by simple proportional reasoning, from the understanding of the meaning of the speed: 18 kilometres travelled in one hour. For one third of the distance, the time is one third of an hour, which is 20 minutes (hence the correct answer A: It took Helen 20 minutes to get to her aunt's house). Information about the percentage of students choosing each multiple choice is available for future analysis through the public databases.*

HELEN THE CYCLIST – *QUESTION 3*

Helen rode her bike from home to the river, which is 4 km away. It took her 9 minutes. She rode home using a shorter route of 3 km. This only took her 6 minutes.

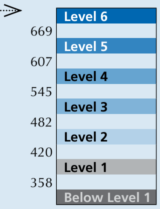
What was Helen's average speed, in km/h, for the trip to the river and back?

Average speed for the trip: ....km/h



Scoring

Description: *Calculate average speed over two trips given two distances travelled and the times taken*
Mathematical content area: *Change and relationships*
Context: *Personal*
Process: *Employ*
Question format: *Constructed response manual*
Difficulty: *696.6*



Full Credit

28

No Credit

Other responses.
28.3 *[Incorrect method: average of speeds for 2 trips (26.67 and 30)].*

Missing.

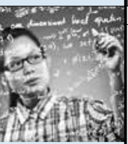
Comment

Question 3 requires a deeper understanding of the meaning of average speed, appreciating the importance of linking total time with total distance. Average speed cannot be obtained just by averaging the speeds, even though in this specific case the incorrect answer (28.3 km/hr) obtained by averaging the speeds (26.67 km/hr and 30 km/hr) is not much different from the correct answer of 28 km/hr. There are both mathematical and real world understandings of this phenomenon, leading to high demands on the fundamental mathematical capabilities of mathematisation *and* reasoning and argumentation *and also* using symbolic, formal and technical language and operations.

For students who know to work from total time (9 + 6 = 15 minutes) and total distance (4 + 3 = 7 km), the answer can be obtained simply by proportional reasoning (7 km in ¼ hour is 28 km in 1 hour), or by more complicated formula approaches (e.g. distance / time = 7 / (15/60) = 420 / 15 = 28). This question has been classified as an employing *process because the greatest part of the demand was judged to arise from the mathematical definition of average speed and possibly also the unit conversion, especially for students using speed–distance–time formulas. It is one of the more difficult tasks of the item pool, and sits in Level 6 on the proficiency scale.*

General comment on this unit

Some indication of the increasing difficulty of the three questions of this unit can be appreciated by looking at the overall strategies for the three questions. In Question 1, two rates are to be compared. In Question 2, the solution strategy goes from speed and distance, to time with a unit conversion. In Question 3, the four quantities have to be combined in a way that students often find counter-intuitive. Instead of combining the distance-time information for each trip, the two distances and the two times are combined, giving new distance and time, and so average speed. In the most elegant solutions, all the arithmetic is simple, but in practice students' methods may often involve more complicated calculation.



■ Figure I.2.56 ■

CLIMBING MOUNT FUJI

CLIMBING MOUNT FUJI

Mount Fuji is a famous dormant volcano in Japan.

CLIMBING MOUNT FUJI – *QUESTION 1*

Mount Fuji is only open to the public for climbing from 1 July to 27 August each year. About 200 000 people climb Mount Fuji during this time.

On average, about how many people climb Mount Fuji each day?

A. 340

B. 710

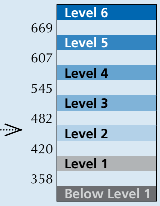
C. 3 400

D. 7 100

E. 7 400

Scoring

Description: *Identify an average daily rate given a total number and a specific time period (dates provided)*
Mathematical content area: *Quantity*
Context: *Societal*
Process: *Formulate*
Question format: *Simple multiple choice*
Difficulty: *464*



Full Credit

C. 3 400

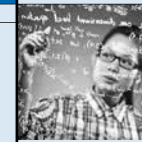
No Credit

Other responses.

Missing.

Comment

Question 1 goes beyond personal concerns of a walker to wider community issues – in this case possibly concerns of use of the public trail. Items classified as societal *involve such things as voting systems, public transport, government, public policies, demographics, advertising, national statistics and economics. Although individuals are involved in these things in a personal way, in the societal context category the focus of problems is more on the community perspective. Allocation to the context category is only carried out in order to ensure a balance across the assessment and is not used for reporting. With minor rewording, presenting the challenges from the point of view of the decisions made by park rangers, this unit could have belonged to the occupational category.*



Question 1 is presented in the simple multiple choice format (choose one out of four). Question 2 requires the answer 11 a.m. and so is a constructed response item with expert scoring needed to ensure that all equivalent ways of writing the time are picked up. Question 3, requiring the number 40 for full score, or the number 0.4 (answering in metres) for partial credit, also had expert scoring.

Question 1 requires calculation of the number of days the trail is open using the given dates, and then calculation of an average. It has been allocated to the quantity *content category because it involves quantification of time and of an average. The formula for average is required and this is indeed a relationship, but in this question the focus is on its use in finding the number of people per day, rather than inherently about the relationship. For this reason, the question is not in the* change and relationships *category. Question 3 has similar characteristics, involving units of length. The correct response to Question 1 is C: 3400.*

CLIMBING MOUNT FUJI – *QUESTION 2*

The Gotemba walking trail up Mount Fuji is about 9 kilometres (km) long.

Walkers need to return from the 18 km walk by 8 p.m.

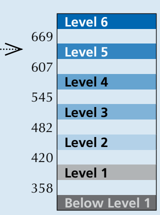
Toshi estimates that he can walk up the mountain at 1.5 kilometres per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times.

Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 p.m.?

....

Scoring

Description: *Calculate the start time for a trip given two different speeds, a total distance to travel and a finish time*
Mathematical content area: *Change and relationships*
Context: *Societal*
Process: *Formulate*
Question format: *Constructed response expert*
Difficulty: *641.6*



Full Credit

11 (a.m.) *[with or without a.m., or an equivalent way of writing time, for example, 11:00]*

No Credit

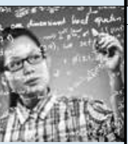
Other responses.

Missing.

Comment

Question 2 is allocated to the change and relationships *category, because here the relationship between distance and time, encapsulated as speed, is paramount. From information about distances and speed, the time to go up and the time to go down have to be quantified, and then used in combination with the finishing time to get the starting time. Had the times to go up and down been given directly, rather than indirectly through distance and speed, then the question could have also belonged in the* quantity *category. Because PISA questions are set in real contexts, they usually involve multiple mathematical topics and underlying mathematical phenomena, so it is necessary to make judgements about the major source of demand in order to categorise them.*

Allocating the process category similarly requires judgement about the major demand of the item. Question 1 has been allocated to the formulating *category, because of the judgement that the major demand in this relatively easy item is to take the two pieces of real world information (open season and total number of climbers), and to set up the mathematical problem to be solved: find the length of the open season from the dates and use it with the information about the total to find the average. Expert judgement is that the major cognitive demand for 15-year-olds lies in this movement from the real world problem to the mathematical relationships, rather than in the ensuing whole number calculations. Question 2 has also been allocated to the formulating process category, because again the major demand is judged to arise from the*



transformation from the real world data to the mathematical problem, identifying all the relationships involved, rather than in carrying out the calculations or in interpreting the answer as a starting time of 11 a.m. In this difficult item, the mathematical structure involves multiple relationships: starting time = finishing time – duration, duration = time up + time down, time up (down) = distance / speed (or equivalent proportional reasoning), time down = half time up, and appreciating the simplifying assumptions that average speeds already include consideration of variable speed during the day and that no further allowance is required for breaks.

CLIMBING MOUNT FUJI – *QUESTION 3*

Toshi wore a pedometer to count his steps on his walk along the Gotemba trail.

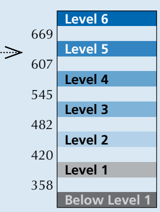
His pedometer showed that he walked 22 500 steps on the way up.

Estimate Toshi's average step length for his walk up the 9 km Gotemba trail. Give your answer in centimetres (cm).

Answer: .... cm

Scoring

Description: *Divide a length given in km by a specific number and express the quotient in cm*
Mathematical content area: *Quantity*
Context: *Societal*
Process: *Employ*
Question format: *Constructed response manual*
Difficulty: *610*



Full Credit

40

Partial Credit

Responses with the digit 4 based on incorrect conversion to centimetres.

- 0.4 *[answer given in metres]*.
- 4 000 *[incorrect conversion]*.

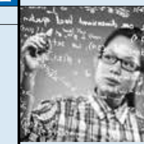
No Credit

Other responses.

Missing.

Comment

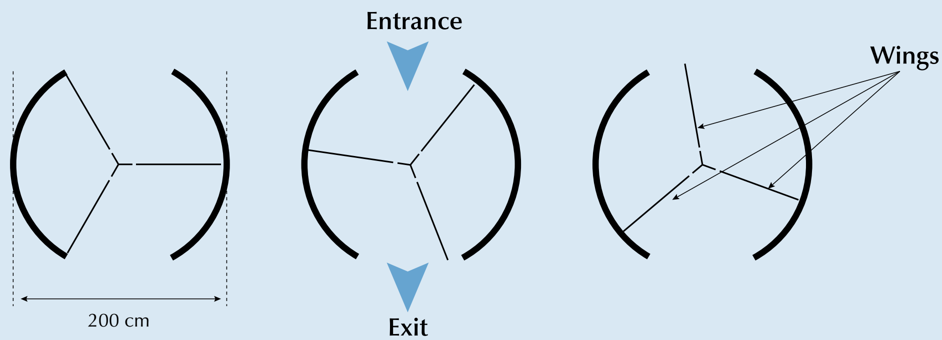
Question 3 has been allocated to the employing *category. There is one main relationship involved: the distance walked = number of steps × average step length. To use this relationship to solve the problem, there are two obstacles: rearranging the formula (which is probably done by students informally rather than formally using the written relationship) so that the average step length can be found from distance and number of steps, and making appropriate unit conversions. For this question, it was judged that the major cognitive demand comes from carrying out these steps; hence it has been categorised in the* employing *process, rather than identifying the relationships and assumptions to be made (the* formulating *process) or interpreting the answer in real world terms.*



■ Figure I.2.57 ■
REVOLVING DOOR

REVOLVING DOOR

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



The stimulus for these three questions concerns a revolving door, which is common in cold and hot countries to prevent heat moving into or out of buildings.

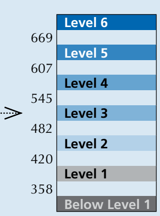
REVOLVING DOOR – *QUESTION 1*

What is the size in degrees of the angle formed by two door wings?

Size of the angle: ....°

Scoring

Description: *Compute the central angle of a sector of a circle*
Mathematical content area: *Space and shape*
Context: *Scientific*
Process: *Employ*
Question format: *Constructed response manual*
Difficulty: *512.3*



Full Credit

120 *[accept the equivalent reflex angle: 240].*

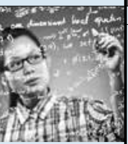
No Credit

Other responses.

Missing.

Comment

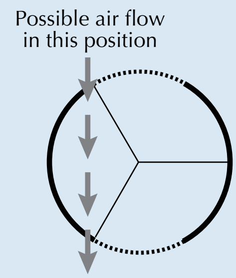
The first question may appear very simple: finding the angle of 120 degrees between the two door wings, but the student responses indicate it is at Level 3. This is probably because of the demand arising from communication, representation *and* mathematisation *as well as the specific knowledge of circle geometry that is needed. The context of three-dimensional revolving doors has to be understood from the written descriptions. It also needs to be understood that the three diagrams in the initial stimulus provide different two-dimensional information about just one revolving door (not three doors) – first the diameter, then the directions in which people enter and exit from the door, and thirdly connecting the wings mentioned within the text with the lines of the diagrams. The fundamental mathematical capability*



of representation *is required at a high level to interpret these diagrams mathematically. This question is allocated to the* space and shape *content category because it requires knowledge that there are 360 degrees in a complete revolution, and because of the requirement for spatial understanding of the diagrams.*

These diagrams give the view from above, but students also need to visualise real revolving doors especially in answering Questions 2 and 3.

REVOLVING DOOR – *QUESTION 2*



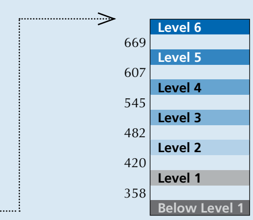
The two door openings (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.

What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: .... cm

Scoring

Description: *Interpret a geometrical model of a real life situation to calculate the length of an arc*
Mathematical content area: *Space and shape*
Context: *Scientific*
Process: *Formulate*
Question format: *Constructed response expert*
Difficulty: *840.3*



Full Credit

Answers in the range from 103 to 105. *[Accept answers calculated as 1/6th of the circumference ($\frac{100\pi}{3}$). Also accept an answer of 100 only if it is clear that this response resulted from using π = 3. Note: Answer of 100 without supporting working could be obtained by a simple guess that it is the same as the radius (length of a single wing).]*

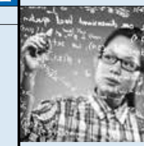
No Credit

Other responses.
- 209 *[states the total size of the openings rather than the size of "each" opening].*

Missing.

Comment

Question 2 was one of the most challenging questions in the survey, lying towards the upper end of Level 6. It addresses the main purpose of revolving doors, which is to provide an airlock between inside and outside the building and it requires substantial geometric reasoning, which places it in the space and shape *content category. The complexity of coding such a multi-step response in so many countries led to this item being assessed only as full credit or no credit. For full credit, the complex geometrical reasoning showing that the maximum door opening is one sixth of the circumference needed to be followed by an accurate calculation in centimetres. The item is classified in the* formulating *process, and it draws very heavily on the* mathematisation *fundamental mathematical capability, because the real situation has to be carefully analysed and this analysis needs to be translated into geometric terms and back again at multiple points to the contextual situation of the door. As the diagram supplied in the question shows, air will pass from the outside to the inside, or vice versa, if the wall between the front and back openings is shorter than the circumference subtended by one sector. Since the sectors each subtend one third of the circumference, and there are two walls, together the walls must close at least two thirds of the circumference, leaving no more than one third for the two openings. Arguing from symmetry of front and back, each opening cannot be more than one sixth of the circumference. There is further geometric reasoning required to check that the airlock is indeed maintained if this opening length is used. The question therefore draws very heavily on the* reasoning and argument *fundamental mathematical capability.*



REVOLVING DOOR – *QUESTION 3*

The door makes 4 complete rotations in a minute. There is room for a maximum of two people in each of the three door sectors.

What is the maximum number of people that can enter the building through the door in 30 minutes?

A. 60

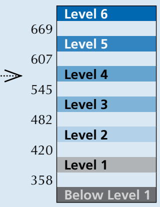
B. 180

C. 240

D. 720

Scoring

Description: *Identify information and construct an (implicit) quantitative model to solve the problem*
Mathematical content area: *Quantity*
Context: *Scientific*
Process: *Formulate*
Question format: *Simple multiple choice*
Difficulty: *561.3*



Full Credit

D. 720

No Credit

Other responses.

Missing.

Comment

Question 3 addresses a different type of challenge, involving rates and proportional reasoning, and it sits within Level 4 on the mathematics proficiency scale. In one minute, the door revolves 4 times bringing 4 × 3 = 12 sectors to the entrance, which enables 12 × 2 = 24 people to enter the building. In 30 minutes, 12 × 30 = 720 people can enter (hence, the correct answer is response option D). The question is allocated to the quantity content category *because of the way in which the multiple relevant quantities (number of people per sector [2], number of sectors per revolution [3], number of revolutions per minute [4], number of minutes [30]) have to be combined by number operations to produce the required number of persons to enter in 30 minutes. The high frequency of PISA items that involve proportional reasoning highlights its centrality to mathematical literacy, especially for students whose mathematics has reached a typical stage for 15-year-olds. Many real contexts involve direct proportion and rates, which as in this case are often used in chains of reasoning. Coordinating such a chain of reasoning requires* devising a strategy *to bring the information together in a logical sequence.*

This item also makes considerable demand on the mathematisation *fundamental mathematical capability, especially in the* formulating *process. A student needs to understand the real situation, perhaps visualising how the doors rotate, presenting one sector at a time, making the only way for people to enter the building. This understanding of the real world problem enables the data given in the problem to be assembled in the right way.*

General comment on this unit

The questions in this unit have been allocated to the *scientific* context category, even though they do not explicitly involve scientific or engineering concepts, as do many of the other items in this category. The scientific category includes items that explain why things are as they are in the real world. Question 2 is a good example of such an essentially scientific endeavour. Formal geometric proof is not required by the question, but in answering this item correctly, the highest students will have almost constructed such a proof.

■ Figure I.2.58 ■

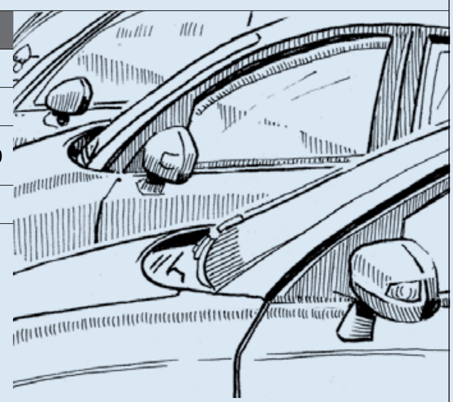
WHICH CAR?

WHICH CAR?

Chris has just received her car driving licence and wants to buy her first car.

This table below shows the details of four cars she finds at a local car dealer.

Model:	Alpha	Bolte	Castel	Dezal
Year	2003	2000	2001	1999
Advertised price (zeds)	4 800	4 450	4 250	3 990
Distance travelled (kilometres)	105 000	115 000	128 000	109 000
Engine capacity (litres)	1.79	1.796	1.82	1.783



WHICH CAR? – *QUESTION 1*

Chris wants a car that meets **all** of these conditions:

- The distance travelled is **not** higher than 120 000 kilometres.
- It was made in the year 2000 or a later year.
- The advertised price is **not** higher than 4 500 zeds.
- Which car meets Chris's conditions?

A. Alpha

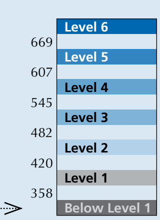
B. Bolte

C. Castel

D. Dezal

Scoring

Description: *Select a value that meets four numerical conditions/statements set within a financial context*
Mathematical content area: *Uncertainty and data*
Context: Personal
Process: Interpret
Question format: *Simple multiple choice*
Difficulty: *327.8*



Full Credit

B. Bolte.

No Credit

Other responses.

Missing.

WHICH CAR? – *QUESTION 2*

Which car's engine capacity is the smallest?

A. Alpha

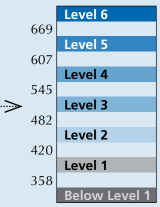
B. Bolte

C. Castel

D. Dezal

Scoring

Description: *Choose the smallest decimal number in a set of four, in context*
Mathematical content area: *Quantity*
Context: *Personal*
Process: *Employ*
Question format: *Simple multiple choice*
Difficulty: *490.9*



Full Credit

D. Dezal.

No Credit

Other responses.

Missing.

WHICH CAR? – *QUESTION 3*

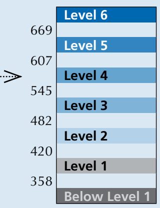
Chris will have to pay an extra 2.5% of the advertised cost of the car as taxes.

How much are the extra taxes for the Alpha?

Extra taxes in zeds: ....

Scoring

Description: *Calculate 2.5% of a value in the thousands within a financial context*
Mathematical content area: *Quantity*
Context: *Personal*
Process: *Employ*
Question format: *Constructed response manual*
Difficulty: *552.6*



Full Credit

120

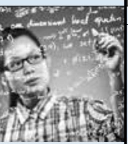
No Credit

Other responses.
- 2.5% of 4 800 zeds *[Needs to be evaluated]*.

Missing.

General comment on this unit

Because buying a car is a situation which many people face in their everyday life, all three questions have been allocated to the *personal* context category. Question 1 and Question 2 are simple multiple choice responses, and Question 3, which asks for a single number, is a constructed response item that does not require expert scoring. Question 1 has been allocated to *uncertainty and data*. The item requires knowledge of the basic row-column conventions of a table, as well as co-ordinated data-handling ability to identify where the three conditions are simultaneously satisfied. The solution also requires basic knowledge of large whole numbers, but the expert judgement is that this knowledge is unlikely to be the main source of difficulty in the item for 15-year-old students. The correct response is B: Bolte.



In contrast, Question 2 has been allocated to the *quantity* content category because it is well known that even at age 15, many students have misconceptions about the base ten and place value ideas required to order "ragged" decimal numbers. Credit is given here for response option D: Dezal.

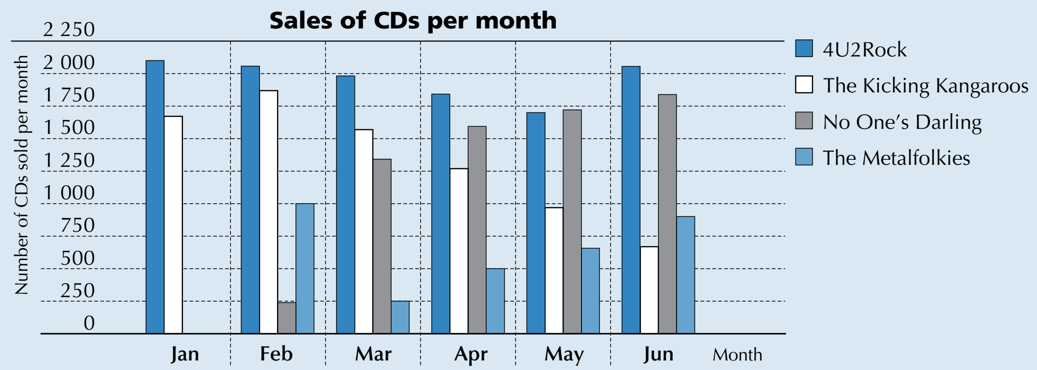
Question 3 is also allocated to the *quantity* content category because the calculation of 2.5% of the advertised cost, 120 zeds, is expected to be a much larger source of cognitive demand than identifying the correct data from the table. The difficulty for this age group in dealing with decimal numbers and percentages is reflected in the empirical results, with Question 1 being an easy item, Question 2 close to the international average and Question 3 above it.

To allocate the items to process categories, it is necessary to consider how the real world situation is involved. Items in the *formulating* category have their major demand in the transition from the real world problem to the mathematical problem. Items in the *employing* category have their major demand within the mathematical world. Items in the *interpreting* category have their major demand in using mathematical information to give a real world solution. Questions 2 and 3 are allocated to the *employing* category. This is because in both of these items, the major source of cognitive demand has been identified as being within mathematics: the concept of decimal notation and the calculation of a percentage. In Question 1, a table of data is presented, and its construction (with the identification of key variables etc.) represents a mathematisation of the real situation. The question then requires these mathematical entities as presented to be interpreted in relation to the real world constraints and situation they represent.

■ Figure I.2.59 ■

CHARTS

In January, the new CDs of the bands 4U2Rock and The Kicking Kangaroos were released. In February, the CDs of the bands No One's Darling and The Metalfolkies followed. The following graph shows the sales of the bands' CDs from January to June.



The three questions making up the unit CHARTS are all of below average difficulty in the main survey. All three items are simple multiple choice, so the demand for communication is only receptive. The unit presents a bar chart showing 6 months of sales data for music. The complication of the bar chart is that it displays four separate data series (four different music bands). Students have to read values from the graphical representation of data and draw conclusions. This is a common task type in the content category *uncertainty and data*. All three items have all been classified in the *societal* context category because it provides information about community behaviour, in this case, aggregated music choices.

CHARTS – *QUESTION 1*

How many CDs did the band *The Metalfolkies* sell in April?

A. 250

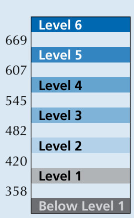
B. 500

C. 1 000

D. 1 270

Scoring

Description: *Read a bar chart*
Mathematical content area: *Uncertainty and data*
Context: *Societal*
Process: *Interpret*
Question format: *Simple multiple choice*
Difficulty: *347.7*

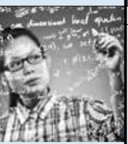


Full Credit

B. 500

No Credit

Other responses.
Missing.



Comment

Question 1, with a difficulty of 347.7, is below Level 1 on the mathematical proficiency scale, being one of the easiest tasks in the PISA 2012 item pool. It requires the student to find the bars for April, select the correct bar for the Metafolkies, and read the height of the bar to obtain the required response selection B (500). No scale reading or interpolation is required. This question is classified in the interpreting *process category.*

CHARTS – *QUESTION 2*

In which month did the band *No One's Darling* sell more CDs than the band *The Kicking Kangaroos* for the first time?

A. No month

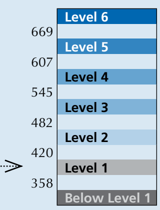
B. March

C. April

D. May

Scoring

Description: *Read a bar chart and compare the height of two bars*
Mathematical content area: *Uncertainty and data*
Context: *Societal*
Process: *Interpret*
Question format: *Simple multiple choice*
Difficulty: *415*



Full Credit

C. April.

No Credit

Other responses.

Missing.

Comment

Question 2 is a little more difficult, and lies near the bottom of Level 3 on the scale. The bars representing two bands need to be identified and the heights compared, starting from January and working through the year. No reading of the vertical scale is required. It is only necessary to make visual comparisons of adjacent bars against a very simple characteristic (which is bigger), –and to identify the correct response option C (April). In comparison with Question 1, Question 2 is a little more demanding of communication *(receptive component),* representation, *and* devising strategies, *and similar on the other fundamental mathematical capabilities. It is also classified in the* interpreting *process category.*

CHARTS – *QUESTION 5*

The manager of *The Kicking Kangaroos* is worried because the number of their CDs that sold decreased from February to June.

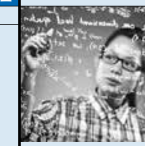
What is the estimate of their sales volume for July if the same negative trend continues?

A. 70 CDs

B. 370 CDs

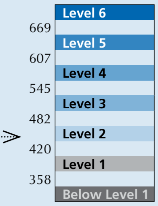
C. 670 CDs

D. 1 340 CDs



Scoring

Description: *Interpret a bar chart and estimate the number of CDs sold in the future assuming that the linear trend continues*
Mathematical content area: *Uncertainty and data*
Context: *Societal*
Process: *Employ*
Question format: *Simple multiple choice*
Difficulty: *428.2*



Full Credit

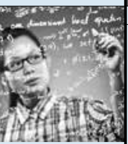
B. 370 CDs.

No Credit

Other responses.
Missing.

Comment

Question 5 requires identifying the data series for the Kangaroos band and observing the negative trend noted in the lead-in to the item stimulus. It involves some work with numbers and also an appreciation that the correct answer to choose may be an approximation to a calculated answer. There are several ways to continue the trend by one more month. A student might work out each monthly decrease and average them, which involves a lot of calculation. A student might take one fifth of the total decrease from February to June. Another student might place a ruler along the tops of the bars for the Kangaroos and find that the July bar would show something between 250 and 500. The correct response option is B (370 CDs), and the task lies in Level 2 on the mathematics scale. The question has been allocated to the Employing *process because it was judged that most students at this level are likely to take the calculation routes, and that carrying these out accurately is likely to present the greatest difficulty for the item.*

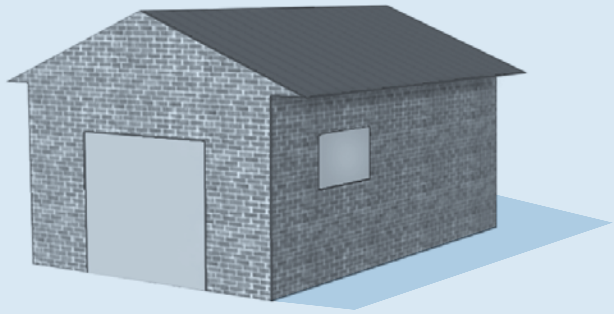


■ Figure I.2.60 ■

GARAGE

A garage manufacturer's "basic" range includes models with just one window and one door.

George chooses the following model from the "basic" range. The position of the window and the door are shown here.

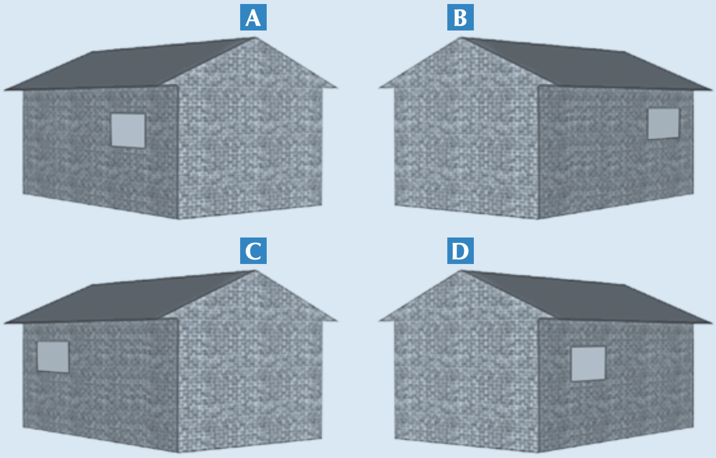


The unit GARAGE consists of two questions, both in the *space and shape* content category because they deal with spatial visualisation and reading building plans, and both in the *occupational* context category, because these questions may arise in the construction, painting or other completion of a building project. Because of the need to derive mathematical information from the diagrams, both questions require activation of the *representation* fundamental mathematical capability.

GARAGE – *QUESTION 1*

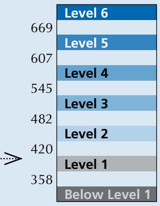
The illustrations below show different "basic" models as viewed from the back. Only one of these illustrations matches the model above chosen by George.

Which model did George choose? Circle A, B, C or D.



Scoring

Description: *Use space ability to identify a 3D view corresponding to another given 3D view*
Mathematical content area: *Space and shape*
Context: *Occupational*
Process: *Interpret*
Question format: *Simple multiple choice*
Difficulty: *419.6*



Full Credit

C. *[Graphic C]*.

No Credit

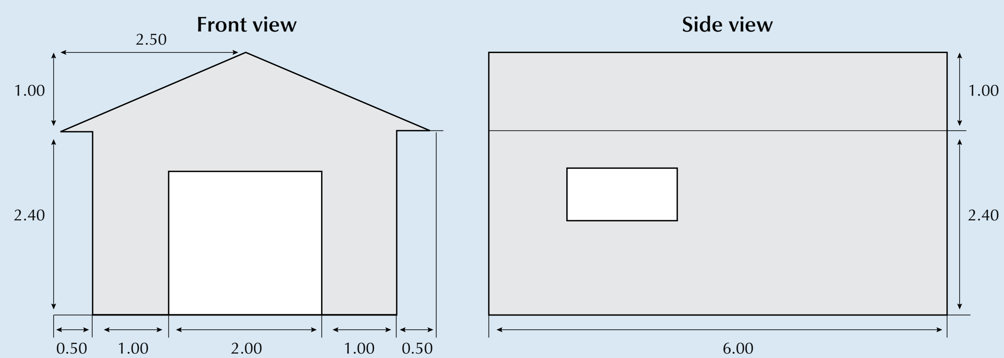
Other responses.
Missing.

Comment

Question 1 lies very close to the Level 1/Level 2 boundary on the proficiency scale. It asks students to identify a picture of a building from the back, given the view from the front. The diagrams must be interpreted in relation to the real world positioning of "from the back", so this question is classified in the interpreting *process. The correct response is C. Mental rotation tasks such as this are solved by some people using intuitive spatial visualisation. Other people need explicit reasoning processes. They may analyse the relative positions of multiple features (door, window, nearest corner), discounting the multiple choice alternatives one by one. Others might draw a bird's eye view, and then physically rotate it. This is just one example of how different students may use quite different methods to solve PISA questions: in this case explicit reasoning for some students is intuitive for others.*

GARAGE – *QUESTION 2*

The two plans below show the dimensions, in metres, of the garage George chose.



Note: Drawing not to scale.

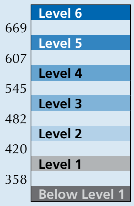
The roof is made up of two identical rectangular sections.

Calculate the total area of the roof. Show your work.

....

Scoring

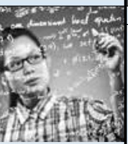
Description: *Interpret a plan and calculate the area of a rectangle using the Pythagorean theorem or measurement*
Mathematical content area: *Space and shape*
Context: *Occupational*
Process: *Employ*
Question format: *Constructed response expert*
Difficulty: *687.3*



Full Credit

Any value from 31 to 33, either showing no working at all or supported by working that shows the use of the Pythagorean theorem (or including elements indicating that this method was used) *[Units (m^2) not required]*.

- $12\sqrt{7.25}$ m^2
- $12 \times 2.69 = 32.28$ m^2
- 32.4 m^2



Partial Credit

Working shows correct use of the Pythagorean theorem but makes a calculation error or uses incorrect length or does not double roof area.

- $2.5^2 + 1^2 = 6$, $12 \times \sqrt{6} = 29.39$ *[correct use of Pythagoras theorem with calculation error].*
- $2^2 + 1^2 = 5$, $2 \times 6 \times \sqrt{5} = 26.8\ m^2$ *[incorrect length used].*
- $6 \times 2.6 = 15.6$ *[Did not double roof area].*

Working does not show use of Pythagorean theorem but uses reasonable value for width of roof (for example, any value from 2.6 to 3) and completes rest of calculation correctly.

- $2.75 \times 12 = 33$
- $3 \times 6 \times 2 = 36$
- $12 \times 2.6 = 31.2$

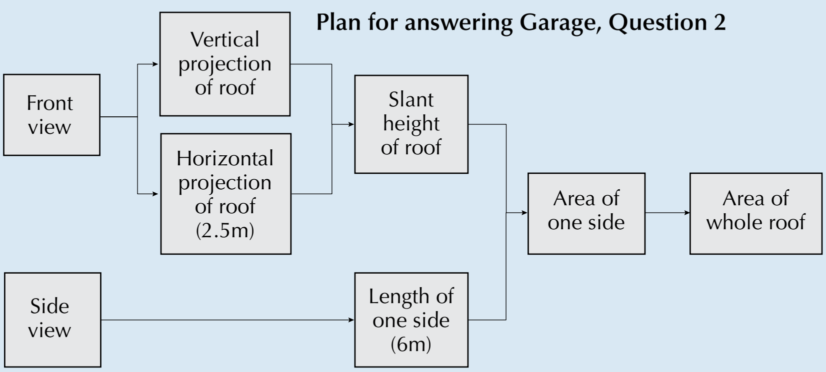
No Credit

Other responses.

- $2.5 \times 12 = 30$ *[Estimate of width of roof lies outside the acceptable range which is from 2.6 to 3].*
- $3.5 \times 6 \times 2 = 42$ *[Estimate of width of roof lies outside the acceptable range which is from 2.6 to 3].*

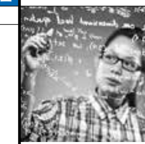
Missing.

Plan for answering Garage, Question 2



Comment

Question 2 requires complicated calculation, with multiple calls upon the mathematical diagrams, and knowing to use Pythagoras's theorem. For this reason, it has been classified in the employing process. There are multiple reasons why this item is at Level 5 for partial credit answers and at Level 6 for full credit answers. Question 2 requires a constructed response, although in this case the explanation of reasoning is only used to award partial credit for incorrect answers, rather than being scored for quality of explanation. There is high level demand for the representation *capability, in understanding and deriving exact information from the front and side views presented.* Mathematisation *is also called upon, especially in reconciling the apparent 1.0 m height of the roof from the side view with the real situation and with the front view. The devising strategies capability is called up at a high level to make a plan to get the area from the information presented. The plan above shows the basic structure of the solution. To carry out such a plan also requires careful monitoring. Future analysis of the data beyond the scope of this first report may show interesting differences between the students who score partial credit.*



Notes

1. The GDP values represent per capita GDP in 2012 at current prices, adjusted for differences in purchasing power among OECD countries.

2. It should be borne in mind, however, that the number of countries involved in this comparison is small, and that the trend line is therefore strongly affected by the particular characteristics of the countries included in the comparison.

3. Spending per student is approximated by multiplying public and private expenditure on educational institutions per student in 2012 at each level of education by the theoretical duration of education at the respective level, up to the age of 15. Cumulative expenditure for a given country is approximated as follows: let *n(0), n(1)* and *n(2)* be the typical number of years spent by a student from the age of 6 up to the age of 15 years in primary, lower secondary and upper secondary education. Let *E(0), E(1)* and *E(2)* be the annual expenditure per student in USD converted using purchasing power parities in primary, lower secondary and upper secondary education, respectively. The cumulative expenditure is then calculated by multiplying current annual expenditure *E* by the typical duration of study *n* for each level of education *i* using the following formula:

$$CE = \sum_{i=0}^{2} n(i) * E(i)$$

4. For this purpose, the respective data were standardised across countries and then averaged over the different aspects.

5. For more details, see Butler and Adams (2007).

6. For trend purposes, Dubai (UAE) and the rest of the United Arab Emirates are counted as separate economies. Dubai (UAE) implemented PISA 2009 in 2009 and the rest of the United Arab Emirates implemented PISA 2009 in 2010, as part of PISA 2009+.

7. As described in more detail in Annex A5, the annualised change takes into account the specific year in which the assessment was conducted. In the case of mathematics, this is especially relevant for the PISA 2009 assessment as Costa Rica, Malaysia and the United Arab Emirates (excluding Dubai) implemented the assessment in 2010 as part of PISA 2009+.

8. Normally, when comparing two concurrent means, the significance is indicated by calculating the ratio of the difference of the means to the standard error of the difference of the means. If the absolute value of this ratio is greater than 1.96, then a true difference is indicated with 95% confidence. When comparing two means taken at different times, with instruments that have a subset of common items, as in different PISA surveys, an extra error term, known as the link error, is introduced, and the resulting statement of significant difference is more conservative. For more details, see Annex A5.

9. By accounting for students' gender, age, socio-economic status, immigrant background and language spoken at home, the adjusted trends allow for a comparison of trends in performance assuming no change in the underlying population or the effective samples' average socio-economic status, age and percentage of girls, students with an immigrant background or students that speak a language at home that is different than the language of assessment.

10. The *PISA index of social, economic and cultural status* is unavailable for Albania in PISA 2012. Albania improved throughout its participation in PISA, but it is impossible to calculate adjusted trends for the country.

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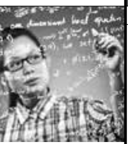

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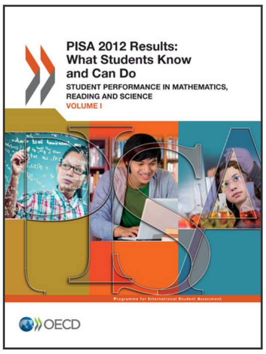

From:

PISA 2012 Results: What Students Know and Can Do (Volume I, Revised edition, February 2014)

Student Performance in Mathematics, Reading and Science

Access the complete publication at:

https://doi.org/10.1787/9789264208780-en

Please cite this chapter as:

OECD (2014), "A Profile of Student Performance in Mathematics", in *PISA 2012 Results: What Students Know and Can Do (Volume I, Revised edition, February 2014): Student Performance in Mathematics, Reading and Science*, OECD Publishing, Paris.

DOI: https://doi.org/10.1787/9789264208780-6-en