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Optimal Monetary  
and Fiscal Stabilisation  
Policies

**Klaus Adam**

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**OPTIMAL MONETARY AND FISCAL STABILISATION POLICIES**

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by **Klaus Adam**

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ABSTRACT / RESUMÉ

**Optimal monetary and fiscal stabilisation policies**

This paper studies optimal stabilisation policies under commitment when monetary policy sets nominal interest rates and fiscal policy decides on public expenditure, income tax rates, and issuance of nominal non-contingent debt. High levels of government debt adversely affect the steady state of the economy and increase aggregate volatility. The latter emerges because debt exposes the government budget to real interest rate risk and thereby induces stronger volatility of taxes and public spending. The optimal variability of fiscal deficits is found to increase with the level of government debt, while the optimal variability of nominal interest rates decreases. Overall, optimal stabilisation policy does not require annual fiscal deficits to deviate by more than 3 percentage points of GDP from their steady state value or nominal interest rates to fall all the way to zero. Only if the standard deviation of economic disturbances is two to three times larger than suggested by post-war evidence do such events occur with non-negligible probability.

*JEL Codes:* E63, E32

*Keywords:* Ramsey optimal policy; non-contingent government debt; government spending; distortionary taxes; interest rate policy

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**Politique optimale de stabilisation monétaire et budgétaire**

Cet article étudie la politique optimale de stabilisation dans des conditions telles que la politique monétaire fixe les taux d'intérêt nominaux et la politique budgétaire détermine les dépenses publiques, les taux de l'impôt sur les revenus et l'émission de la dette nominale non contingente. Un niveau élevé d'endettement public a des effets négatifs sur l'état stationnaire de l'économie et accroît la volatilité globale. Cette volatilité tient à ce que la dette expose le budget de l'État à un risque de taux d'intérêt réel et provoque donc une plus grande instabilité de l'impôt et des dépenses publiques. On constate que la variabilité optimale des déficits budgétaires s'accroît en fonction du niveau de la dette publique, contrairement à la variabilité des taux d'intérêt nominaux, qui diminue. Au total, une politique optimale de stabilisation n'exige pas que le déficit budgétaire annuel s'écarte de plus de 3 points de PIB de sa valeur à l'état stationnaire, ni que les taux d'intérêt nominaux tombent totalement à zéro. C'est seulement si l'écart type des perturbations économiques est deux à trois fois supérieur aux résultats observés depuis la fin de la guerre que de tels événements se produisent, avec une probabilité non négligeable.

*Codes JEL :* E63, E32

*Mots clés :* Politique optimale de Ramsey ; dette publique non contingente ; dépenses publiques ; distorsions fiscales ; politique de taux d'intérêt.

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## OPTIMAL MONETARY AND FISCAL STABILISATION POLICIES

Klaus Adam<sup>1</sup>

### 1. Introduction

1. The purpose of this paper is to determine optimal monetary and fiscal stabilisation policies under commitment using a stylized dynamic equilibrium model with nominal rigidities. The model considers three stabilisation instruments that are generally deemed relevant for the conduct of cyclical stabilisation policy, namely (1) monetary policy defined as control of the short-term nominal interest rate, (2) fiscal policy taking the form of spending decisions on public goods, and (3) fiscal policy determining whether to finance public expenditure via taxes or the issuance of government debt. The main innovation with respect to most of the existing literature is that the present paper treats government spending as an endogenous decision variable which can be used for stabilisation purposes, rather than as an exogenous stochastic process, as is common in the public finance literature (Schmitt-Grohe and Uribe, 2004).

2. The economic model in this paper is based on earlier work by Adam and Billi (2008 and 2009), but extended to allow for distortionary income taxation and for government debt dynamics at the same time. Moreover, this paper considers fully optimal stabilisation policies while the earlier work was concerned with time-consistent (or discretionary) policymaking and the design of institutions that would allow overcoming the distortions generated by the lack of commitment.

3. The present paper determines the optimal monetary and fiscal stabilisation policies under commitment in response to a range of economic disturbances. Specifically, it considers adverse technology shocks, shocks that increase the price elasticity of product demand (deflationary demand shocks), and discount factor shocks that cause a temporary desire of households to shift consumption into the future (thrift shocks). With respect to adverse technology shocks specifications that go beyond traditional low order autoregressive shock specifications are considered. In particular, we allow for ARMA(1,5) technology processes which allow to specify hump shaped shock impulses which include anticipated components (news shocks). The flexibility this offers is used to define a hump-shaped adverse and persistent technology shock, which is intended to capture some of the implications of the recent financial crisis. Interestingly, the optimal stabilisation response to persistent adverse technology shocks is often to reduce government spending and income taxes. Reduced taxes help to limit the drop in labour supply and private consumption, while the cut in government spending helps containing the increase in government debt. The latter is desirable because the interest burden on government debt has to be financed by

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1. Klaus Adam is Professor of Economics at Mannheim University This is one of the background papers for the OECD's project on counter-cyclical economic policy. The main paper was issued as the *OECD Economics Department Working Paper* No. 760. Thanks go to Balázs Égert, Jorgen Elmeskov, Peter Hoeller, Oliver Röhn, Jean-Luc Schneider and Douglas Sutherland for valuable comments and suggestions.

distortionary taxation, so that higher debt levels would imply higher taxes and lower consumption levels in the long run.

4. Besides determining the optimal stabilisation response to shocks, the paper also assesses to what extent optimal stabilisation policies would be constrained by the presence of a zero floor on nominal interest rates or whether fiscal deficits optimally move by more than 3% points of GDP from their steady state value.<sup>2</sup> In addition, the effects of differences in the initial government debt to GDP ratio for the conduct of optimal stabilisation policy are assessed.

5. The paper shows for the baseline calibration that optimal stabilisation policies do not imply negative nominal interest rates or swings in the annual fiscal balance by more than 3 percentage points of GDP from its baseline value, provided initial government debt ratios are in the range between zero and 60% of GDP and shocks are in their normal range. Higher government debt levels, however, expose the government budget to real interest rate risk because movements in real interest rates have greater budgetary implications whenever debt levels are higher. Government deficits therefore optimally become more variable the higher is the initial level of government debt. This effect has adverse consequences for the volatility of taxes and thereby for the volatility of consumption and labour supply. Moreover, since monetary stabilisation policy affects the economy through changes in the real interest rate, monetary stabilisation policy itself is a source of real interest rate risk for the government budget. High debt levels therefore make monetary stabilisation policy less desirable (or more costly). As a result, nominal interest rate variability optimally decreases with the initial debt level. Higher government debt levels thus imply that under optimal policy the zero lower bound on nominal interest rates constrains optimal monetary policy less but that the swings in the government balance increase in size. Overall, however, nominal interest rates remain virtually always positive and the variability of budget balances remains almost always within a 3% bound from its steady state value.

6. The robustness of these latter findings is assessed. While increasing the degree of nominal rigidity or increasing the elasticity of labour supply will not overturn the baseline findings, increasing the standard deviation of economic disturbances by a factor of two to three causes the zero floor on nominal interest rates to become binding with non-negligible likelihood and fiscal deficits to display swings that exceed 3% of GDP.

7. The paper is organized as follows. Section 2 describes the economic model and derives the implementability conditions summarizing optimal private sector behaviour. After determining the first best allocation in section 3, section 4 describes the optimal policy problem and the numerical solution strategy. It also derives analytical results regarding the steady state outcomes. Section 6 determines the impulse responses of the economy to supply and demand disturbances and the optimal monetary and fiscal stabilisation policies. Section 7 discusses the implications of alternative initial debt levels for the steady state outcome and the optimal stabilisation policies. The paper then turns to the issue of whether monetary policy is constrained by the existence of a zero lower bound on nominal interest rates and of whether government deficits deviate by more than 3% from their baseline values. Section 8 discusses the results for the baseline calibration and section 9 assesses the robustness of these findings to a range of modelling assumptions. Section 10 briefly discusses the implications of lack of commitment and a conclusion summarizes.

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2. The latter is of interest because the Stability and Growth Pact requires European Union countries to keep their annual fiscal deficits below a level of 3% of GDP.

## 2. Description of the economic model

8. The next sections adapt the sticky price model presented in Adam and Billi (2008) to the more relevant setting with distortionary income taxes and credible repayment promises for government debt. Besides presenting the model ingredients, this section derives the implementability constraints characterizing optimal private sector behaviour, *i.e.*, derives the optimality conditions determining households' consumption and labour supply decisions and firms' price setting decisions.

### 2.1. Private sector

There is a continuum of identical households with preferences given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t e^{\varepsilon_t} u(c_t, h_t, g_t) \right] \quad (1)$$

where  $c_t$  denotes consumption of an aggregate consumption good,  $h_t \in [0,1]$  denotes labour supply, and  $g_t$  public goods provision by the government in the form of aggregate consumption goods and  $\varepsilon_t$  is a shock to the discount factor. Throughout the paper we impose the following conditions.

**Condition 1**  $u(c, h, g)$  is separable in  $c$ ,  $h$ , and  $g$ , and  $u_c > 0$ ,  $u_{cc} < 0$ ,  $u_h < 0$ ,  $u_{hh} \leq 0$ ,  $u_g > 0$ ,  $u_{gg} < 0$ .

Each household produces a differentiated intermediate good. Demand for that good is given by

$$y_t d \left( \frac{\tilde{P}_t}{P_t} \right)$$

where  $y_t$  denotes (private and public) demand for the aggregate good,  $\tilde{P}_t$  is the price of the good produced by the household, and  $P_t$  is the price of the aggregate good. The demand function  $d(\cdot)$  satisfies

$$\begin{aligned} d(1) &= 1 \\ \frac{\partial d}{\partial (\tilde{P}_t/P_t)}(1) &= \eta_t \end{aligned}$$

where  $\eta_t \in (-\infty, -1)$  is the price elasticity of demand for the differentiated goods. This elasticity is time-varying and follows an exogenous stochastic process with unconditional mean  $\eta < -1$ . The time varying demand elasticity induces fluctuations in the monopolistic mark-up charged by firms: a high value for  $\eta_t$ , for example, will have deflationary effects because it indicates that consumers have become more price sensitive, which induces firms to reduce their prices. Importantly, the previously stated assumptions about the demand function are consistent with optimizing individual behaviour when private and public consumption goods are Dixit-Stiglitz aggregates of the goods produced by different households.



The household chooses  $\tilde{P}_t$  and then hires the necessary amount of labour  $\tilde{h}_t$  to satisfy the resulting product demand, *i.e.*,

$$z_t \tilde{h}_t = y_t d \left( \frac{\tilde{P}_t}{P_t} \right) \quad (2)$$

where  $z_t$  is an aggregate technology shock which follows an exogenous stochastic process and has unconditional mean  $z = 1$ . Following Rotemberg (1982) we introduce sluggish nominal price adjustment by assuming that firms face quadratic resource costs for adjusting prices according to

$$\frac{\theta}{2} \left( \frac{\tilde{P}_t}{\tilde{P}_{t-1}} - 1 \right)^2$$

where  $\theta > 0$ . The flow budget constraint of the household is then given by

$$P_t c_t + B_t = R_{t-1} B_{t-1} + P_t \left[ \frac{\tilde{P}_t}{P_t} y_t d \left( \frac{\tilde{P}_t}{P_t} \right) - w_t \tilde{h}_t - \frac{\theta}{2} \left( \frac{\tilde{P}_t}{\tilde{P}_{t-1}} - 1 \right)^2 \right] + P_t w_t h_t (1 - \tau_t) \quad (3)$$

where  $B_t$  denotes nominal government bonds that pay  $B_t R_t$  in period  $t+1$ ,  $w_t$  is the real wage paid in a competitive labour market, and  $\tau_t$  is a labour income tax.<sup>3</sup>

9. Although nominal government bonds are the only available financial instrument, adding complete financial markets for claims between households would make no difference for the analysis: since households have identical incomes in a symmetric price setting equilibrium, there exists no incentive to actually trade such claims. One should note that the model also abstracts from money holdings. This should be interpreted as the “cashless limit” of an economy with money (Woodford, 1998). Money thus imposes only a lower bound on the gross nominal interest rate, *i.e.*,

$$R_t \geq 1 \quad (4)$$

in each period. Abstracting from money entails that seigniorage revenues generated in the presence of positive nominal interest rates are ignored. Given the size of these revenues in relation to GDP in industrialized economies, this does not seem to be an important omission for the analysis conducted here.<sup>4</sup>

Finally, a no Ponzi scheme constraint on household behaviour is imposed, *i.e.*,

$$\lim_{j \rightarrow \infty} E_t \left[ \left( \prod_{i=0}^{t+j-1} \frac{1}{R_i} \right) B_{t+j} \right] \geq 0 \quad (5)$$

---

3. Considering income or consumption taxes, instead, would be equivalent to a labour income tax plus a lump sum tax (on profits).

4. As emphasized by Leeper (1991), however, seigniorage may nevertheless be an important marginal source of revenue.

The household's problem consists of choosing state-contingent processes  $\{c_t, h_t, \tilde{h}_t, \tilde{P}_t, B_t\}_{t=0}^{\infty}$  so as to maximize (1) subject to (2), (3) and (5) taking as given  $\{y_t, P_t, w_t, R_t, g_t, \tau_t\}_{t=0}^{\infty}$  as well as the exogenous stochastic disturbance processes  $\{\eta_t, z_t, \varepsilon_t\}_{t=0}^{\infty}$ .

Using equation (2) to substitute  $\tilde{h}_t$  in (3) and letting the Lagrange multiplier on (3) be given by  $\beta^t \lambda_t e^{\varepsilon_t} / P_t$ , the first order conditions of the household's problem are then equations (2), (3), and (5) holding with equality and also

$$u_{c,t} = \lambda_t \quad (6)$$

$$u_{h,t} = -\lambda_t w_t (1 - \tau_t) \quad (7)$$

$$e^{\varepsilon_t} \lambda_t = \beta E_t \left[ e^{\varepsilon_{t+1}} \lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \right] \quad (8)$$

$$0 = e^{\varepsilon_t} \lambda_t \left( y_t d(r_t) + r_t y_t d'(r_t) - \frac{w_t}{z_t} y_t d'(r_t) - \theta \left( \Pi_t \frac{r_t}{r_{t-1}} - 1 \right) \frac{\Pi_t}{r_{t-1}} \right) \\ + \beta \theta E_t \left[ e^{\varepsilon_{t+1}} \lambda_{t+1} \left( \frac{r_{t+1}}{r_t} \Pi_{t+1} - 1 \right) \frac{r_{t+1}}{r_t^2} \Pi_{t+1} \right]$$

where

$$r_t = \frac{\tilde{P}_t}{P_t}$$

denotes the relative price. Furthermore, there is the transversality constraint

$$\lim_{j \rightarrow \infty} E_t \left( \beta^{t+j} e^{\varepsilon_{t+j}} u_{c,t+j} \frac{B_{t+j}}{P_{t+j}} \right) = 0 \quad (9)$$

which has to hold at each contingency.

## 2.2. Government

10. The government consists of two authorities. First, there is a monetary authority which controls the nominal interest rates on short-term nominal bonds through open market operations. Since we consider a cashless limit economy, the open market operations are infinitesimally small allowing us to abstract from seigniorage revenue. Second, there is a fiscal authority deciding on the level of government spending, labour income taxes and on debt policy. The government provides public goods  $g_t$  and pays interest on

outstanding debt. The level of public goods provision is a choice variable of the government. The government finances current expenditure by labour income taxes and by issuing new debt so that its budget constraint is given by

$$\frac{B_t}{P_t} + \tau_t w_t h_t = g_t + \frac{R_{t-1}}{\Pi_t} \frac{B_{t-1}}{P_{t-1}} \quad (10)$$

11. The government can credibly commit to repay its debt. The government debt instruments are assumed to be nominal and not state-contingent, consistent with the type of debt typically issued by governments around the globe. These features imply, however, that monetary policy decisions affect the government budget through two channels: first, the nominal interest rate policy of the monetary authority influences directly the nominal return the government has to offer on its instruments; second, nominal interest rate decisions also affect the price level and thereby the real value of outstanding government debt. Thus, to the extent that monetary policy can affect the real interest rate or the price level, it will affect the government budget, as is the case in Diaz-Gimenez *et al.* (2008). In what follows we assume that government debt and tax policies are such that the no-Ponzi constraint (5) and the transversality constraint (9) are both satisfied.

### 2.3. Rational expectations equilibrium

12. In a symmetric equilibrium the relative price is given by  $r_t = 1$  for all  $t$ . The private sectors' optimality conditions can then be condensed into a (non-linear) Phillips curve

$$e^{\varepsilon_t} u_{c,t} (\Pi_t - 1) \Pi_t = e^{\varepsilon_t} \frac{u_{c,t} z_t}{\theta} h_t \left( 1 + \eta_t + \frac{u_{h,t}}{u_{c,t} (1 - \tau_t)} \frac{\eta_t}{z_t} \right) + \beta E_t \left[ e^{\varepsilon_{t+1}} u_{c,t+1} (\Pi_{t+1} - 1) \Pi_{t+1} \right] \quad (11)$$

and a consumption Euler equation

$$e^{\varepsilon_t} u_{c,t} = \beta E_t \left[ e^{\varepsilon_{t+1}} u_{c,t+1} \frac{R_t}{\Pi_{t+1}} \right] \quad (12)$$

Using (6) and (7) and defining  $b_t = \frac{B_t}{P_t}$ , the government budget constraint can be expressed as

$$b_t - \frac{\tau_t}{1 - \tau_t} \frac{u_{h,t}}{u_{c,t}} h_t = g_t + \frac{R_{t-1}}{\Pi_t} b_{t-1} \quad (13)$$

**Definition 1** (Rational Expectations Equilibrium) *Given the initial outstanding debt level  $(R_{-1} b_{-1})$ , a Rational Expectations Equilibrium (REE) consists of a sequence of government policies  $\{R_t \geq 1, \tau_t, g_t, b_t\}_{t=0}^{\infty}$  and private sector choices  $\{c_t, h_t, \Pi_t\}_{t=0}^{\infty}$  satisfying equations (11) and (12), the market clearing condition*

$$c_t + \frac{\theta}{2} (\Pi_t - 1)^2 + g_t = z_t h_t \quad (14)$$

and the government budget constraint (13), as well as the no-Ponzi constraint (5) and the transversality condition (9).

### 3. First best allocation

13. The first best allocation, which takes into account only household preferences and the constraints imposed by the production technology, satisfies

$$u_{g,t} = u_{c,t} = -\frac{u_{h,t}}{z_t}$$

14. It thus turns out to be optimal to equate the marginal utilities of private and public consumption to the marginal disutility of work where the latter is scaled by labour productivity. This simple allocation rule is optimal because it is equally costly to produce the public and the private consumption goods.

### 4. Optimal monetary and fiscal policy

15. This section describes the monetary and fiscal policy problem. It is important to note that – due to the existence of a number of important economic distortions – policy can generally not achieve the welfare maximizing allocation determined in the previous section. First, market power by firms generally implies that wages fall short of their marginal product, so that labour supply and therefore output is too low relative to the optimal allocation.<sup>5</sup> Second, the requirement to finance government expenditure and interest payments on outstanding government debt with distortionary income taxes additionally depresses labour supply and output. Third, the presence of nominal rigidities may prevent the price system from providing the appropriate scarcity signals. Monetary and fiscal policy will seek to minimize the effects of all these distortions. As will be seen below, this will involve reducing government consumption below its first best level so as to reduce the adverse labour supply consequences of income taxes.

16. The optimal policy problem (Ramsey problem) which takes into account the existence of all these distortions is given by

$$\begin{aligned} \max_{\{c_t, h_t, \Pi_t, R_t \geq 1, \tau_t, g_t, B_t / P_t\}_{t=0}^{\infty}} & E_0 \left[ \sum_{t=0}^{\infty} \beta^t e^{\varepsilon_t} u(c_t, h_t, g_t) \right] \\ \text{s. t.:} & \text{Equations (11), (12), (13), (14) for all } t \\ & R_{-1} b_{-1} \text{ given} \end{aligned} \quad (15)$$

---

5. This assumes non-negative income tax rates, as are required when government debt is non-negative.

The Lagrangian of the problem is

$$\begin{aligned}
& \max_{\{c_t, h_t, \Pi_t, R_t \geq 1, \tau_t, g_t, b_t\}_{t=0}^{\infty}} \min_{\{\gamma_t^1, \gamma_t^2, \gamma_t^3, \gamma_t^4\}_{t=0}^{\infty}} \\
& E_0 \left[ \begin{aligned}
& \sum_{t=0}^{\infty} \beta^t e^{\varepsilon_t} u(c_t, h_t, g_t) \\
& + \beta^t \gamma_t^1 \left( e^{\varepsilon_t} u_{c,t} (\Pi_t - 1) \Pi_t - e^{\varepsilon_t} \frac{u_{c,t} z_t}{\theta} h_t \left( 1 + \eta_t + \frac{u_{h,t}}{u_{c,t} (1 - \tau_t)} \frac{\eta_t}{z_t} \right) \right. \\
& \quad \left. - \beta e^{\varepsilon_{t+1}} u_{c,t+1} (\Pi_{t+1} - 1) \Pi_{t+1} \right) \\
& + \beta^t \gamma_t^2 \left( e^{\varepsilon_t} \frac{u_{c,t}}{R_t} - \beta e^{\varepsilon_{t+1}} \frac{u_{c,t+1}}{\Pi_{t+1}} \right) \\
& + \beta^t \gamma_t^3 \left( z_t h_t - c_t - \frac{\theta}{2} (\Pi_t - 1)^2 - g_t - x \right) \\
& + \beta^t \gamma_t^4 \left( b_t - \frac{\tau_t}{1 - \tau_t} \frac{u_{h,t}}{u_{c,t}} h_t - g_t - \frac{R_{t-1}}{\Pi_t} b_{t-1} \right)
\end{aligned} \right] \quad (16)
\end{aligned}$$

Assuming that the exogenous processes  $\eta_t$  and  $z_t$  follow Markov processes, one can recursively the dual of the Lagrangian by using the exogenous shocks and the variables  $R_{t-1} b_{t-1}$  and  $\lambda_{1,t-1}$  as the state variables:

$$\begin{aligned}
V(R_{t-1} b_{t-1}, \gamma_{t-1}^1, \gamma_{t-1}^2, \eta_t, z_t) &= \min_{(\gamma_t^1, \gamma_t^2, \gamma_t^3, \gamma_t^4)} \max_{(c_t, h_t, \Pi_t, R_t \geq 1, \tau_t, g_t, b_t)} \\
& \left( \begin{aligned}
& e^{\varepsilon_t} u(c_t, h_t, g_t) \\
& + \gamma_t^1 e^{\varepsilon_t} \left( u_{c,t} (\Pi_t - 1) \Pi_t - \frac{u_{c,t} z_t}{\theta} h_t \left( 1 + \eta_t + \frac{u_{h,t}}{u_{c,t} (1 - \tau_t)} \frac{\eta_t}{z_t} \right) \right) \\
& - \gamma_{t-1}^1 e^{\varepsilon_t} u_{c,t} (\Pi_t - 1) \Pi_t \\
& + \gamma_t^2 e^{\varepsilon_t} \frac{u_{c,t}}{R_t} - \gamma_{t-1}^2 e^{\varepsilon_t} \frac{u_{c,t}}{\Pi_t} \\
& + \gamma_t^3 \left( z_t h_t - c_t - \frac{\theta}{2} (\Pi_t - 1)^2 - g_t - x \right) \\
& + \gamma_t^4 \left( b_t - \frac{\tau_t}{1 - \tau_t} \frac{u_{h,t}}{u_{c,t}} h_t - g_t - \frac{R_{t-1}}{\Pi_t} b_{t-1} \right) \\
& + \beta e^{\varepsilon_{t+1}} E_t [V(R_t b_t, \gamma_{t-1}^1, \gamma_{t-1}^2, \eta_{t+1}, z_{t+1})]
\end{aligned} \right) \quad (17)
\end{aligned}$$

s.t. :

$$\eta_{t+1} = \mu_\eta + \rho_\eta \eta_t + \alpha_{\eta,0} \varepsilon_{\eta,t+1} + \alpha_{\eta,1} \varepsilon_{\eta,t} + \alpha_{\eta,2} \varepsilon_{\eta,t-1} + \alpha_{\eta,3} \varepsilon_{\eta,t-2} + \alpha_{\eta,4} \varepsilon_{\eta,t-3} + \alpha_{\eta,5} \varepsilon_{\eta,t-4}$$

$$z_{t+1} = \mu_z + \rho_z z_t + \alpha_{z,0} \varepsilon_{z,t+1} + \alpha_{z,1} \varepsilon_{z,t} + \alpha_{z,2} \varepsilon_{z,t-1} + \alpha_{z,3} \varepsilon_{z,t-2} + \alpha_{z,4} \varepsilon_{z,t-3} + \alpha_{z,5} \varepsilon_{z,t-4}$$

$$\varepsilon_{t+1} = \rho_\varepsilon \varepsilon_t + \alpha_{\varepsilon,0} \varepsilon_{\varepsilon,t+1} + \alpha_{\varepsilon,1} \varepsilon_{\varepsilon,t} + \alpha_{\varepsilon,2} \varepsilon_{\varepsilon,t-1} + \alpha_{\varepsilon,3} \varepsilon_{\varepsilon,t-2} + \alpha_{\varepsilon,4} \varepsilon_{\varepsilon,t-3} + \alpha_{\varepsilon,5} \varepsilon_{\varepsilon,t-4}$$

$$\gamma_{-1}^1 = \gamma_{-1}^2 = 0$$

$$R_{-1} b_{-1} \text{ given}$$

17. Note that a fairly general ARMA(1,5) specification is allowed for the shock processes in (17). The shock innovations  $\varepsilon_{\eta,t}, \varepsilon_{z,t}, \varepsilon_{\varepsilon,t}$  are iid normal independent random variables with standard deviation  $\sigma_\eta, \sigma_z$  and  $\sigma_\varepsilon$ , respectively. The ARMA(1,5) specification allows for standard autoregressive shock processes ( $\rho_i > 0, \alpha_{i,0} > 0$ , and  $\alpha_{i,j} = 0$  for all  $j > 0$ ), as well as the presence of “news” shocks, *i.e.* shocks that are anticipated in advance ( $\rho_i = 0, \alpha_{i,0} = 0$ , and  $\alpha_{i,j} \neq 0$  for some  $j > 0$ ). Consider the

case of technology shocks, for example, and suppose that  $\alpha_{z,j} = 0$  for all  $j$ , except that  $\alpha_{z,4} = 1$ . Then the productivity disturbance  $\varepsilon_{z,t}$  will affect productivity only in quarter  $t + 4$ , but has no implications for productivity up until then. Since agents are assumed to know the current shocks, the productivity shock  $\varepsilon_{z,t}$  represents news in period  $t$  about productivity conditions in period  $t + 4$ . The combination of news and standard autoregressive components ( $\rho_i > 0$  and  $\alpha_{i,j} \geq 0$  for some or all  $j$ ) will allow for general hump shaped shock processes and this flexibility in the shock specification will be exploited later on.

#### 4.1. Numerical solution strategy

18. The recursive optimization problem (17) can be brought into the standard format used by the CompEcon toolbox of Miranda and Fackler (2002). This toolbox is used to numerically compute the Ramsey steady state (analytic results are provided in the next section) and then to quadratically approximate the objective function in (17) around its steady state. The resulting linear quadratic optimization problem is solved using the `lqapprox` routine provided by the toolbox and determine linearly approximate optimal equilibrium dynamics. The impulse responses and the optimal stabilisation policies that are shown in the remaining part of the paper assume that the economy starts at a Ramsey steady state before the disturbance hits. This implies that initially and absent any shocks, the policymakers have no incentive to engage in “surprise measures”, as would be optimal when determining time-zero optimal Ramsey policy. Specifically, it implies that the monetary authority does not try to reduce the real value of outstanding nominal government debt through a temporary increase in the inflation rate.

#### 4.2. Monetary and fiscal policy in steady state: analytical results

19. We now consider a Ramsey steady state with a constant government debt level  $b$ . The implied steady state government deficit is then equal to zero, if one abstracts from economic growth. Taking into account positive growth rates, however, the steady state deficit to GDP ratio in the economy with growth is given by the following relation

$$\frac{d}{y} = g \frac{b}{y}$$

where  $g$  denotes the growth rate of the economy,  $b/y$  the debt to GDP ratio, and  $d/y$  the deficit to GDP ratio. Thus, assuming an annual growth rate of 2% and a debt to GDP ratio of 60%, the steady state deficit equals 1.2% of GDP. Given that the steady state outcome is used as the baseline and results on optimal stabilisation policy are presented in terms of deviations from this baseline, it is important to keep this aspect in mind when interpreting the results.

From the Euler equation (12) follows that constant consumption implies a constant real interest rate given by

$$\frac{R_{t-1}}{\Pi_t} = \beta^{-1}$$

so that the government budget constraint implies

$$\tau_t w_t h_t = g_t + \tilde{x} \tag{18}$$

with  $\tilde{x}$  given by

$$\tilde{x} = -(1 - \beta^{-1})b$$

and denoting the interest rate payments on outstanding government debt. From an economic point of view, interest payments involve just income redistribution, but in an environment with distortionary taxation such redistribution is costly to provide. It follows from the appendix that the Ramsey steady state satisfies

$$\Pi = 1 \quad (19)$$

$$R = \beta^{-1} \quad (20)$$

Equation (19) shows that it is optimal to implement price stability in the absence of shocks. This holds independently of the level of outstanding government debt and shows that it is suboptimal to use inflation in steady state with the objective to reduce the real value of outstanding government debt. Clearly, this does not imply that inflation is also always equal to zero following economic disturbances. Moreover, if for some reason the central bank were to target a positive inflation rate on average, then the results in Ascari and Ropele (2007) suggest that the model behaviour in response to shocks will be close to the one implied by targeting zero inflation, provided the alternatively targeted inflation rate remains in a range not too far from zero, e.g., in the 1-2% range per annum. Equation (20) gives the nominal interest rate consistent with price stability. Since  $\beta < 1$ , nominal interest rates are positive.

Appendix A.1. also shows that is optimal to have

$$-u_h < u_g \quad (21)$$

Equation (21) demonstrates that it is optimal to reduce public spending to a level below that suggested by consumer preferences and technological feasibilities, *i.e.*, below the first best allocation determined in section 3. The economic rationale for restraining spending on public goods provision can be seen from the following equation which is derived in Appendix A.1:

$$-u_h = \left( \frac{1 + \eta}{\eta} - \frac{g + \tilde{x}}{h} \right) u_c \quad (22)$$

It shows that there exists a wedge between the marginal utility of (private) consumption and the disutility of labour.<sup>6</sup> This wedge is due to the monopoly power of firms, which leads to the price mark-up  $\frac{1+\eta}{\eta}$ , and due to the need to finance public expenditure and interest rate payments through distortionary taxation. Reducing public spending below its first best level reduces the required labour tax rates and therefore helps reducing the wedge between the marginal utility of private consumption and the marginal utility of leisure and ultimately increases welfare. These arguments also suggest that in an economy with a higher stock of real government debt, *i.e.* a higher level for  $\tilde{x}$ , there are stronger incentives to reduce public consumption below its first best level (*ceteris paribus*) because taxes are high already due to a high interest rate burden.

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6. This is true whenever  $\tilde{x} > 0$ , *i.e.*, whenever government debt is non-negative.

## 5. Model calibration

20. We assume the following preference specification, which satisfies condition 1 and is consistent with balanced growth,

$$u(c_t, h_t, g_t) = \log(c_t) - \omega_h \frac{h_t^{1+\varphi}}{1+\varphi} + \omega_g \log(g_t) \quad (23)$$

with  $\omega_h > 0$ ,  $\omega_g \geq 0$  and  $\varphi \geq 0$  denoting the inverse of the Frisch labour supply elasticity.

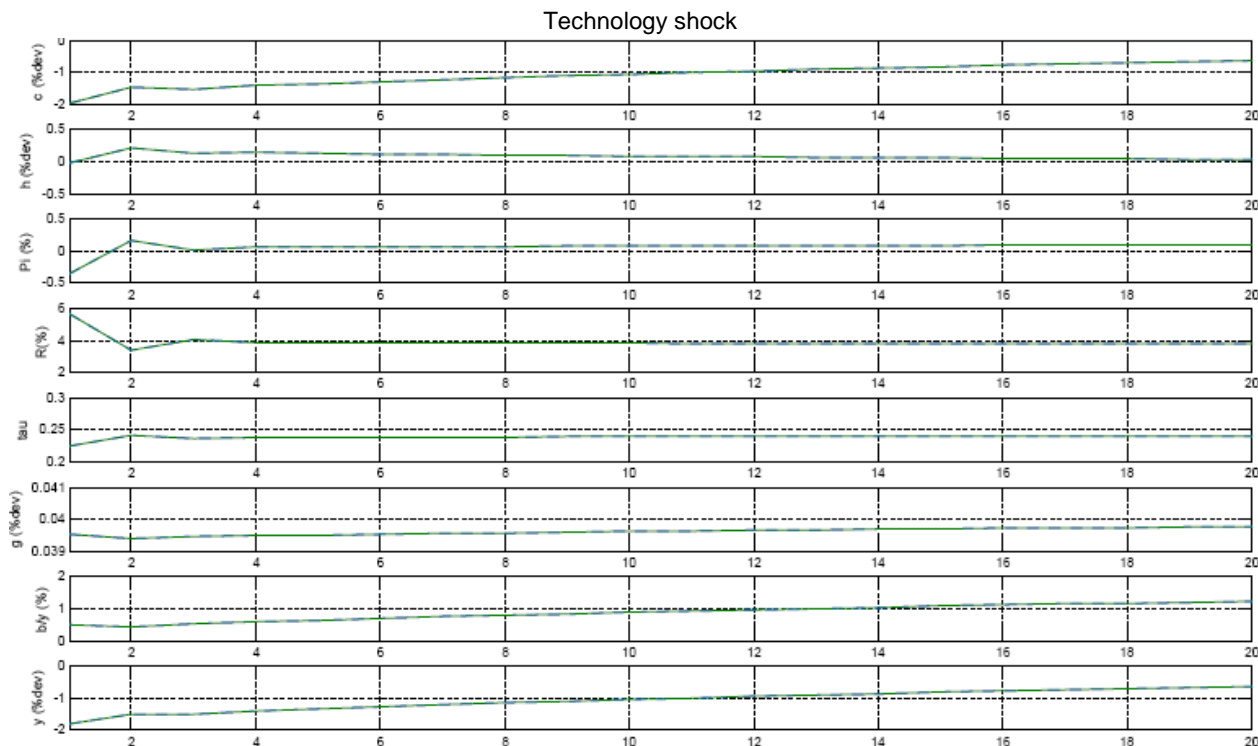
The calibration of the model is summarized in Table 1, following Adam and Billi (2008). The quarterly discount factor is chosen to match the average ex-post US real interest rate, 3.5%, during the period 1983:1-2002:4. The value for the elasticity of demand implies a gross mark-up equal to 1.2. The elasticity of labour effort is assumed to be one ( $\varphi = 1$ ) and the values of  $\omega_h$  and  $\omega_g$  are chosen such that in the Ramsey steady state without government debt, agents work 20% of their time and it is optimal to spend 20% of total output on public goods. Appendix A.2 provides details how the parameters have to be chosen to achieve this. The price stickiness parameter is selected such that the log-linearized version of the Phillips curve (11) is consistent with the estimates of Sbordone (2002), as in Schmitt-Grohé and Uribe (2004).

## 6. Optimal stabilisation policies

21. This section uses the setup developed above to determine the optimal stabilisation responses of monetary and fiscal policy to various economic disturbances. We first consider negative supply shocks (a persistent drop in aggregate productivity) and then demand side shocks. On the demand side we consider two scenarios: a deflationary demand shock resulting from the fact that consumers' product demand becomes temporarily more price sensitive (the price elasticity  $\eta_t < 0$  falls temporarily below its steady state value) and a negative demand shock resulting from consumers becoming more thrifty (the discount factor temporarily increases).



Figure 1. Impulse response to a negative technology shock



### 6.1. Negative supply shocks

22. The aim of this section is to study optimal stabilisation policy in response to negative technology shocks. We start by considering a standard technology shock process as in Adam and Billi (2008) and thereafter consider a shock that replicates more closely the real effects of the recent economic and financial crisis.

#### 6.1.1. Standard technology shock process

23. To quantify the economic effects of technology changes the AR(1) specification for technology from Adam and Billi (2008) is employed who set  $\rho_z = 0.96$  and  $\sigma_z = 0.006$ .<sup>7</sup> We consider the effects of a negative three standard deviation innovation to technology, which implies that technology drops on impact by approximately 1.8% points below its trend.<sup>8</sup> It then gradually recovers over time. The impulse responses under optimal stabilisation policy are shown in Figure 1. The responses of consumption ( $c$ ), hours ( $h$ ), government spending ( $g$ ), and GDP ( $y$ ) are expressed in per cent deviation from the pre-shock steady state values.<sup>9</sup> Inflation ( $\pi$ ) and interest rates ( $R$ ) are expressed in annualized percentage rates. Taxes ( $\tau$ ) are expressed in per cent and the government debt variable in terms of per cent of GDP

7. This requires setting  $\alpha_{z,0} = 1$  and  $\alpha_{z,i} = 0$  for  $i > 0$ .

8. The trend growth of technology is an unmodeled constant.

9. These are  $c = 0.16$ ,  $h = 0.2$ ,  $g = 0.04$ ,  $y = 0.2$ .

( $b/y$ ). The initial steady state tax level is 24% and the initial debt level is set equal to zero. These conventions apply to all figures in this paper unless otherwise stated.

24. From the second period onwards private and public consumption as well as total output fall persistently and roughly in proportion to the technology shock. In the first period, however, these variables fall by more. To avoid an even stronger drop of output and consumption on impact, government spending is reduced slightly less proportionately in the first period and tax rates temporarily fall. Since real wages drop (relative to the unmodelled growth trend) following the technology shock, these policy measures give rise to an increase in the debt to GDP ratio of about 0.5% in the 4 quarters following the shock and by a total of about 1.1% after 5 years. Hours worked actually increase slightly following the shock. This is due to the negative wealth effect of a negative productivity shock: as consumers feel poorer they consume less consumption goods and less leisure, *i.e.*, work more. Regarding nominal variables, inflation stays positive throughout the sample, except on impact where it is slightly negative, and nominal interest rates never reach the zero floor.

25. In summary, the real variables (private consumption and output) react roughly proportionately to aggregate technology, the budget balance deteriorates by about 0.5% of GDP because tax income falls and public consumption is cut more sluggishly than suggested by the drop in technology. Nominal interest rates remain bounded away from the zero floor.

26. It might come as a surprise that it is optimal to reduce government spending following the drop in output. This is so because production is not very efficient, so that it is also not a good time to produce goods for government consumption. Moreover, not cutting government spending would lead to a stronger debt increase. Since interest payments on debt have to be financed by distortionary taxation, higher debt levels would imply higher taxes and lower consumption levels (private and public) in the long run. To avoid these adverse long-run consequences, government spending is reduced following a deterioration in aggregate technology. Therefore, the overall implications for the long run levels of consumption and output are negligible.

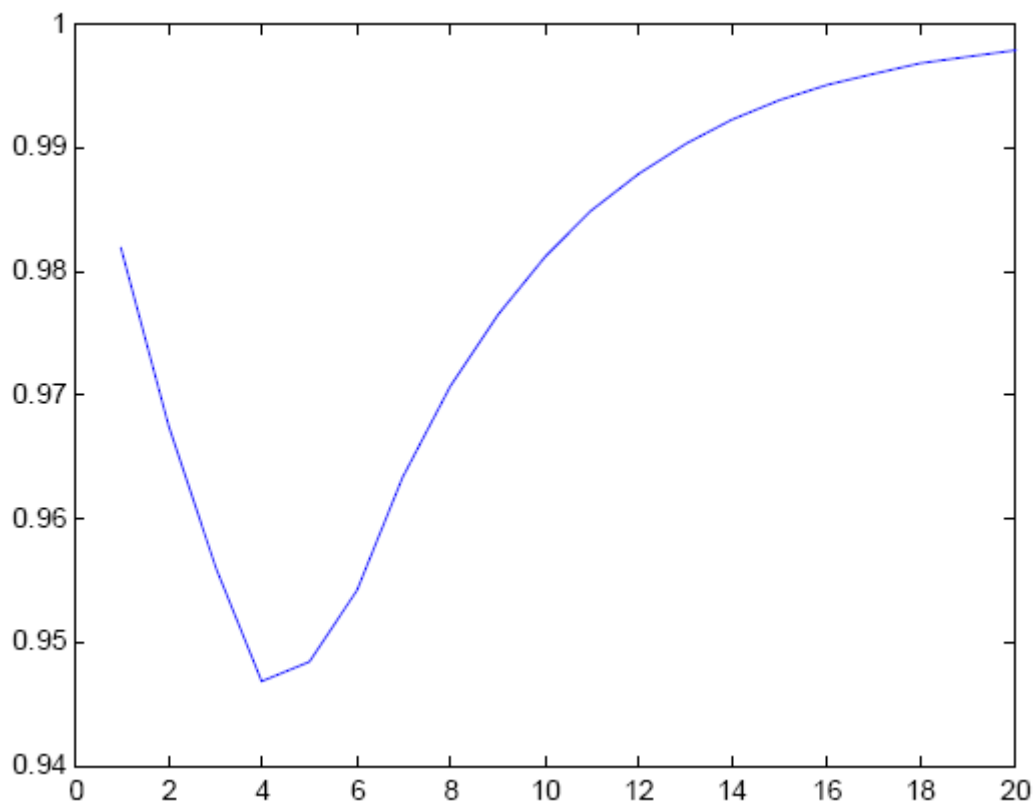
### 6.1.2. A recession shock

27. The recent experience following the financial crisis put an end to the view that the major industrialized economies are experiencing a period of “Great Moderation”. Therefore, this section considers the effects of shocks that take a different form and size than the standard technology shocks considered above. Based on the view that financial intermediation is one ingredient contributing to the economy’s total factor productivity, the real consequences of a financial crisis are analysed by studying the effects of a large and persistent negative technology shock. The considered shock process is depicted in Figure 2.<sup>10</sup> Unlike in the previous section, it takes the form of a negative hump to capture the fact that some of the deterioration of economic outcomes was predictable over the course of the crisis. Specifically, technology drops on impact by 2% and deteriorates further over the next 3 quarters, reaching an overall low slightly under -5%. From the fifth quarter the situation then gradually recovers and the economy roughly reaches its pre-shock productivity level after about five years.

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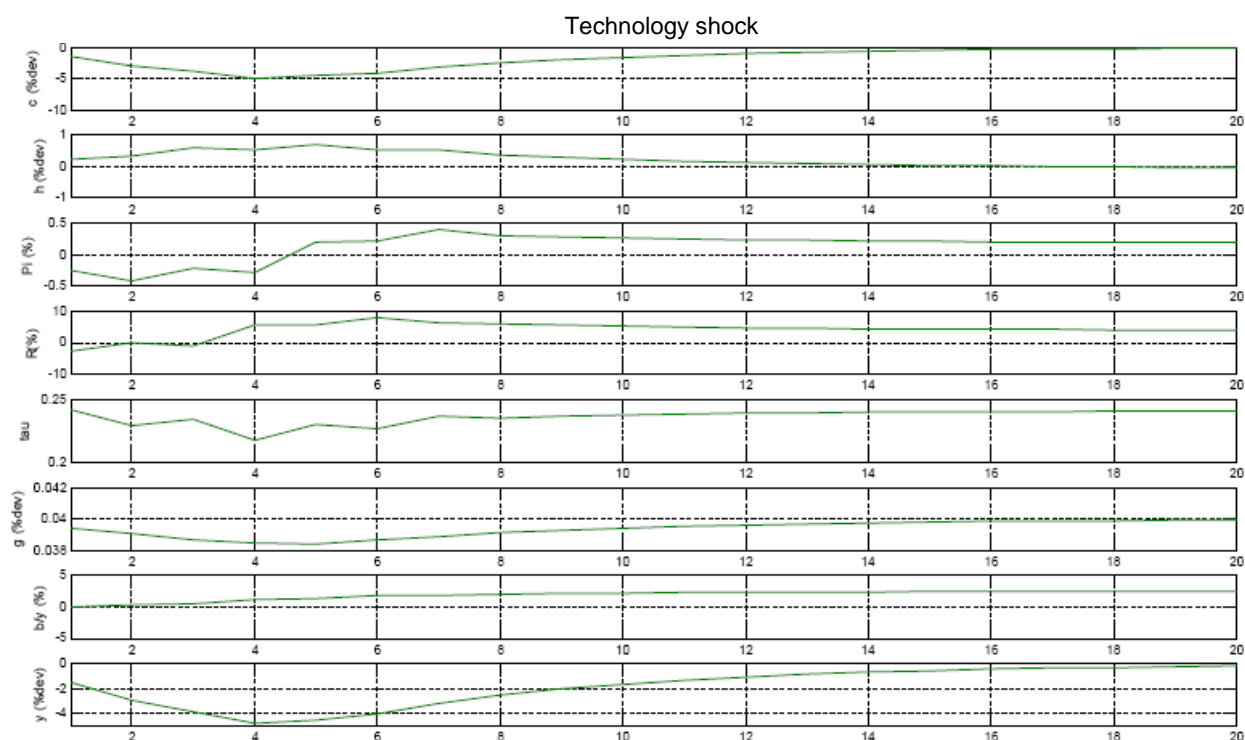
10. It assumes  $\rho_z = 0.8$ ,  $\alpha_{z,0} = \alpha_{z,1} = \alpha_{z,2} = \alpha_{z,3} = 1$ ,  $\alpha_{z,4} = 0.5$ , and  $\alpha_{z,5} = 0.25$ .

Figure 2. Persistent negative technology shock



28. Figure 3 depicts the impulse responses under optimal stabilisation policy. The figure shows that private consumption and output drop persistently and roughly in proportion to the shock. Unlike for the standard technology shock, however, inflation becomes negative for about 4 quarters and monetary policy lowers nominal rates strongly. Indeed, the zero lower bound on nominal rates becomes binding for about 4 quarters. Fiscal policy now responds by persistently reducing income taxes and by lowering spending, with the spending reduction being again less strong than the fall in technology. A government deficit then emerges because tax revenues fall also because of falling real wages. The debt to GDP ratio increases over five years from 0% to about 2.5% with an increase of about 1% in the first and 0.8% in the second year after the shock. Note that employment is slightly up. This occurs because the negative wealth effect of the adverse productivity disturbance causes households to consume less consumption goods and less leisure, *i.e.*, to work more. The somewhat unrealistic equilibrium path for hours could be eliminated at the cost of introducing more complex preference specifications that eliminate the wealth effect on labour supply (Greenwood *et al.*, 1988).

Figure 3. Impulse responses to a persistent negative technology shock



29. The overall government debt increase implies that taxes in the long run will be slightly higher to be able to finance the interest payments on the outstanding debt. Yet, the quantitative importance of the long run level effect is again negligible.

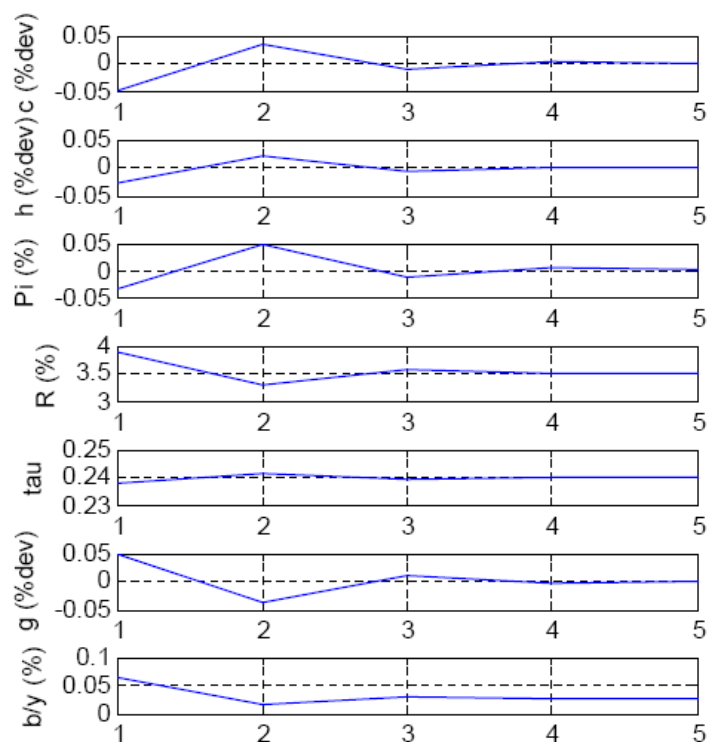
## 6.2. Demand side shocks

### 6.2.1. Deflationary demand shock (increase in price elasticity of demand)

30. This section considers the optimal response to a deflationary demand shock. The underlying scenario is one where consumers' product demand becomes temporarily more price sensitive. Adam and Billi (2008) estimate such a shock process for the great moderation period using US data and find it to be iid with a standard deviation of  $\sigma_{\eta} = 0.15\%$ . Based on these findings the effects of a 3 standard deviation negative innovation to this process is considered. The resulting impulse responses under optimal monetary and fiscal stabilisation policy are depicted in Figure 4. Overall the economic effects are short-lived and the quantitative implications of such shocks rather limited. On impact, consumption and hours (and therefore output which is proportional to hours in this scenario) all fall by small amounts and inflation drops slightly below baseline. Fiscal policy reduces taxes and increases spending slightly. The effects of the shock last longer than the shock itself because stabilisation policy uses the intertemporal margin to improve on the economic outcomes. For example, monetary policy creates an economic expansion in the second period so that the resulting inflation helps to stabilise inflation in the first period through the anticipation effects implicit in the Phillips curve. The debt/GDP ratio increases by a small amount as fiscal policy seeks to sustain output via slight spending increases and lower income taxes. Overall, the quantitative implications

of mark-up shocks appear fairly limited and this remains true even if the standard deviation of the shocks were considerably larger than their estimated value.

Figure 4. **Impulse responses to a deflationary demand shock**



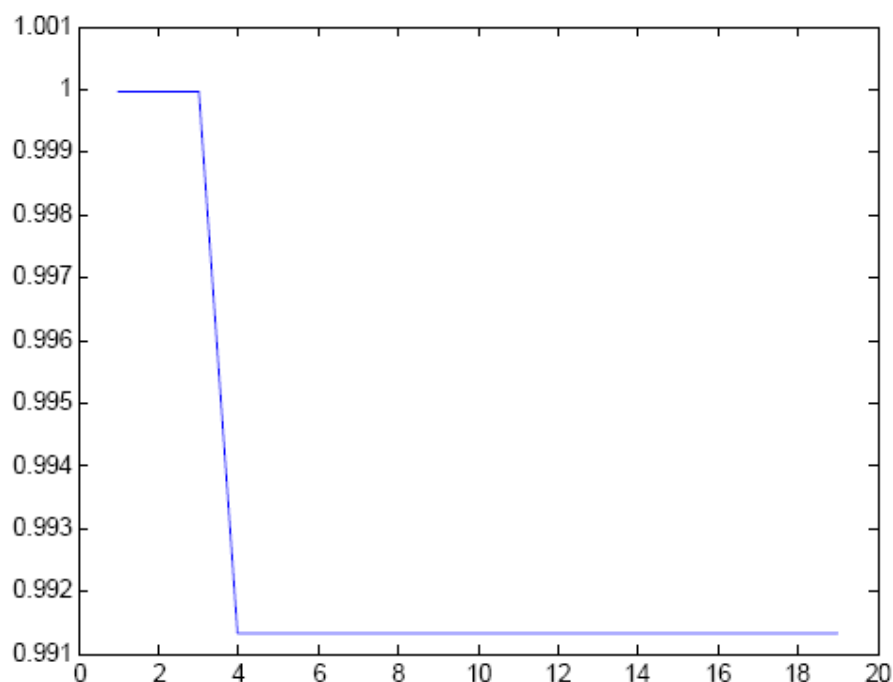
### 6.2.2. Thrift shock (increase in the discount factor)

31. We now consider a shock that causes consumers to become temporarily more thrifty. Specifically, it is assumed that consumers' discount factor increases for three quarters to a value of one – consistent with no preference for early consumption – and then reverts to its baseline value, as is illustrated in Figure 5.<sup>11</sup> Everything else equal such an event induces agents to postpone consumption into the future, as their desire for future consumption has increased relative to current consumption.

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11. The model implied effective time discount factor is  $\beta E_t[e^{\varepsilon_{t+1}-\varepsilon_t}]$ . We choose a shock specification such that  $E_t[e^{\varepsilon_{t+1}-\varepsilon_t}]$  equals  $\beta^{-1}$  for three quarters and is equal to one thereafter.

Figure 5. Evolution of the discount factor (“thrift shock”)



32. The impulse responses under optimal stabilisation policy are depicted in Figure 6. Importantly, fiscal policy reduces government spending and increases taxes when the shock emerges and monetary policy optimally lowers nominal interest rates. The former allows the fiscal authority to accumulate net claims against the private sector (who is now indebted with the government). The interest income earned by the government will allow it to lower taxes in the future, thereby sustaining a higher steady state consumption (private and public) and a higher output level in the long run. Clearly, this comes at the cost of reduced private and public consumption in the short and medium term but achieves the intertemporal consumption shift that is desirable under the shift in intertemporal consumption preferences that is considered.

33. The quantitative implications of the thrift shock turn out to be substantial, *e.g.*, [the debt to GDP ratio moves by 1% point over four quarters. Overall, Figure 6 may severely understate the quantitative implication of such shocks, as it assumes that nominal interest rates can become negative: see the impulse response for nominal interest rates in the first period.

## 7. The effects of steady state debt levels

34. This section discusses the implications of positive government debt levels for economic outcomes. The steady state implications of positive government debt levels are considered first and then the implication for optimal stabilisation policies.

### 7.1. Steady state effects

35. Since interest payments on debt have to be financed by distortionary income taxes, the presence of government debt has important implications for the long-run (steady state) outcomes. These are illustrated in Table 1 which shows that higher debt to GDP ratios cause output, private consumption and government spending to be significantly lower than in the absence of debt. This is so because the tax rates are more than 10% (16%) higher when the debt to GDP ratio is 60% (100%) compared to the economy

without government debt. While the effects of debt on private and public consumption and hours (and thus output) are approximately linear in the debt level, they are non-linear in the tax rates and also in welfare terms. Specifically, the welfare costs of an additional unit of government debt increases with the already existing debt level.

Figure 6. Impulse response following a thrift shock

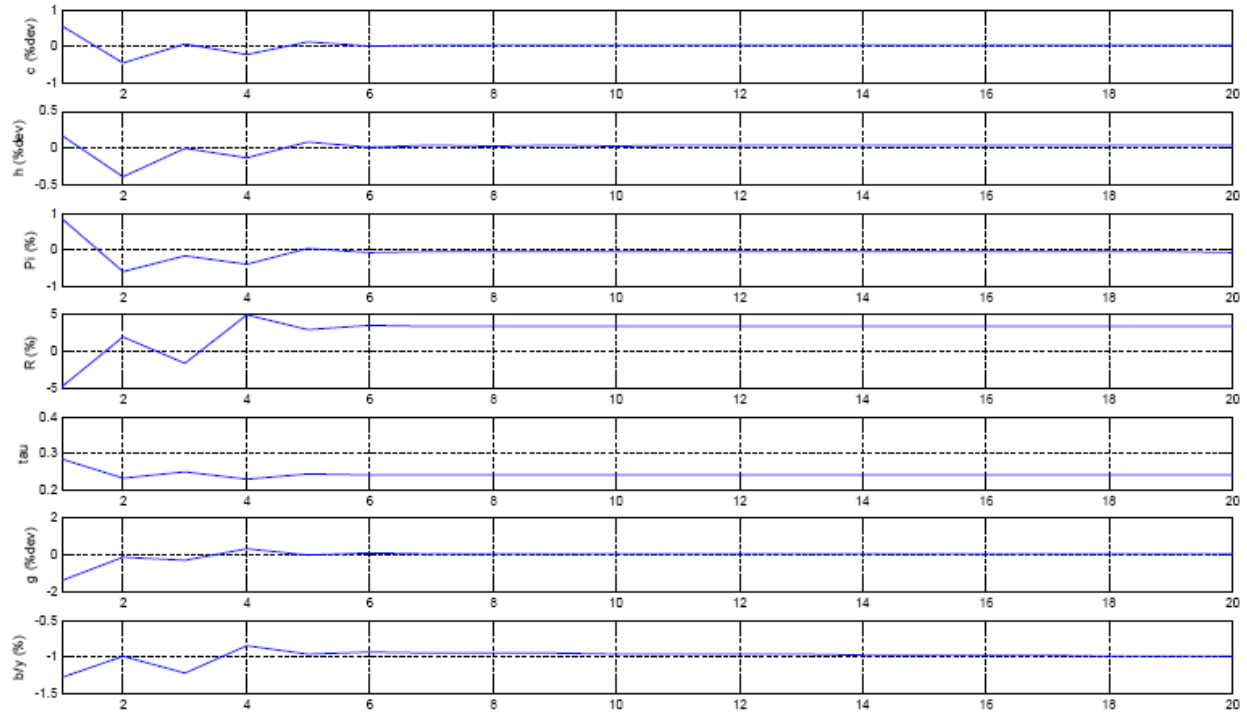


Table 1. Steady state effects of government debt

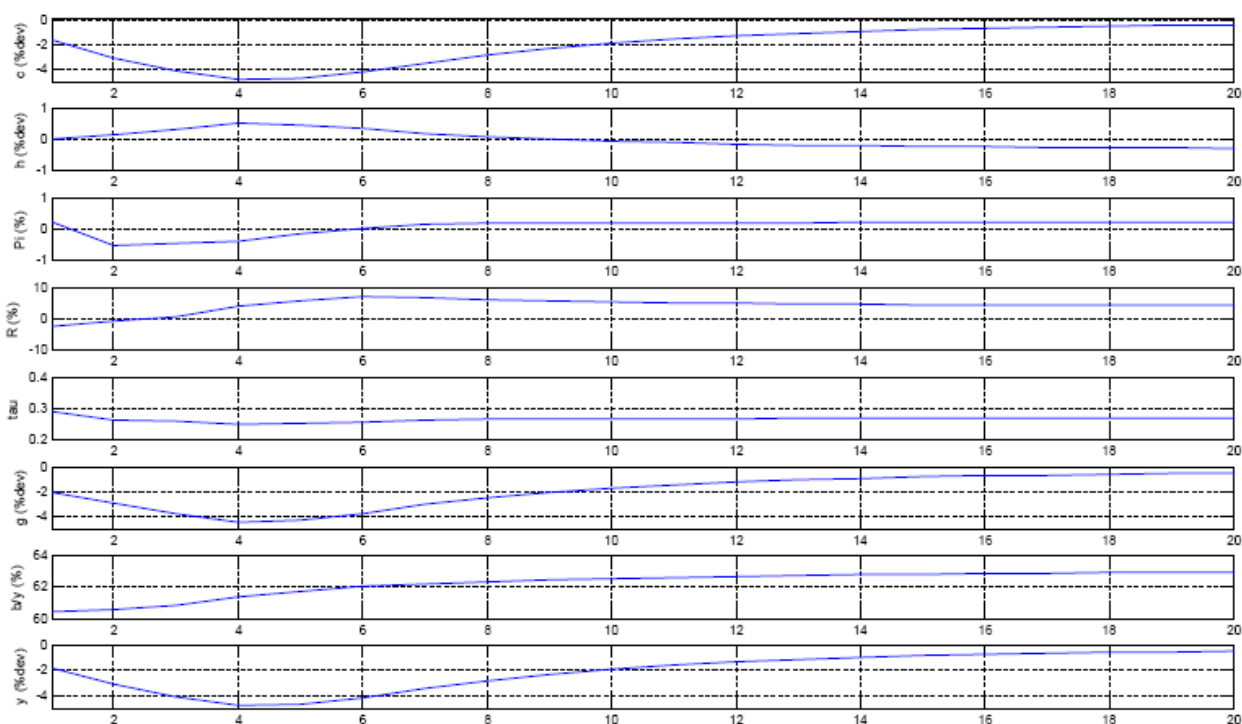
	priv.cons( <i>c</i> )	hours( <i>h</i> )	gov.cons( <i>g</i> )	taxes( $\tau$ )
Zero debt	0.16	0.2	0.04	24%
60% debt/GDP	0.1575	0.1966	0.0392	26.5%
Change wrt zero debt	-1.58%	-1.68%	-2.10%	+10.2%
100% debt/GDP	0.1558	0.1944	0.0386	28.0%
Change wrt zero debt	-2.61%	-2.78%	-3.47%	+16.8%

## 7.2. Implications for stabilisation policy

36. The implications of the presence of government debt for the conduct of optimal stabilisation policies are considered, assuming that the government debt to GDP ratio is initially at 60% and compared to the baseline outcome without government debt.

37. Figure 7 depicts the optimal impulse response to the recession shock. Some noteworthy differences arise when comparing the outcome to those in Figure 3. With a substantial amount of accumulated government debt it is optimal to reduce government spending more in response to the negative technology shock, despite starting from a lower steady state spending level already. For example, the initial cut in government spending is slightly above 2% and now exceeds the drop in technology (1.8%) while previously government consumption was falling less strongly than technology. Nevertheless, the total increase in government debt resulting from the shock now exceeds that occurring when initial debt is zero. In particular, over 5 years the debt to GDP ratio now increases by close to 3% while with a zero initial debt level the increase over this period was limited to less than 2.5%. This occurs because the turnaround of the economy starting from the fourth quarter onwards implies that real interest rates start to increase again. With higher debt this real rate increase generates a stronger increase in the interest burden and therefore the terminal government debt level. As a result, taxes, output and consumption will remain significantly below their starting values in the long run. As is clear from Figure 7, the negative long-run implications are now much larger than in the case without government debt.

Figure 7. High debt scenario: impulse response to a persistent, negative technology shock

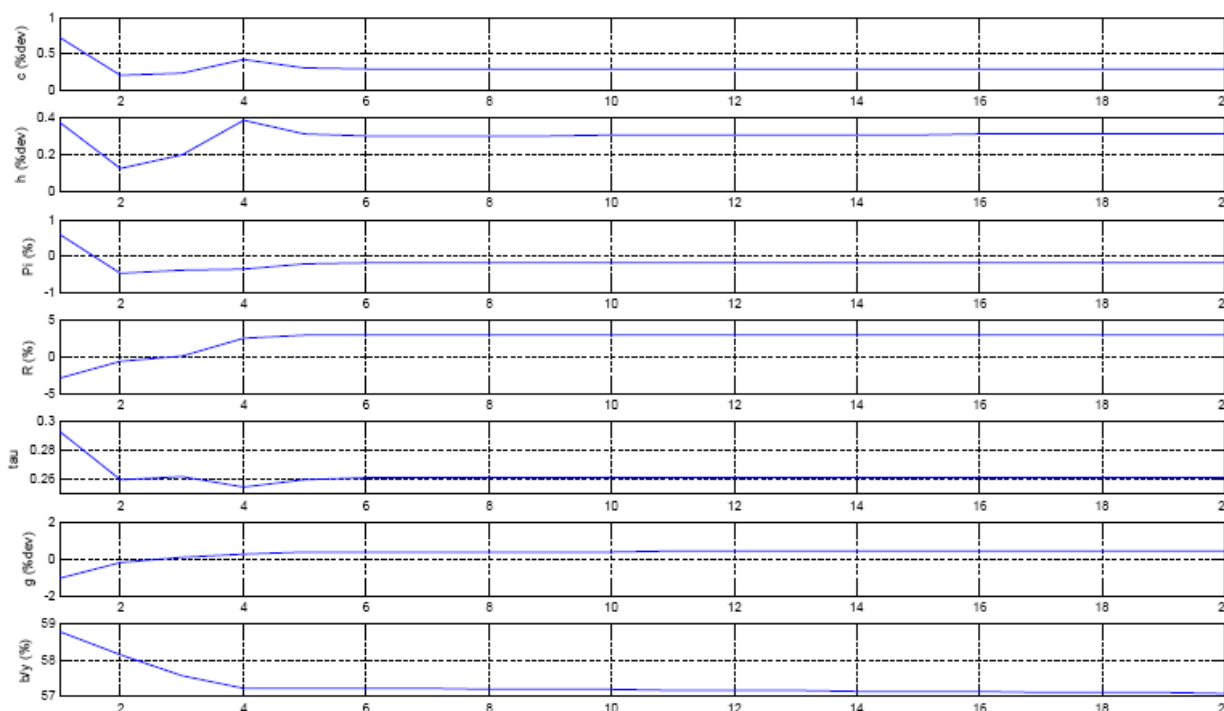


38. The impulse responses in the case with a standard technology shock process are altered in a very similar manner, so that they are not discussed. The quantitative implications of deflationary demand shocks still remain muted, even if the government debt to GDP ratio is significantly above zero.



39. Finally, the effects of a thrift shock are discussed, which are shown in Figure 8. As in the case with no government debt, the policymaker shifts consumption into the future at the expense of current consumption. Since consumers' desire to save reduces real interest rates, the shock also substantially reduces the interest burden on outstanding debt. This allows the government to reduce the level of outstanding debt by about 3% over 5 years, which is more than three times the claims accumulated when starting from a zero initial debt level. As a result, hours and consumption now increase much more. This is highly desirable because high debt and taxes imply that labour supply and consumption start from a rather depressed level.

Figure 8. High debt scenario: impulse response to a thrift shock



### 7.3. Discussion

40. Overall, the message from this section is that higher government debt levels give rise to a larger exposure of the government budget to real interest rate risks. Since economic disturbances have implications for real interest rates, they move debt levels by more whenever the initial debt level is higher. Movements in debt in turn give rise to movements in distortionary taxes which have important implications for labour supply and other variables. This is illustrated in Table 2, which reports the standard deviation of private consumption, hours and government consumption as a function of the initial debt to GDP ratio.<sup>12</sup> While the standard deviation of private consumption is roughly unchanged, higher debt levels significantly increase the standard deviation of hours worked and of government consumption. This shows that high levels of government debt not only have negative implications for the average level of output and government consumption, but also increase the volatility of these variables.

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12. The reported standard deviations are the average standard deviation across 1 000 model simulations with a length of 100 quarters.

Table 2. **Standard deviation effects of government debt**

	priv.cons( <i>c</i> )	hours( <i>h</i> )	gov.cons( <i>g</i> )
Zero debt	$2.3 \cdot 10^{-3}$	$2.9 \cdot 10^{-4}$	$5.1 \cdot 10^{-4}$
60% debt/GDP	$2.4 \cdot 10^{-3}$	$5.0 \cdot 10^{-4}$	$6.0 \cdot 10^{-4}$
100% debt/GDP	$2.3 \cdot 10^{-3}$	$7.2 \cdot 10^{-4}$	$6.7 \cdot 10^{-4}$

## 8. Safety margins for monetary and fiscal policy

41. This section simulates the economy under the optimal monetary and fiscal stabilisation policy and asks the following questions: Does optimal policy require that nominal interest rates fall all the way to zero? Does optimal policy give rise to swings in government deficits that deviate more than 3% from their baseline value?<sup>13</sup> It is thereby assumed that the economy is hit by the standard technology and demand shocks (price elasticity shocks). This section discusses the baseline calibration while the next section considers the robustness of these findings to various assumptions. As before, two economic settings are considered, one with a zero initial government debt level and one where the debt to GDP ratio equals 60%.

42. The main finding of this section is that higher debt levels require larger safety margins for fiscal policy, *i.e.*, the optimal variability of the debt to GDP ratio increases, if the initial level of debt is higher, but require lower safety margins for monetary policy, *i.e.*, the optimal variability of nominal rates falls as the initial debt level increases.

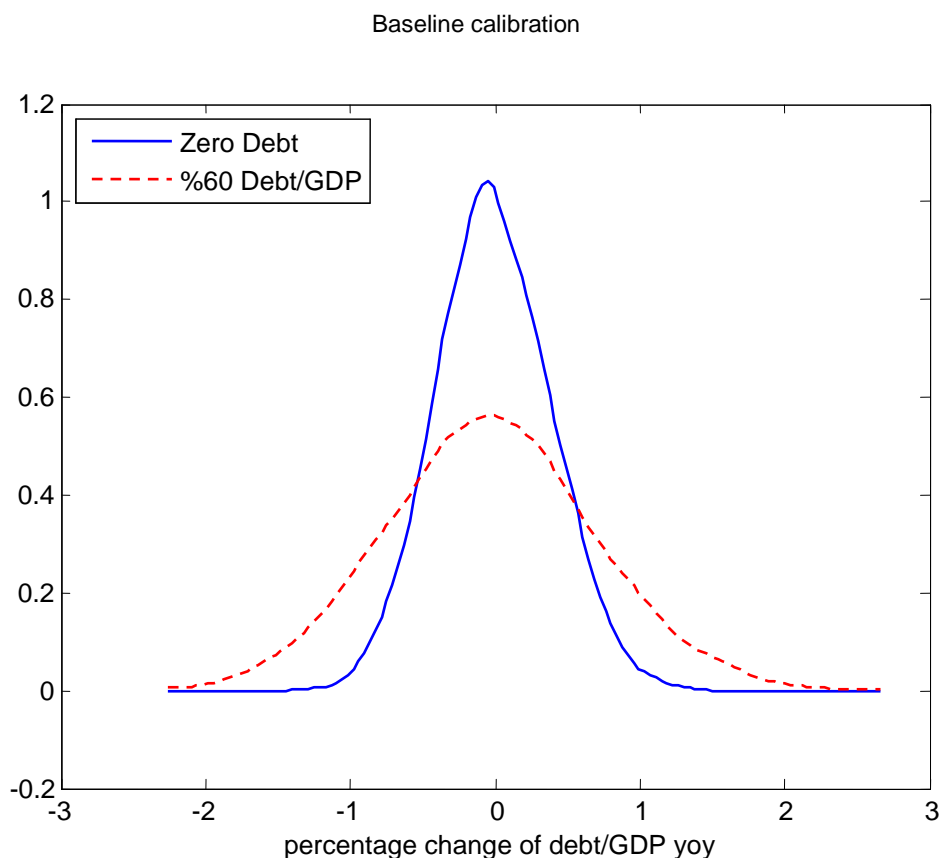
43. Using the baseline model calibration reported in Table 1 we compute the model-implied distribution of the year on year changes in the debt to GDP ratio that can be expected over the course of 20 quarters. The distributions are shown in Figure 9, once for the case where initial debt equals zero and once for the case where initial debt equals 60% of GDP.<sup>14</sup> Independently of the initial debt level, the government deficit virtually never deviates by more than 3% from its baseline value. Figure 9 also shows that the distribution of debt changes widens as the government debt level increases. This occurs because higher debt levels imply that the government budget is subject to greater real interest rate risk. While the effects of higher debt on the variability of government deficits are sizable, the deficit swings remain nevertheless well contained within the 2% range. Since the baseline deficit to GDP ratio equals -1.2% when the state growth rate of is 2% and debt is 60% of GDP, see the discussion in section 4.2, optimal fiscal stabilisation policy almost never implies a violation of the 3% deficit limits imposed under the Stability and Growth Pact in Europe.

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13. Recall that the baseline (or steady steady state) deficit to GDP ratio need not be equal to zero, as is explained at the beginning of section 4.2.

14. The distribution is computed by simulating the model 1 000 times for a length of 20 quarters starting the simulations either with a zero initial debt level or one corresponding to 60% of GDP.

Figure 9. Safety margins for fiscal policy (baseline)



44. Figure 10 depicts the corresponding distributions for the nominal interest rate. It shows that under optimal monetary and fiscal policy the zero lower bound is never binding, independently of the debt level. Interestingly, a higher debt to GDP ratio implies that monetary policy is setting nominal interest rates in a less variable way. This is optimal because it attenuates the real interest rate risk for the government budget stemming from economic disturbances. Monetary policy therefore optimally contributes less to stabilizing the economy when the government debt level is high.

### 9. Safety margins: robustness analysis

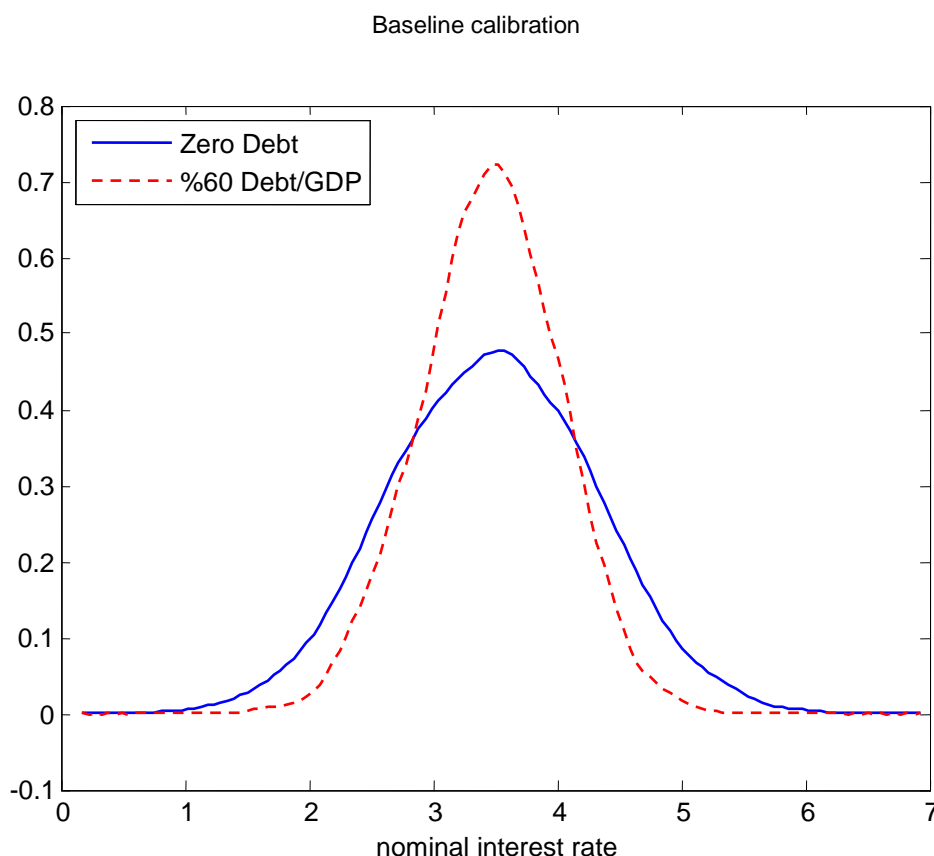
45. This section evaluates the robustness of the baseline findings to various underlying assumptions. We start by considering to what extent larger economic disturbances affect the analysis and then discuss the effects of alternative model parameterizations.

#### 9.1. The effect of different shock sizes

46. The robustness of the previous findings to the assumed variability of the economic disturbances is evaluated. Obviously smaller disturbances do not pose a problem, as these cause correspondingly smaller distributions for the debt changes and the nominal interest rate. The implications of a larger standard deviation for the shocks are thus considered. This is partly motivated by the consideration that the historical size of economic disturbances may misrepresent the magnitude of shocks that could occur in the

future, given that the 25 years predating the recent crisis were characterized by exceptionally low levels of aggregate volatility.

Figure 10. **Safety margins for monetary policy (baseline)**

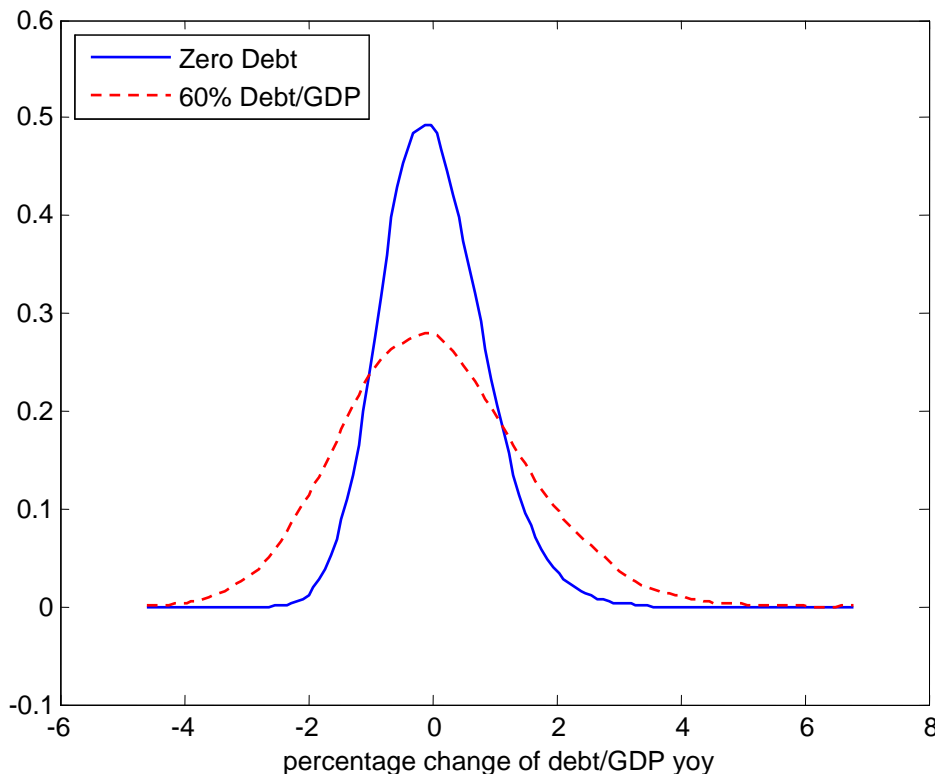


47. As a simple robustness test the standard deviation of the innovations to technology and the demand shocks are multiplied by a factor of two and three.

48. Figure 11 depicts the implied distribution of the year on year changes in the debt to GDP ratio under optimal policy for the case with double standard deviations. For the 60% debt ratio there is now a non-negligible quarterly probability of 2.4% that optimal stabilisation policy implies a change in the debt to GDP ratio by more than 3% from its baseline value. This number increases to about 8.2% when the standard deviation of shocks is increased by a factor of three (Figure 13). If the steady state growth rate of the economy is 2%, then the baseline deficit to GDP ratio equals -1.2% already, so that larger shock standard deviations considerably increase the likelihood that European countries violate the prescriptions of the Stability and Growth Pact under the optimal stabilisation policy. The situation is, however, considerably different for lower government debt ratios. For the case with a zero debt ratio, the probability of an increase in the debt to GDP ratio of more than 3% is virtually zero in the case with double sized standard deviations. It increases to just 1.0% with a threefold increase in the shock sizes. Since the baseline deficit to GDP ratio is zero when debt is zero, this shows that lower debt ratios allow for tighter fiscal safety margins. Overall, the likelihood of events leading to government deficits that violate the Stability and Growth Pact limits is then relatively low, even if shocks are 2 to 3 times as large as suggested by the baseline calibration.

Figure 11. **Safety margins for fiscal policy (double sized shocks)**

Double standard deviation of shocks



49. A larger standard deviation of the shocks also has implications for the distribution of nominal interest rates. Figure 12 depicts the distribution of nominal rates for the case with double sized shocks. While the zero lower bound constraint is not an issue under a high initial debt level, it becomes slightly binding under the low debt scenario, where monetary stabilisation policy is used more because it has less adverse implications for the variability of fiscal deficits. The overall likelihood of reaching the zero lower bound in any quarter is about 1.2% and increases to 6.7% under the triple-sized shock (Figure 14). With a triple-sized shock the zero lower bound constraint also becomes an issue in the 60% debt scenario where the likelihood of reaching the zero lower bound then reaches 1.1% per quarter. This shows that higher standard deviations cause the zero lower bound on nominal rates to become a binding constraint under optimal policy, especially for countries which can afford high real interest rate variations due to low levels of government debt.

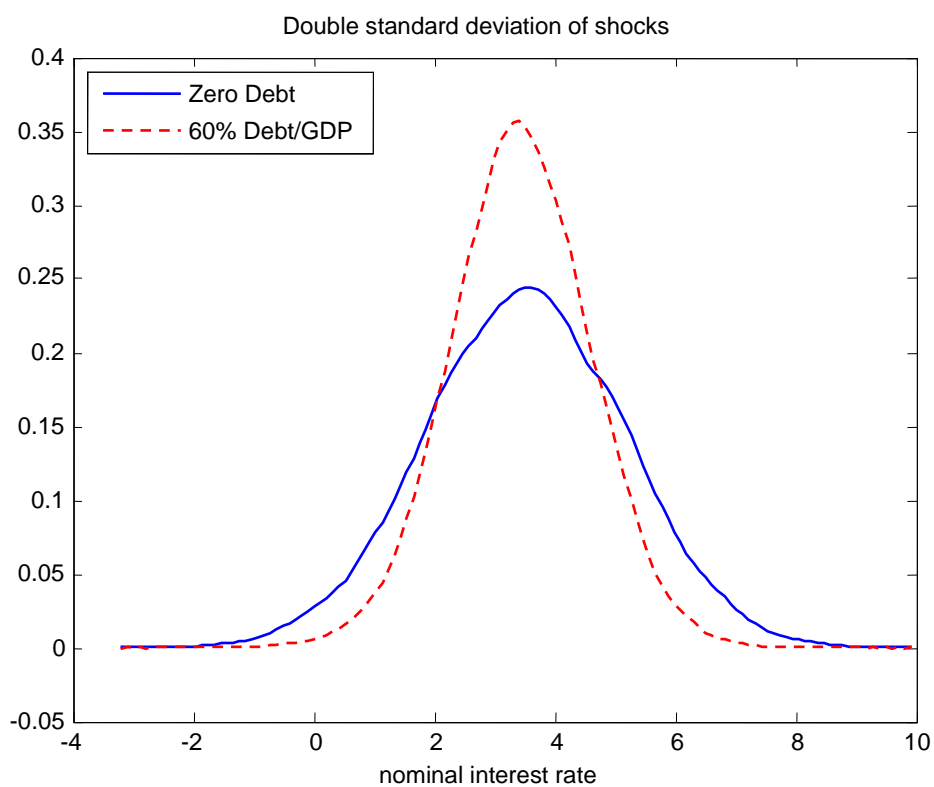
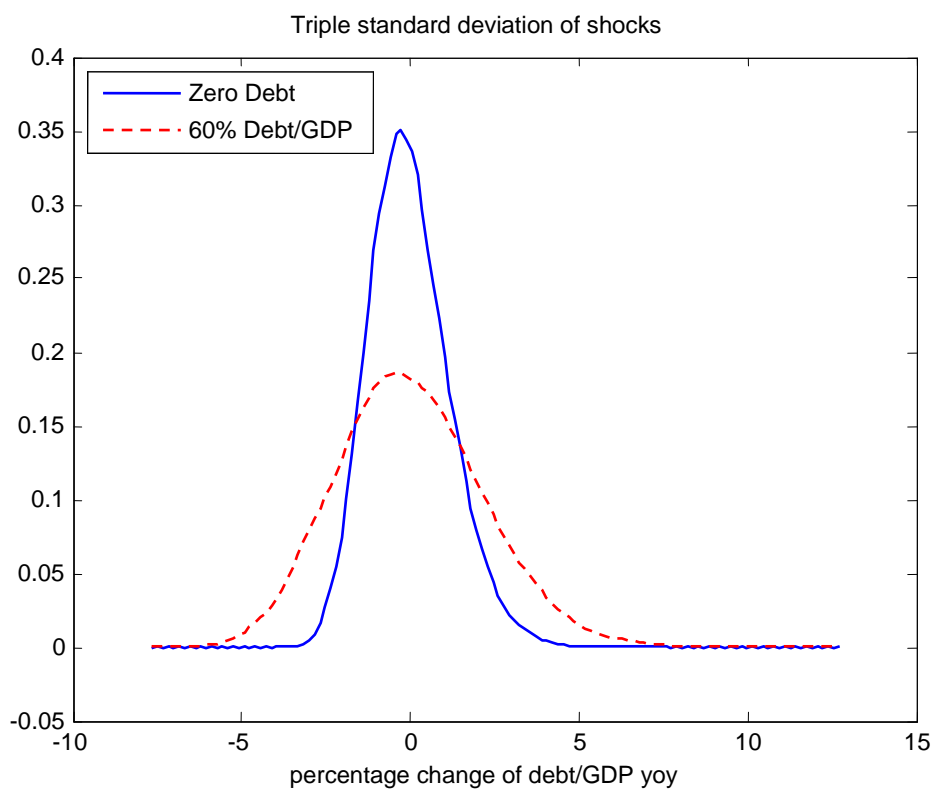
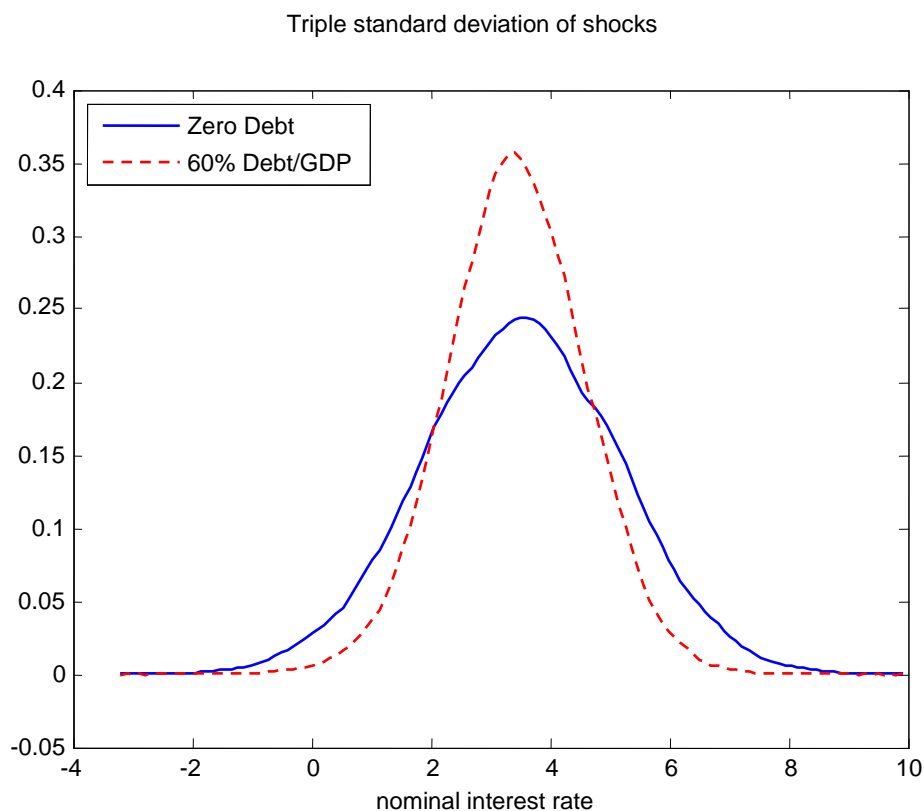
Figure 12. **Safety margins for monetary policy (double-sized shocks)**Figure 13. **Safety margins for fiscal policy (triple-sized shocks)**

Figure 14. Safety margins for monetary policy (triple-sized shocks)



### 9.2. Robustness with respect to alternative parameterizations

50. This section explores the robustness of the baseline result with respect to a selected number of model parameters. Specifically, it considers the implication of higher nominal rigidity and of a more elastic labour supply.

51. Consideration of stronger nominal price rigidities is motivated by the observation that the model is calibrated to US data. Price studies at the micro-level, *e.g.*, those conducted at the European Central Bank within the Price Dynamics Network (see Alvarez *et al.*, 2006 for a summary) have documented that prices tend to be changed less often in European Union countries than in the United States. Therefore, this section considers the implications of increasing the price stickiness parameter from its baseline value of  $\theta = 17.5$  to  $\theta = 21$ . This corresponds to a 20% increase in the resource cost of inflation over the baseline value.

52. Figures 15 and 16 depict the distribution of debt ratio changes and nominal interest rates implied by this higher value for nominal rigidities. Both distributions widen compared to the baseline distributions. While the distribution for the changes in the debt ratio is altered only in minor ways, the distribution of nominal interest rates widens considerably: with stronger nominal rigidities, interest rate policy becomes a more effective stabilisation tool. As a result there is now a positive probability of about 0.14% that nominal interest rates hit the zero lower bound constraint. The robustness analysis thus suggests that optimal monetary policy is more likely to be constrained by the zero floor on nominal rates when nominal rigidities are higher.

Figure 15. Safety margins for fiscal policy: higher price stickiness

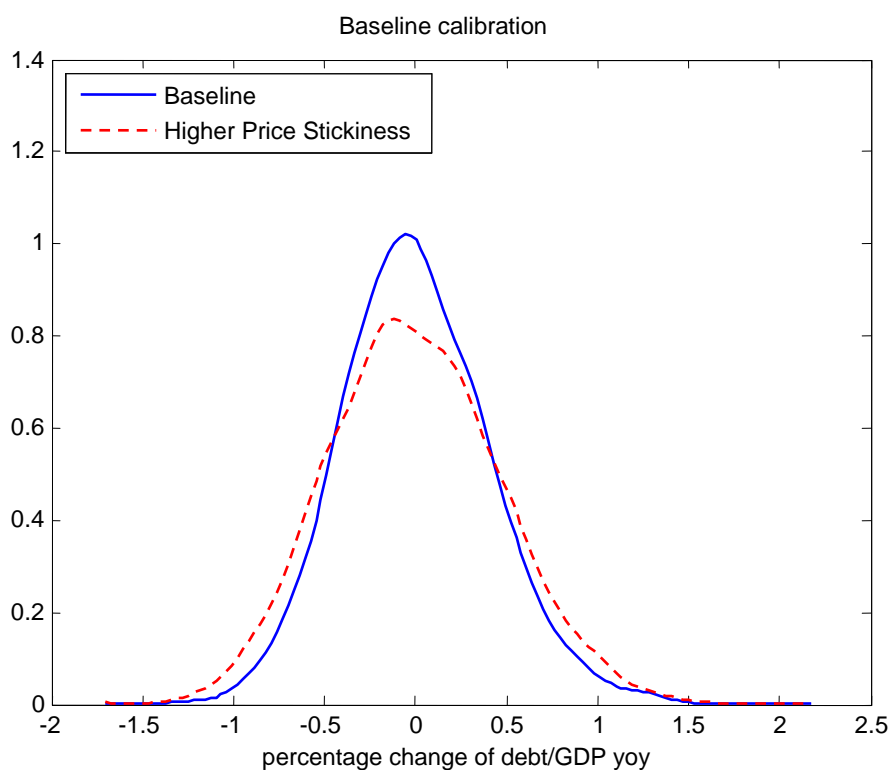
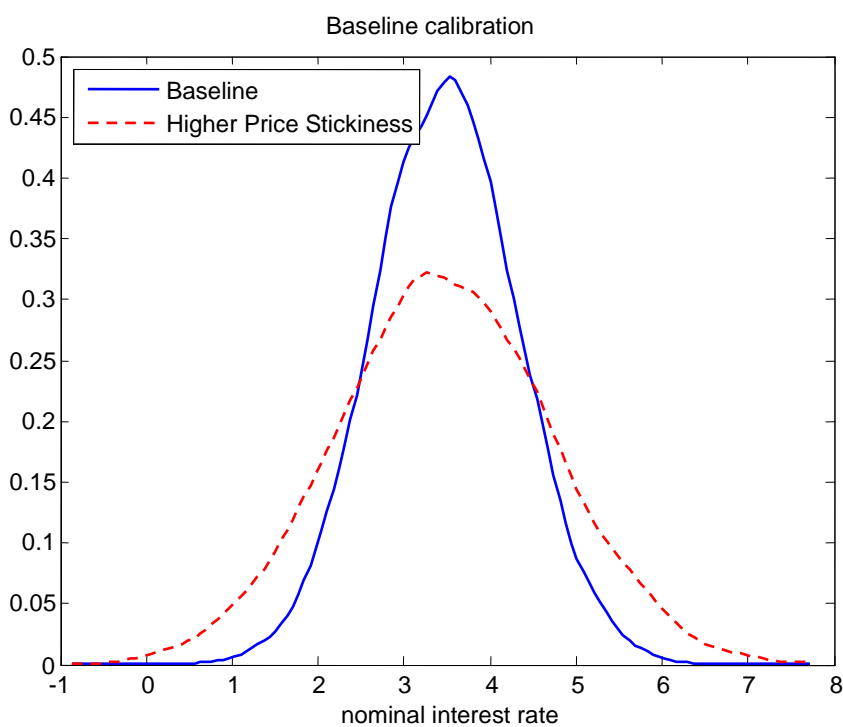


Figure 16. Safety margins for monetary policy: higher price stickiness





53. Next, the implications of a more elastic labour supply are considered. While the baseline calibration assumed a Frisch labour supply elasticity equal to one, the effects of increasing the Frisch elasticity to two, which requires setting  $\varphi = \frac{1}{2}$ , are assessed. Higher values of the Frisch labour supply elasticity help the model in matching the empirically estimated impulse responses to monetary policy shocks that have been documented in the VAR literature. This is so because more elastic labour supply causes marginal costs (*i.e.* real wages) to react less strongly to nominal demand conditions. Labour supply elasticity values close to 2 are also not uncommon in the DSGE literature, *e.g.*, Smets and Wouters (2007). Figures 17 and 18 depict the implications of an increased labour supply elasticity. They show that the variability of changes in the debt ratio increases considerably compared to the baseline, albeit deviations of the deficit to GDP ratio of more than 3% still remain very unlikely. The distribution of interest rates, however, narrows significantly. With a more elastic labour supply, agents are more willing to adjust labour (or leisure) in response to economic disturbances. This helps to stabilise private consumption and implies less variable real interest rates and therefore a lower variability of the nominal rate.

54. The robustness analysis thus suggests that optimal monetary policy is less likely to be constrained by the zero floor on nominal rates when the labour supply elasticity is higher, but that the chances of fiscal deficit swings exceeding 3% increases slightly.

Figure 17. **Safety margins for fiscal policy: higher labour supply elasticity**

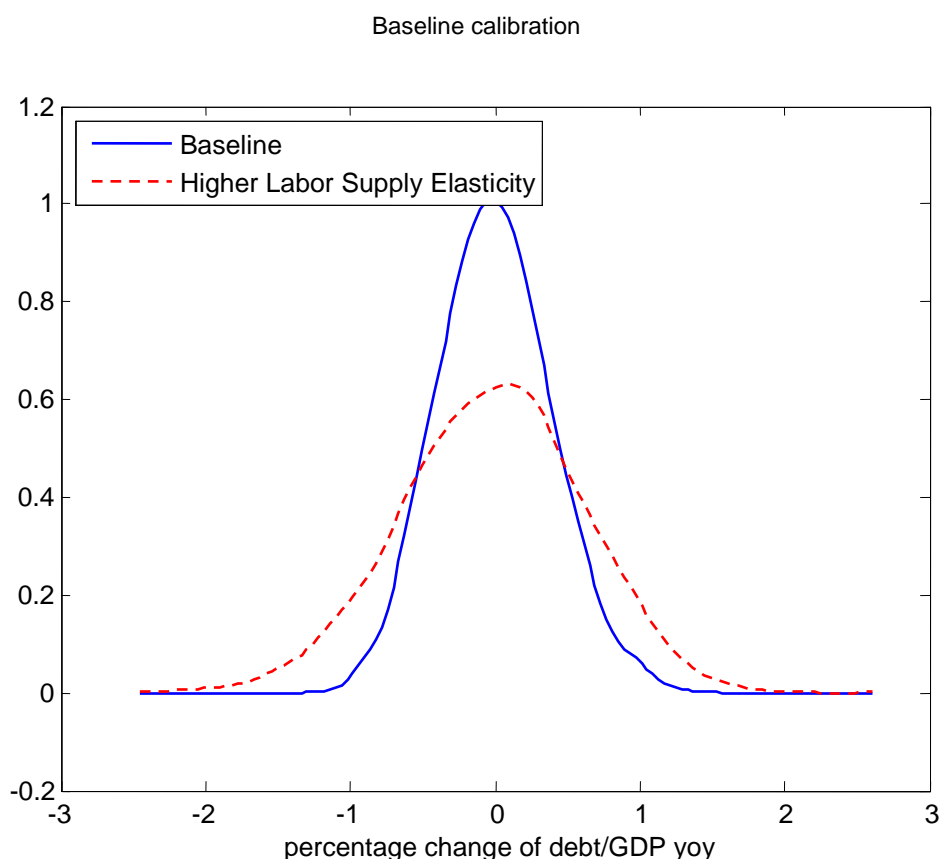
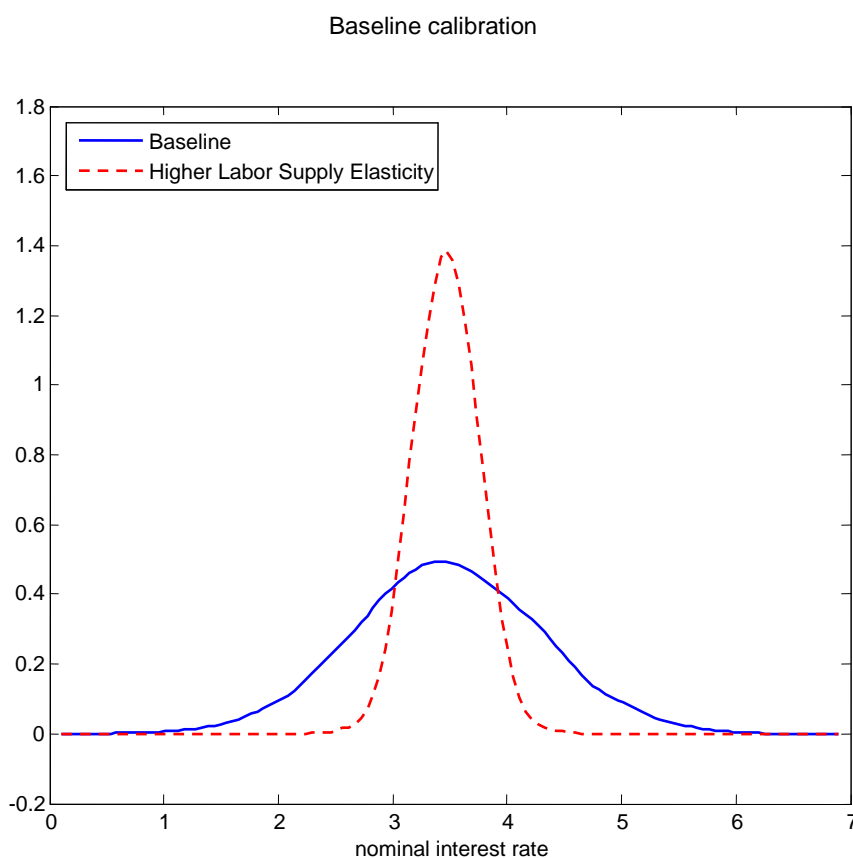


Figure 18. **Safety margins for monetary policy: higher labour supply elasticity**

## 10. The implications of lack of commitment

55. The analysis of optimal stabilisation policies conducted in this paper assumes that monetary and fiscal policymakers can make credible plans on how to conduct policy in the future. The ability to credibly commit to future plans greatly helps in stabilising the economy following economic disturbances because it allows policymakers to steer the private sector's expectations. Since the private sector is forward-looking, private sector expectations influence current economic decisions. Therefore, the ability to commit to future policies allows policymakers to intertemporally “smooth” the effects of shocks through the management of expectations.

56. Importantly, however, the policymakers' incentives to stick to the initially envisaged plans may vanish over time, *i.e.*, optimal plans may not be time consistent. Specifically, some future policy measure may have been chosen with the objective to favourably influence private sector expectations and thus private sector decisions at some earlier date. Yet, once the future has arrived and the private sector's earlier decisions have been taken, the incentives to stick to the initially envisaged plans have changed. For this reason this section briefly discusses the economic implications of not being able to commit to future policies. A fully fledged analysis of the lack of commitment is, however, well beyond the scope of this paper.

57. The first issue to note is that without commitment the steady state of the economy will be adversely affected. These findings go back to the work of Kydland and Prescott (1977) and Barro and

Gordon (1983). For the economic model analyzed in this paper, Adam and Billi (2008) show that monetary and fiscal policymakers both face the temptation to stimulate output if they cannot commit to future policies. This is so because the monopoly power of firms implies that the economy's steady state output level falls short of its efficient level. As a result, fiscal policy engages in too much fiscal spending and monetary policy gives rise to too high inflation rates. Adam and Billi (2009) show that these effects can be quantitatively large when the fiscal authority must use distortionary income taxes to finance government expenditure, as has been assumed in the present paper.

58. The implications of lack of commitment on the stabilisation outcomes in response to economic disturbances has not been analyzed widely before for the economic model considered in this paper. From the analysis of Adam and Billi (2008) it becomes clear, however, that the policymakers' response to shocks is restricted to take on the same pattern as the shock itself. Therefore, if the shock takes the form of a downward hump, as is the case with the persistent technology shock in section 6.1.2., then policy must take on the form of a positive or negative hump. As becomes clear from Figure 3, the optimal interest rate response is not of this form. Instead, it is optimal to first lower nominal rates, then to increase them temporarily above the steady state level and then to let them slowly return to steady state. This suggests that the optimal stabilisation policies are time inconsistent and cannot be implemented, unless policymakers can credibly commit to future plans. The steady state and the stabilisation outcomes therefore both deteriorate under lack of commitment.

## 11. Summary

59. This paper determines optimal monetary and fiscal stabilisation policies and analyzes the implications of different levels of government debt for the steady state outcomes of the economy and for optimal stabilisation policy. It shows that high levels of government debt result in inferior steady states and also give rise to more volatile developments for consumption, hours and public goods provision. Overall, the required safety margins for fiscal policy increase with the initial government debt level and those for monetary policy decrease. The zero lower bound, however, is virtually never reached. Also, for our baseline calibration the swings in the fiscal deficits remain well contained within 3% from the baseline value and basically never violate the limits imposed under the Stability and Growth Pact. Yet, when government debt to GPD ratios are high and the standard deviation of shocks much larger than suggested by the baseline scenario, fiscal deficits may violate the limits imposed by the Stability and Growth Pact rather frequently.

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## Appendix

### A.1. Ramsey steady state

To simplify matters taxes and the government budget constraint are eliminated from the Lagrangian (16). Note that the FOCs (6) and (7) imply

$$\frac{-u_{h,t}}{u_{c,t}} = (1 - \tau_t)w_t = w_t - \tau_t w_t$$

and from steady state version of the government budget (18) we have

$$\tau_t w_t = \frac{g_t + \tilde{x}}{h_t}$$

Substituting the latter equation into the former gives the following expression for the real wage

$$w_t = -\frac{u_{h,t}}{u_{c,t}} + \frac{g_t + \tilde{x}}{h_t}$$

which allows expressing the Phillips curve without reference to taxes. The simplified constant debt version of the Lagrangian (16) is then

$$\begin{aligned} & \max_{\{c_t, h_t, \Pi_t, R_t \geq 1, g_t\}_{t=0}^{\infty}} \min_{\{\gamma_t^1, \gamma_t^2, \gamma_t^3\}} \sum_{t=0}^{\infty} \beta^t e^{\epsilon t} u(c_t, h_t, g_t) & (24) \\ & + \beta^t \gamma_t^1 \left( e^{\epsilon t} u_{c,t} (\Pi_t - 1) \Pi_t - e^{\epsilon t} \frac{u_{c,t} z_t h_t}{\theta} \left( 1 + \eta + \eta \left( \frac{u_{h,t}}{u_{c,t}} - \frac{g_t + \tilde{x}}{h_t} \right) \right) \right. \\ & \quad \left. - \beta e^{\epsilon t+1} [u_{c,t+1} (\Pi_{t+1} - 1) \Pi_{t+1}] \right) \\ & + \beta^t \gamma_t^2 \left( e^{\epsilon t} \frac{u_{c,t}}{R_t} - \beta e^{\epsilon t+1} \left[ \frac{u_{c,t+1}}{\Pi_{t+1}} \right] \right) \\ & + \beta^t \gamma_t^3 \left( h_t - c_t - \frac{\theta}{2} (\Pi_t - 1)^2 - g_t \right) \end{aligned}$$

The FOCs consist of the three constraints and

$$c_t : e^{\epsilon t} u_{c,t} + (\gamma_t^1 - \gamma_{t-1}^1) e^{\epsilon t} u_{cc,t} (\Pi_t - 1) \Pi_t - \gamma_t^1 e^{\epsilon t} \frac{u_{cc,t} \tilde{z}_t h_t}{\theta} \left[ 1 + \eta - \eta \frac{g_t + \tilde{x}}{h_t} \right] + \gamma_t^2 e^{\epsilon t} \frac{u_{cc,t}}{R_t} - \gamma_{t-1}^2 e^{\epsilon t} \frac{u_{cc,t}}{\Pi_t} - \gamma_t^3 = 0 \quad (25)$$

$$h_t : e^{\epsilon t} u_{h,t} - \gamma_t^1 e^{\epsilon t} \frac{u_{c,t}}{\theta} \left( 1 + \eta + \eta \left( \frac{u_{h,t}}{u_{c,t}} - \frac{g_t + \tilde{x}}{h_t} \right) \right) + h_t \eta \left( \frac{u_{hh,t}}{u_{c,t}} + \frac{g_t + \tilde{x}}{(h_t)^2} \right) + \gamma_t^3 = 0 \quad (26)$$

$$\Pi_t : e^{\epsilon t} (\gamma_t^1 - \gamma_{t-1}^1) (2\Pi_t - 1) + e^{\epsilon t} \gamma_{t-1}^2 \left( \frac{-u_{c,t}}{(\Pi_t)^2} \right) - \gamma_t^3 \theta (\Pi_t - 1) = 0 \quad (27)$$

$$R_t : \gamma_t^2 e^{\epsilon t} \frac{-u_{c,t}}{(R_t)^2} = 0 \quad (28)$$

$$g_t : u_{g,t} + \gamma_t^1 \frac{u_{c,t}}{\theta} \eta - \gamma_t^3 = 0 \quad (29)$$

We now impose steady state conditions by dropping time subscripts. From (28)

$$\gamma^2 = 0$$

so that (27) gives

$$\Pi = 1$$

and from (12) one obtains

$$R = \frac{1}{\beta}$$

From (11) one obtains

$$\frac{-u_h}{u_c} = \frac{1 + \eta}{\eta} - \frac{g + \tilde{x}}{h} \quad (30)$$

which is equation (22) in the main text. Since  $u_h < 0$  and  $u_c > 0$  the previous equation implies

$$1 + \eta - \eta \frac{g + \tilde{x}}{h} < 0 \quad (31)$$

In the steady state equations (25), (26) and (29) simplify to

$$u_c - \gamma^1 \frac{u_{cc}h}{\theta} \left[ 1 + \eta - \eta \frac{g + \tilde{x}}{h} \right] - \gamma^3 = 0 \quad (32)$$

$$u_h - \gamma^1 \frac{\eta}{\theta} \left( u_{hh}h + u_c \frac{g + \tilde{x}}{h} \right) + \gamma^3 = 0 \quad (33)$$

$$u_g + \gamma^1 \frac{\eta}{\theta} u_c - \gamma^3 = 0 \quad (34)$$

where  $\gamma^3 > 0$  denotes the marginal utility of relaxing the resource constraint. The previous FOCs indicate the alternative possible uses of additional resources, namely private consumption (equation [32]), leisure (equation [33]) and public consumption (equation [34]). Since public consumption is only one of three possible uses of resources, it must be the case that  $\gamma^3 \geq u_g$  in the optimum. Equation (34) therefore implies that  $\gamma^1 \geq 0$ . Combining equations (32) and (34) to eliminate  $\gamma^3$  then gives

$$u_c = \frac{u_g + \gamma^1 \frac{u_{cc}h}{\theta} \left[ 1 + \eta - \eta \frac{g + \tilde{x}}{h} \right]}{(1 - \gamma^1 \frac{\eta}{\theta})}$$

From  $\gamma^1 \geq 0$  and (31) it follows that

$$\begin{aligned} u_c &= \frac{u_g + \gamma^1 \frac{u_{cc}h}{\theta} \left[ 1 + \eta - \eta \frac{g + \tilde{x}}{h} \right]}{(1 - \gamma^1 \frac{\eta}{\theta})} \\ &\leq u_g + \gamma^1 \frac{u_{cc}h}{\theta} \left[ 1 + \eta - \eta \frac{g + \tilde{x}}{h} \right] \\ &\leq u_g \end{aligned}$$

and since  $u_c > -u_h$ , we have  $u_g > -u_h$ , as claimed in the main text.

## A.2. Utility parameters and Ramsey steady state

Here we show how the utility parameters  $\omega_h$  and  $\omega_g$  are determined by the Ramsey steady state values. Let variables without subscripts denote their steady state values and consider the Ramsey steady state with constant debt from appendix A.1. Since  $\Pi = 1$  the Phillips curve constraint in (24) implies

$$1 + \eta - \eta \left( \omega_h h^\varphi c - \frac{g + \tilde{x}}{h} \right) = 0 \quad (35)$$

which delivers

$$\omega_h = \frac{\left( \frac{1+\eta}{\eta} - \frac{g+\tilde{x}}{h} \right)}{h^\varphi c} \quad (36)$$



and allows to determine the steady state values of  $u_h$  and  $u_{hh}$ . Adding up equations (32) and (33) then delivers

$$\gamma^1 = \frac{u_c + u_h}{\left( \frac{\eta}{\theta} \left( u_{hh}h + u_c \frac{g+\tilde{x}}{h} \right) + \frac{u_{cc}h}{\theta} \left[ 1 + \eta - \eta \frac{g+\tilde{x}}{h} \right] \right)}$$

and (33) gives

$$\gamma^3 = -u_h + \gamma^1 \frac{\eta}{\theta} \left( u_{hh}h + u_c \frac{g+\tilde{x}}{h} \right)$$

It then follows from (34)

$$\omega_g = g \left( \gamma^3 - \gamma^1 \frac{\eta}{\theta} u_c \right)$$

Table 1. **Baseline calibration**

Parameter Definition	Assigned Value	Parameter Code
quarterly discount factor	$\beta = 0.9913$	beta
price elasticity of demand	$\eta = -6$	eta
degree of price stickiness	$\theta = 17.5$	theta
1/elasticity of labor supply	$\varphi = 1$	varphi
fiscal waste	$x = 0$	x
utility weight on labor effort	$\omega_h = 19.792$	omegah
utility weight on public goods	$\omega_g = 0.2656$	omegag
technology shock process persistence	$\rho_z = 0.95$	rhoz
mark-up shock process persistence	$\rho_\eta = 0$	rhoeta
discount factor shock process		epsilon
quarterly s.d. technology shock innovation	$\text{s.d}(\varepsilon_{zt}) = 0.6\%$	sdz
quarterly s.d. mark-up shock innovation	$\text{s.d}(\varepsilon_{\eta t}) = 0.154\%$	sdeta

### A.3. VAR representation of ARMA process

The ARMA(1,5) specification of the stochastic shock processes has to be translated into a VAR(1) representation in order to feed it into the CompEcon toolbox. For example the process for  $z$  in equation (17) can be written as (see pg. 223 in Lütkepohl [1993]):

$$\begin{bmatrix} z_{t+1} \\ \varepsilon_{t+1} \\ \varepsilon_t \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \\ \varepsilon_{t-3} \end{bmatrix} = \begin{bmatrix} \mu_z \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_z & \alpha_{z,0} & \alpha_{z,1} & \alpha_{z,2} & \alpha_{z,3} & \alpha_{z,4} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_t \\ \varepsilon_t \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \\ \varepsilon_{t-3} \\ \varepsilon_{t-4} \end{bmatrix} + \begin{bmatrix} \alpha_z \varepsilon_{t+1} \\ \varepsilon_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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