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ON CITY SIZE DISTRIBUTION:
EVIDENCE FROM OECD FUNCTIONAL URBAN AREAS

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Abstract

An increasing amount of empirical evidence documents that city-size distribution within a country follows a power law, often in the form of Zipf's law. This paper provides new comparative evidence on city size distribution across OECD countries. It uses a database where urban agglomerations are consistently identified across different countries, through an algorithm based on population density and commuting patterns. The paper investigates whether Zipf's law fits well with data. A robustness check is carried out using a traditional administrative definition of cities. Results show that Zipf's law describes well city size distribution not only at country level, but also at wider spatial scales. The law does not fit as well with the data when using a traditional administrative definition of cities.

JEL classification: R12, O40

Key words: city size distribution, Zipf's law, rank-size rule, metropolitan areas

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1. Introduction

The empirical regularity that city-size distribution follows a power law has attracted the attention of economists and scientists from the beginning of the XX Century (Auerbach, 1913). In the context of city-size, a power law states that, within a given urban system, the frequency of cities having a certain population varies as a power of the population itself. Under the hypothesis of a Pareto probability distribution, the log(rank)-log(size) relationship is linear and when the coefficient is equal to -1 it means that we are under the Zipf's law (Zipf's, 1949). This law implies that the largest city is twice as large as the second largest city, three times the third one and so on along the whole urban hierarchy.

The relevance of Zipf's law in the context of city-size distribution is relevant in at least two respects. The first is connected with the desire to understand why activities distribute across space following a specific pattern. This has been synthesized by the Krugman's claim that a so stable regularity is "spooky" and should find a theoretical explanation (Krugman, 1996: 40). Another reason for the relevance of this strong statistical regularity in the city-size distribution is that the validity of Zipf's law, together with its stability over time, may set some constraints in the patterns of urban growth. More specifically, the growth trajectories of cities – both in the short and in the long run – could not change the overall city-size distribution. In other words, some cities can grow and some other decline changing their position within the hierarchy of the whole urban system. However, after these slow movements along the urban hierarchy, the size distribution of cities remains stable (Duranton, 2007). Another reason for the analysis of the shape of city size distribution is to try to understand whether there are different levels of economic efficiency for different shapes of the urban systems (number of cities and their sizes) (Storper, 2013), although this question is not the object of this paper.

During the last decade, there have been some attempts to provide a theoretical foundation to such regularity in the city-size distribution. Many of these are founded on random growth models, hence on the hypothesis that there is not any dependence between the growth of cities and their initial size. Accordingly, the current size of cities is a function of their past positive and negative shocks. Within this literature, Gabaix (1999) proposes a model where variations in city size are caused by random amenity shocks. Similar approaches, where city size is modelled with productivity shocks, are those by Eeckhout (2004) or Rossi-Hansberg and Wright (2007), while Duranton (2007) develops a model where changes in city size are driven by innovation shocks. In a more recent paper, Lee and Li (2013) developed a model where a Zipfian city size distribution is determined by many random factors jointly, which may be more or less correlated one with the other. From a more static perspective, Behrens *et al.* (2010) propose a model where differences in city-size are explained by small differences in their productivity. The latter is in turn dependent on the talent of residents and on the balance between agglomeration economies and congestion costs. Finally, Hsu (2012), following a Christallerian approach that also considers the location of cities in the geographic space, develops a model where a power law in the city-size distribution is generated from the presence of scale economies in the production of goods.

There is a wide amount of empirical literature aimed at testing the validity of Zipf 's law. On the whole, the Zipf's law was found to describe well city-size distribution and its fitting improves when only cities above a certain threshold – e.g. 100,000 inhabitants – are taken into account (Gabaix and Ioannides, 2004). In fact, when considering the whole set of cities, a log-normal distribution has a better fit than a Paretian distribution. In addition, Zipf's law was found to be sensitive to the definition of cities. More specifically, functionally defined urban areas seem to better fit to the rank size rule than administratively defined cities (Cheshire, 1999; Rosen and Resnick, 1980). In studying the city-size statistical distribution of US cities, Berry and Okulicz-Kozaryn (2012) found that once the units of observations are properly defined, the goodness of fit of Zipf's law increases. However, there are only few empirical tests that have been carried out to show that results are affected by the way in which urban units are defined (Rozenfeld et al., 2011). At the same time, most of the empirical work is on US cities. There are few cross-country analyses on city-size distribution. Among these, it is worth mentioning Rosen and Resnick (1980) and Soo (2005), which perform cross-country comparison of rank-size rule using both administrative cities and functionally defined urban areas.

This work provides evidence on the fit of Zipf's law for city size distribution at country level and OECD area. In principle, city-size distribution should be analysed at the urban system level. However, it is at least questionable how to identify an appropriate boundary of an urban system (Cheshire, 1999: 1344). Literature focuses mostly on national boundaries, but an urban system should be identified on the base of maximum interaction among cities and the minimum interaction with other urban systems (Vapnarsky, 1969). In an era of globalisation, national urban systems could be considered too small, since cities are now connected internationally in a global network of socio-economic relationships. This makes worthwhile to investigate whether city-size distribution obey to Zipf's law even at wider spatial scales than the national one, like for example at the level of continent or of the whole set of OECD countries.

This analysis is carried out by using functional definition of cities. The units of analysis – functional urban areas (FUAs) – are consistently identified across countries, using a methodology recently proposed by OECD (2012). In addition, results are compared with those obtained by using traditional administrative city boundaries. To our knowledge, this paper represents the first attempt to analyse city-size distribution across different countries by using a unique definition of FUAs and by comparing the results with administratively defined cities. In addition, the paper verify whether Zipf's law fits also in urban systems that go beyond the boundaries of single countries, like at the level of continents or at the whole system of the OECD countries.

The rest of the paper is organized as follows. Section 2 reviews the empirical literature of Zipf's law, underlying the main findings and open issues. Section 3 describes the data and the method with which the analysis is carried out. Section 4 estimated the rank-size equation for different OECD countries, underlying differences and peculiarities. Section 5 verifies whether Zipf's law fits data considering urban systems wider than country, like at continent level and at the whole OECD level. Finally, Section 6 compares the main results with those obtained using administratively-defined cities and Section 7 concludes.

2. Zipf's law literature: a reminder

The first condition under which Zipf's law holds is that the size distribution of cities must be approximated by a Pareto distribution, as in [1]:

$$y = \frac{a}{S^{\zeta}}, \qquad [1]$$

where S is the city size in terms of population; y is the number of cities with population greater than S; a is a positive constant equal to the population of the largest city. The Zipf's law hold in a special case of this Pareto distribution that is verified when ζ=1. In this case the size of a city times the number of cities with larger size (rank) is constant. More specifically, when ζ=1 the size of the largest city is twice as large as the size of the second largest cities, three times as large of the third one and so on along the whole urban hierarchy.

Zipf's law can be approximated empirically by a deterministic rule called rank-size rule. This can be identified by log transforming [1], obtaining the following linear equation:

$$\ln (y) = \ln (a) - \zeta \ln (S) \qquad [2]$$

where ζ can be estimated with OLS and, under Zipf's hold, it should be close to 1.

However, before trying to verify the validity of Zipf's law, several issues should be considered. First, rank-size rule is an approximation of the Zipf's law. Hence, the latter can still hold when the rank-size rule is only partially verified (Gabaix and Ioannides, 2004). Second, rank-size rule approximates well the Zipf's law when large cities are taken into account, but not the smallest ones. A lower threshold in the size of cities is necessary to make a linear interpolation fitting well with data. In fact, if small towns are also included in the analysis, then it was found that the city-size distribution is not anymore similar to a Pareto distribution, appearing closer to a lognormal one (Eeckhout, 2004). This is the reason why the magnitude of ζ is highly sensitive to the truncation point of data. Related with this, some other authors argued that the best approximation of city-size distribution is a combination of a power law distribution (for large cities) and a lognormal distribution for the smallest cities (Parr and Suzuki, 1973; Levy, 2009).

A third point to be underlined is related to the OLS properties to estimate ζ. As argued by Gabaix and Ioannides (2004), OLS estimation of ζ is likely to be downward biased, especially for small samples, since the size of the largest cities will appear too big. In addition, OLS standard errors can be also underestimated and, as a consequence, the Zipf's law can be rejected too often. In this respect, Gabaix and Ibragimov (2011) propose a simple way to overcome this problem in OLS estimation by running $\log(y\text{-}1/2) = a - \zeta \log(S)$. In addition, they propose to substitute the standard error with the asymptotical one, equal to $(2/n)^{1/2}\zeta$. Another possible way to estimate ζ could be by using the Hill's estimator, which is the maximum likelihood estimator under the hypothesis that city-size distribution follows a power law perfectly. However, given what has been highlighted above, this hypothesis may be too strong to completely rely on the Hill estimator. In addition, Hill's estimator still underestimates standard errors and it is likely to be biased for small samples (Gaibaix and Ioannides, 2004: 2349).

Finally, the validation of Zipf's law should not rely too much on the statistical acceptance or rejection of the hypothesis that $\zeta = 1$. As already argued, this rule cannot be taken too strictly, since rank-size rule is an approximation of the Zipf's law behind it. Hence, as Gabaix and Ioannides (2004) suggest, the empirical debate on Zipf's law should focus more on to what extent such a law fits well data rather than its pure statistical acceptance of rejection. In this respect, an estimated ζ in a range close enough to 1 – it may be between 0.8 and 1.2 – already indicates a certain success of the Zipf's law in describing the city-size distribution in an urban system.

3. Data and methods

3.1 Units of analysis

In order to do international comparison it is fundamental to use comparable units of analysis. In fact, it is well known that different countries have different definitions of cities and these differences can in turn limit to a substantial extent cross-country comparative analyses. This issue is well perceived by policy makers and much work has been done in order to provide a common definition of urban areas that allows for international comparative analyses. In this context, OECD (2012) recently proposed a unique methodology to identify FUAs in different countries and provided data for 29 countries.

FUAs are composed by a core and a commuting hinterland, consistently with most of the algorithms that are used for the same aim (Cheshire and Hay, 1989). The OECD's methodology applied in this work starts with the identification of the urban core/s of each area. The cores are identified using residential net density thresholds for each 1-square-kilometre cell of a regular grid structure. For European countries, population grid data are provided by the Joint Research Centre for the European Environmental Agency (EEA), while for all the other countries, harmonised gridded population data are provided by Landscan.[1] More specifically, all cells with a population of at least 1,500 inhabitants have been selected as urban core

1. Source: http://www.ornl.gov/sci/landscan/landscan_references.shtml.

cells.[2] Then, the final identification of the urban core is made by aggregating all contiguous LAU2 regions whose share of area covered by urban core cells is higher than 50% and whose total population is higher than 50,000 inhabitants.[3]

The second step consists in verifying whether two or more cores are parts of a single polycentric metropolitan region, instead of considering each core as the centre its own region. This approach makes it possible to detect those regions whose spatial structure is more complex than the traditional monocentric one composed by a single urban core and a surrounding hinterland. Hence, two or more cores are considered as part of the same functional region if at least 15% of resident population in one core commute to the other core. Data on commuting flows at the municipal (LAU2) level are hence needed to catch the relations among different urban cores.

The same data are used for the third and last step of the procedure, which is aimed at identifying the areas of influence of the cores (hinterland). In this respect, all municipalities whose shares of resident population that commute to the core exceed 15% are considered as composing the hinterland of the metro region. This threshold can be seen as arbitrary in certain respects, but is consistent with that used by other official country-based methodologies[4] and followed a sensitivity analysis. A full description of this methodology with results can be found on OECD (2012).

2. There are exceptions to this threshold for non European countries like Australia, United States and Canada. Please see OECD (2012) for more details.

3. Although there is a minimum threshold of 50,000 inhabitants for the identification of the urban core, it can happen that the total population of the FUA is slightly lower than that amount. This may happen either because less than 50% of municipal area is covered by urban core cells (so that a municipality is excluded) or because of differences in population data between the Census and gridded population data.

4. See, for example, Office of Management and Budget (1998) for the United States.

Table 1 Population in OECD functional urban areas: basic statistics by country

Country	Country code	N. of FUAs	Average pop. In 2001	Std. dev.	Minimum population	Maximum population
Austria	AT	6	742 213	850 919	243 858	2 454 241
Belgium	BE	11	552 942	633 100	118 732	2 273 476
Canada	CAN	34	635 658	1 103 443	75 385	5 450 470
Chile	CL	26	438 978	1 138 009	49 503	5 929 563
Czech Rep.	CZ	16	294 148	405 105	72 858	1 682 032
Denmark	DK	4	727 887	794 685	269 774	1 915 285
Estonia	EE	3	248 685	247 259	73 275	531 481
Finland	FI	7	372 637	440 979	115 903	1 356 482
France	FR	83	457 885	1 195 887	80 123	10 900 000
Germany	DE	109	481 033	628 987	78 946	4 334 215
Greece	GR	9	621 646	1 175 550	70 006	3 671 587
Hungary	HU	10	497 272	808 262	134 433	2 790 878
Ireland	IE	5	418 651	561 272	90 743	1 413 073
Italy	IT	74	395 804	727 784	50 190	3 867 226
Japan	JP	76	1 262 851	4 179 645	125 814	32 700 000
Korea	KR	45	864 218	3 007 069	45 262	20 100 000
Luxembourg	LU	1	388 217	.	388 217	388 217
Mexico	MEX	77	748 054	1 984 947	117 829	17 200 000
Netherlands	NL	35	333 772	416 115	59 569	2 175 368
Norway	NO	6	338 455	371 323	72 758	1 073 554
Poland	PL	58	362 511	536 363	65 175	2 881 670
Portugal	PT	13	426 444	740 392	63 470	2 650 467
Slovak Republic	SK	8	247 154	196 904	113 259	689 848
Slovenia	SI	2	381 648	212 694	231 250	532 045
Spain	ES	76	358 422	740 752	47 652	5 533 488
Sweden	SE	12	392 312	506 827	96 883	1 838 377
Switzerland	CH	10	409 945	333 676	116 145	1 114 737
United Kingdom	UK	101	425 160	1 063 771	82 384	10 600 000
United States	US	262	725 646	1 688 834	40 373	16 100 000

Source: Author's elaboration based on OECD (2012).

Table 1 summarizes the number of the identified FUAs as well as their basic population descriptive statistics for each of the 29 OECD countries that have been included in this analysis. The whole dataset includes 1,179 FUAs of different size, ranging from 40 thousands to almost 33 million inhabitants (Tokyo, Japan). The number of observations by country is also very diverse, reflecting the size of each country and ranging from one single observation in the case of Luxembourg to 262 units in the case of U.S.

4. City-size distribution at country level

This section provides evidence on the extent to which Zipf's law fits well data on city-size distribution at country level. Consistently with most of the literature, only countries with at least 20 cities are considered. Hence this part of the analysis is carried out for 12 OECD countries only. Tables 2 reports the OLS estimated ζ coefficients both applying the rank-size equation and that proposed by Gabaix and

Ibragimov (2011) (G-I), where rank is in the ln(*rank-1/2*) form. The table provides also the squared-R and the *t*-test under the null hypothesis that the coefficient estimated is equal to 1, hence that Zipf's law holds.

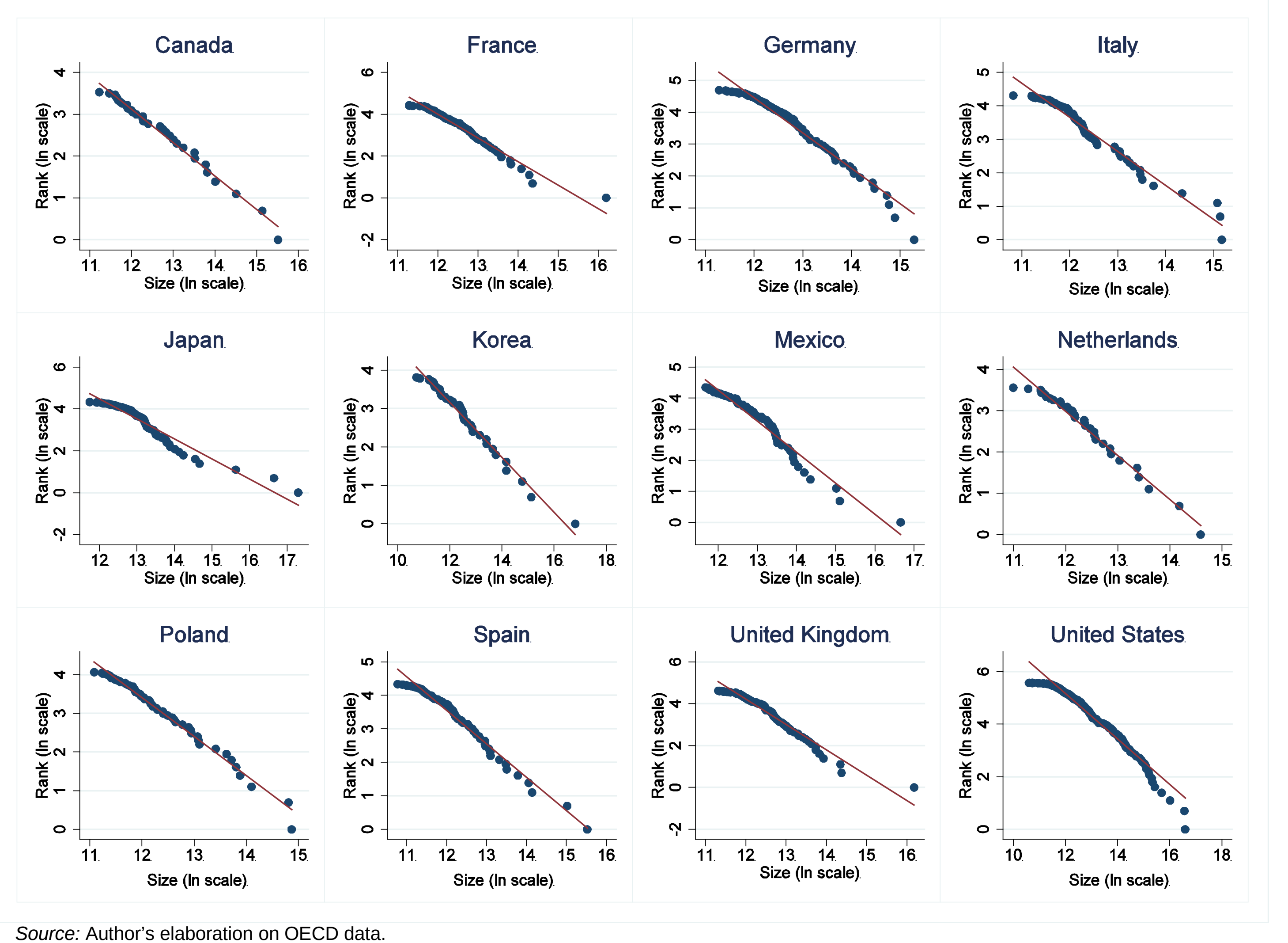
Looking at results, it is quite clear that notwithstanding the type of equation estimated (traditional Zipf's equation or G-I corrected version), the estimated coefficients are always close to 1. However, while the *t* test accepts the hypothesis that $\zeta=1$ only in 3 cases out of 12, the same hypothesis is always accepted using the G-I correction. Gabaix and Ioannides (2004: 2350) suggest that Zipf's law should not be tested, but just estimated, since it should be evaluated for its capacity to fit well data. The high squared-R values (Table 2) and a look at Zipf plots in Figure 1 support the hypothesis that Zipf's law well describe the rank-size relationship of urban systems in OECD countries.

Table 2 Results of OLS regression for Eqs. [2] and for its corrected version (G-I)

	rank-size			(rank-1/2)-size		
Country	ζ coeff. (rank-size)	sq.R	t-test ζ=-1	ζ coeff. (G-I)	sq.R	t-test ζ=-1
Canada	-0.798	0.99	169.12***	-0.887	0.97	0.541
Chile	-0.832	0.94	14.55***	-0.962	0.96	0.144
France	-1.128	0.97	33.47***	-1.209	0.97	1.114
Germany	-1.106	0.96	26.59***	-1.161	0.95	1.024
Italy	-1.019	0.97	0.71	-1.092	0.96	0.511
Japan	-0.957	0.93	1.9	-1.035	0.94	0.210
Korea	-0.715	0.99	467.37***	-0.788	0.99	1.273
Mexico	-0.999	0.95	0	-1.072	0.95	0.418
Netherlands	-1.067	0.98	5.46**	-1.187	0.96	0.660
Poland	-1.008	0.99	0.26	-1.087	0.97	0.433
Spain	-0.997	0.98	0.03	-1.068	0.97	0.391
United Kingdom	-1.215	0.96	73.66***	-1.292	0.97	1.606
United States	-0.864	0.97	183.6***	-0.888	0.96	1.449

Source: Author's elaborations on OECD data.

Figure 1 Zipf plot for 12 OECD countries using consistently identified functional areas

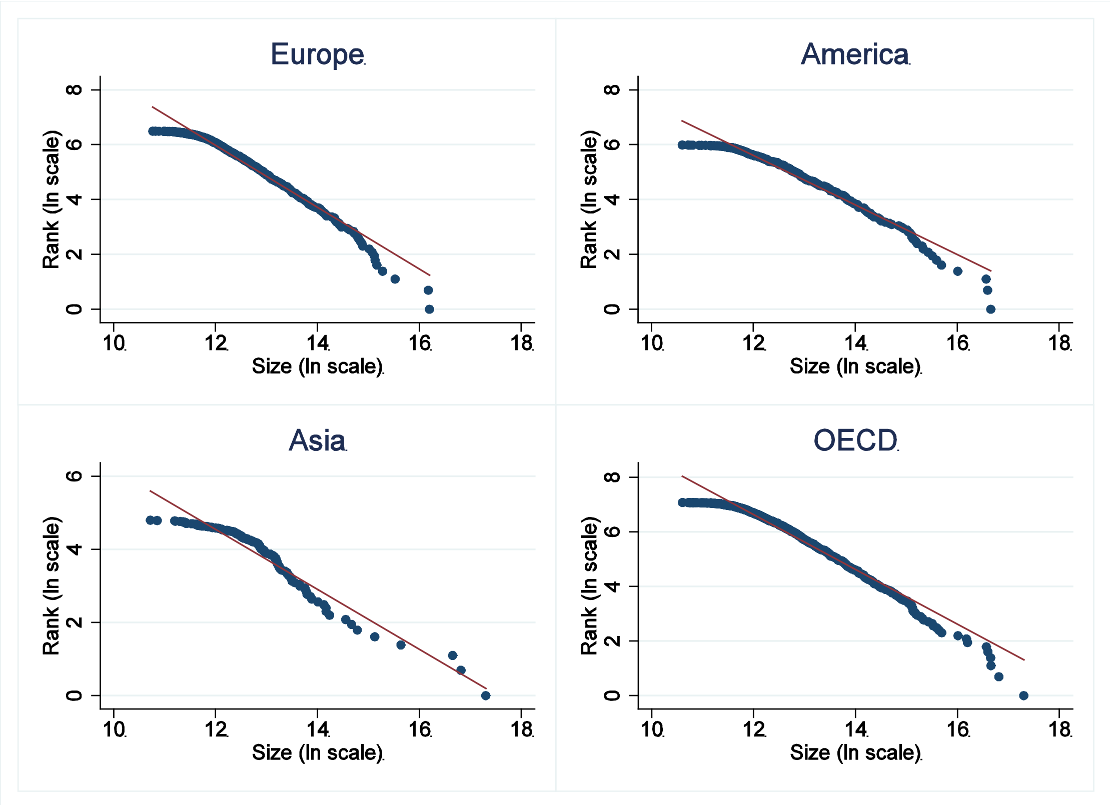


Source: Author's elaboration on OECD data.

5. City-size distribution beyond the country-level

The previous section has shown that Zipf's law fits well the city-size distribution in most OECD countries. Although empirical analysis (Cheshire and Magrini, 2009) show that – even in integrated markets such as Europe – national boundaries strongly affect economic adjustments and spatial disparities, the zipfian shape of city-size distribution is not necessarily confined at country level. Gibrat law, and therefore Zipf law, tends to hold at different spatial scales. Giesen and Südekum (2011) found that Zipf law fits data on German city-size distribution at both national and regional level. By using the OECD definition of FUAs, this paper provides some evidence on the city-size distribution for aggregation of countries. More specifically, OECD countries are aggregated by continent as well as at the level of the whole urban system of the OECD in order to see whether the Zipf law still fits well data.

Figure 2 Zipf plots for Europe, America, Asia and OECD functional urban areas



Source: Author's elaborations on OECD data.

Figure 2 shows the Zipf plots for Europe, America, Asia and the whole sample of OECD countries. With the exception of Asia – Japan and Korea – all the plots confirm an almost perfect linear relationship in most part of the distribution. The estimated coefficients are reported in Table 3, which shows that, even when aggregating countries by continent, the Zipf law fits well data. The coefficient associated to city-size ranges from a minimum of 0.82 for Asia to a maximum of 1.13 for Europe. Consistently to what argued by ESPON (2006), Europe emerged as the most polycentric continent, while Asia is the one with the highest urban primacy, where the largest cities dominate more strongly the whole urban system. This is measured by the magnitude of the coefficient associated to the logarithm of the city-size: the higher the coefficient in absolute value, the more balanced the spatial structure of the urban system. Surprisingly, the Zipf's law fits very well data on city size distribution when the whole urban system at the OECD level is considered. In this case, the hypothesis of a coefficient exactly equal to -1 cannot be rejected with the standard specification. When the G-I correction is used, the statistical validity of Zipf's law is accepted also for the aggregation of American and Asian countries.

Table 3 OLS results for eqs. [2] and its corrected version (G-I) by continent

	rank-size			(rank-1/2)-size		
	ζ coeff. (rank-size)	Sq.R	t-test ζ =-1	ζ coeff. (G-I)	sq.R	t-test ζ =-1
America	-0.903	0.97	129.98***	-0.921	0.96	1.21
Asia	-0.820	0.93	75.06***	-0.864	0.93	1.22
Europe	-1.127	0.97	274.63***	-1.143	0.97	2.27**
OECD (29)	-1.005	0.97	1.07	-1.015	0.97	0.35

Source: Author's elaborations on OECD data.

6. Does the city definition matter?

This section verifies whether there are substantial differences in the shape of the city-size distribution when using comparable definitions of cities based on economic functions rather than traditional administrative boundaries. The latter have been compared with functional urban areas in some of the 29 countries included in this analysis, with the exception of Canada, US, UK, Ireland, Japan, Korea and Portugal. The latter were excluded because determining the proper administrative boundaries at city level was not obvious. By considering the remaining countries, the first main evidence is that, even using traditional administrative definition of cities, Zipf's law approximates well the actual city-size distribution along the urban hierarchy. This is true both for countries and for larger geographical domains. In fact, the estimated coefficients of equation 2 when using administrative definitions of cities are always included between 0.8 and 1.2 (Table 4). Again, the statistical validity of the Zipf's law is more likely to be accepted using the G-I correction.

A closer look at the estimates reported in Table 4 helps highlighting some of the differences that emerge when using different city-definitions. First, when the OECD functional definition of cities is used, the estimated coefficients are always closer to -1 with respect to the case where administrative units are used. This is especially true when city-size distribution is observed beyond the boundaries of single countries. In these cases, two factors might be at the basis of the better fit of Zipf's law with functionally defined cities. First, the actual size of cities should be better defined when economic self-organisation is taken into account, for example through commuting flows. Second, when city boundaries are consistently identified following everywhere the same functional approach, there should be higher comparability, since national administrative boundaries can be very different. For example, the average size of municipal boundaries ranges from less than 2,000 inhabitants for France, Czech Republic and Slovak Republic to more than 50,000 inhabitants for Denmark, Lithuania and UK.

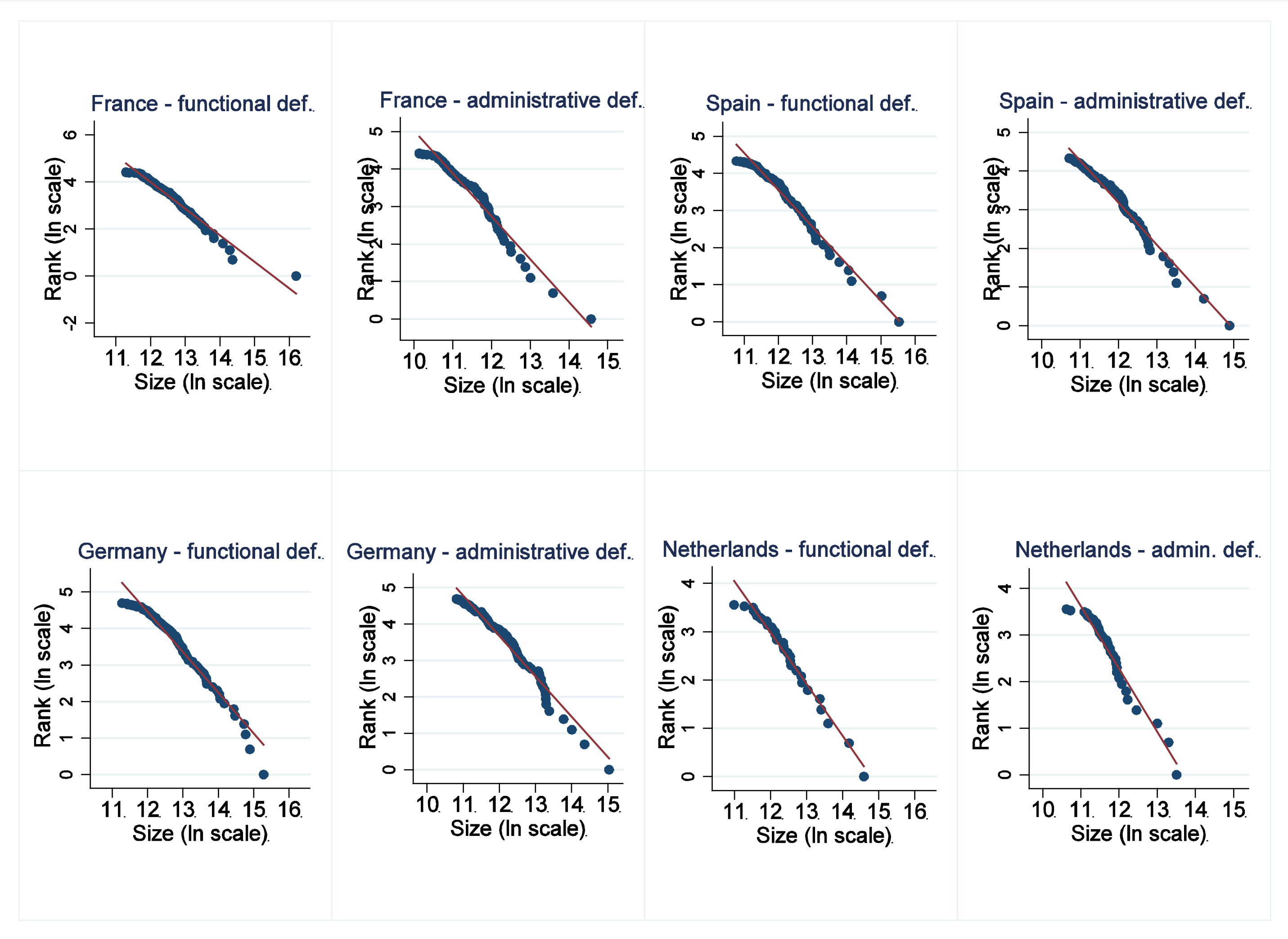
Table 4 OLS results for eqs [2] by type of city definition

	Functional definition of city			Administrative definition		
	coeff. (rank-size)	sq.R	t-test ζ=-1	coeff. (rank-size)	sq.R	t-test ζ=-1
OECD (22)	-1.10	0.96	131.01***	-1.15	0.95	212.86***
America	-1.02	0.94	0.52	-1.17	0.90	19.95***
Europe	-1.11	0.97	148.98***	-1.19	0.97	419.9***
France	-1.13	0.97	33.47***	-1.14	0.97	34.61***
Germany	-1.11	0.96	26.59***	-1.11	0.97	32.33***
	coeff. (rank-1/2)-size	sq.R	t-test ζ=-1	coeff. (rank-1/2)-size	sq.R	t-test ζ=-1
OECD (22)	-1.11	0.96	1.83*	-1.17	0.94	2.56**
America	-1.07	0.93	0.49	-1.23	0.88	1.33
Europe	-1.12	0.96	1.81*	-1.21	0.96	2.89***
France	-1.10	0.81	0.58	-1.22	0.96	1.16
Germany	-1.08	0.82	0.56	-1.17	0.96	1.05

Source: Author's elaborations on OECD data.

Another difference in using administrative rather than functionally defined city-boundaries is visible in Figure 3. In most of the cases, it emerges that a linear relationship between log(rank) and log(size) of cities fits better by using a functional definition of cities rather than an administrative one. This is already visible by the average higher squared-R reported in Table 4. In summary, a functional definition of cities does not change substantially the shape of the city-size distribution, but it increases, on average, the goodness of fit of the Zipf's law both at country level and at wider geographical scales.

Figure 3 Zipf plot: administrative vs. functional definition of city



6 Concluding remarks

This paper provided new statistical evidence on the shape of the city-size distribution in OECD countries. Such evidence is based on a consistent definition of the units of analysis (OECD, 2012), which are identified through functional criteria, and allow accounting for a robust definition of the actual economic size of cities. Evidence was found that, on the whole, Zipf's law fits well data on city size distribution. This is true in most of the OECD countries, but also, and even more, for wider geographical entities, including continents and the whole OECD urban system. In other words Zipf's law approximates well the city-size distribution at different scales, which are not confined to national boundaries and that include, according to existing literature (Giesen and Südekum, 2011), sub-national boundaries. The statistical validity of Zipf's law increases substantially using the corrected equation proposed by Gabaix and Ibragimov (2011). Results have been compared with those obtained when cities are considered in their traditional administrative boundaries, as defined at a national basis. In this respect, evidence shows that the overall picture does not change substantially when administrative cities are used, but the fit of Zipf's law is lower.

A possible step ahead is to look at whether the shape of the city-size distribution has changed over time, in a long time horizon. This would require an adjustment of city boundaries in the different points in time, which is not an easy task if the actual economic size of cities is to be taken into account. Extending this analysis over time would also make it possible to investigate whether the empirical regularity of the city-size distribution (Zipf's law) is associated with the independency of city growth from city size (Gibrat law) for different countries using comparable spatial units.

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