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MEASURING THE JOINT DISTRIBUTION OF HOUSEHOLD'S INCOME, CONSUMPTION AND WEALTH USING NESTED ATKINSON MEASURES[1]

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ABSTRACT

Following recommendations from the Stiglitz-Sen-Fitoussi Commission (2009), this paper proposes the use of a new methodology to measure the joint distribution of households' income, consumption and wealth. Based on a multidimensional extension of the Atkinson generalized mean framework, the paper justifies the application of this methodology based on a set of standard and acknowledged properties, proving that this is the sole methodology satisfying them all simultaneously. The derived multidimensional index has an intuitive structure, which allows evaluating the overall material conditions of households under different perspectives and with varying sensitivity to distributionnal issues. Under its general form, the index encompasses a class of sub-indices that impose various restrictions on its parameters; the paper discusses the extent to which different restrictions on parameters affect the multidimensional assessments of various population groups, and provides some empirical illustrations using those different specifications. The question addressed by the multi-dimensional measure presented here is whether the joint consideration of household income, consumption and wealth modifies substantially the picture of material living standards of different individuals and groups relative to the one provided by income alone. Based on the dataset used here, the paper provides strong evidence on the importance of such a multidimensional assessment.

Keywords: material conditions, multidimensional measure, generalized mean, inequality

RÉSUMÉ

Dans le prolongement des recommandations de la Commission Stiglitz-Sen-Fitoussi (2009), cet article propose une nouvelle approche pour la mesure de la distribution jointe du revenu, de la consommation et de la richesse. En se basant sur une extension multidimensionnelle des moyennes généralisées *à la* Atkinson, l'utilisation d'une telle structure est justifiée sur la base de propriétés standards utilisées dans la cadre de la théorie de la mesure du bien-être. Il est en outre démontré que la structure sélectionnée est la seule forme possible satisfaisant l'ensemble des propriétés simultanément. Cette nouvelle mesure revêt une structure relativement intuitive permettant d'évaluer le bien-être matériel des ménages selon différentes perspectives et avec une sensibilité ajustable aux inégalités, ce dans les différentes dimensions considérées. Sous sa forme générale, l'index contient en outre une variété de spécifications se traduisant par différentes restrictions paramétriques; l'impact de ces restrictions ainsi que leur signification économique sont discutés, théoriquement et empiriquement. La question centrale adressée par cette nouvelle mesure est ainsi de savoir si l'appréciation jointe du revenu, de la consommation et de la richesse est à même d'altérer significativement la mesure des conditions de vie matérielles des ménages, en comparaison de l'approche standard se focalisant sur le revenu uniquement. Sur la base des données utilisées, ce document conclue sur la pertinence de cette nouvelle approche.

Mots-clés: Bien-être matériel, mesure multidimensionnelle, moyennes généralisées, inégalités

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Introduction

1. Well-being is a multi-dimensional concept, and measuring it requires a multi-dimensional framework. This is one of the key recommendations of Stiglitz et al. (2009). Building on academic research and concrete measurement initiatives, this report identified the most relevant dimensions of well-being that ought to be considered for a complete assessment of a society's welfare: material living standards, health, education, personal activities (including work), political voice and governance, social connections, environmental quality, insecurity and subjective well-being. The proposals laid to rest the conventional approach that relies on income as the single, sufficient determinant of people's well-being, a criticism rooted stongly in Sen's capabilities framework. The report also highlighted the need to look at distributions and not only at aggregates or average measures of different achievements.

2. The appraisal of the material living standards also requires a multi-dimensional approach. Indeed, and despite the fact that material living standards of individuals are a joint function of income, consumption and wealth, income alone is most often used in practice. This is clearly limiting since it is quite possible for income to be small but for wealth to be large (or *viceversa*). From a poverty perspective, headcounts of the income-poor could lead to the inclusion of "false positives", *e.g.* people with income under the poverty threshold but with moderate or high wealth, such as business owners whose current income may not be representative of their economic resources. Moreover, both wealth and income may be small but the provision of in-kind transfers by governments or relatives outside the household may meet, partially or completely, people's consumption needs. So both income and wealth determine consumption possibilities, while low levels of income and wealth may not always imply a low level of consumption. These examples, relying fundamentally on the absence of perfect correlation between the three dimensions of material living standards at the level of each person, point toward the necessity for an integrated multi-dimensional framework to get a better understanding of material living standards.

3. Multi-dimensional well-being measures have a long tradition in economics and statistics, beginning with seminal articles by Kolm (1977) and Atkinson *et al.* (1982). However, it is only recently that this literature has taken off, with the recognition by policymakers of the need for tools which encompass several dimensions. Probably the most famous initiative in this field has been the Human Development Index (HDI) introduced by the United Nation Development Programme in 1990, an aggregate indicator based on the average of individual circumstances for three dimensions (health status, education and income). Its simple methodology makes the HDI readily comprehensible and thus popular. However, this indicator fails to meet the challenge of multidimensional measurement as it ignores inequality in each of the three dimensions. Moreover the HDI ignores that in a multidimensional context there are two distincts forms of inequality. The first is identical to the single-dimensional case and considers the spread of the distributions for each dimension. The second form of inequality appears specifically in a multi-dimensional setting, and it relates to the statistical dependence between the dimensions: when all dimensions are strongly correlated, higher (lower) achievement in one dimension leads to higher (lower) achievements in the others. Clearly, an inequality sensitive multi-dimensional framework has to take both forms of inequalities into account.

4. This paper builds on the proposal of Foster *et al.* (2005). The first section proposes and theoretically justifies a methodology to analyse the joint distribution of household income, consumption and wealth in a multi-dimensional, inequality-sensitive framework: this methodology maps these three variables into a single index, which is called here the Material Condition Index (MCI). This paper demonstrates that such an index is particularly well suited for well-being dimensions that share the same unit of measurement, and so is specifically appealing for the analysis of households' material living standards. In a second section, these theoretical results will be illustrated by a numerical example. Finally, Section 3 provides an empirical application, based on a French household survey that jointly records people's income, consumption and wealth; these empirical illustrations show that the use of the MCI as a measure of households' material living standards provides a different picture to one based solely on income, at both the aggregate and the microeconomic level. The last section concludes.

The nested Atkinson measures

5. Measuring households' material living standards requires two major steps: first, to identify the key dimensions of the material space; second to choose a specific procedure for aggregation according to some desirable properties in order to map the dimension into a single statistic.

6. Most of the time, the usual practice is to choose a single dimension (typically income) and to summarize it by a key statistic: the mean, median or some fractile's thresholds of its distribution are the most widely used measures. Others, more advanced practices, wrap at the same time information on efficiency (the general level achieved) and equity (the spread of the distribution's achievement) according to some general concept. Probably the most famous example of such practice is that proposed by Atkinson (1970). This framework discounts the mean income level by a measure of the spread of the distribution, according to a utilitarian welfare concept. Stated otherwise, the Atkinson's measure sees inequality as a "waste": the discount factor can be conceived as the share of total income that is destroyed if each individual in the distribution has equal income and the resulting distribution is socially indifferent. Atkinson's measure makes use of the generalized mean, an income standard used also by Blackorby *et al.* (1981) for the uni dimensional case. The index presented in this paper is a multidimensional generalization of the Atkinson's income standard.

Preliminaries

7. Let $M = \{1,..., m\}$ be the set of relevant dimensions chosen in the material space, with $m \in \mathbb{N}$ and $m \geq 2$. The set of units of observations (likely households or individuals) is $N = \{1,..., n\}$ with $n \in \mathbb{N}$ and $n \geq 2$. A distribution matrix defined in the strictly positive subset of the Euclidean $n \times m$ space $X \in \mathbb{R}_{++}^{n \times m}$ represents a specific distribution of outcomes for the n units in the m dimensions. Let X_n be the set of all distribution matrices of size n, and let $\mathbf{X} = \cup_{n=1}^{\infty} X_n$ be the set of all distribution matrices. The nm^{th} term x_n^m of a distribution matrix is the achievement of the unit n in dimension m; its n^{th} row $x_n^{\cdot}$ is the transposed vector of the m achievements of unit n; and its m^{th} row $x_{\cdot}^m$ is the vector of the achievements of the n units in dimension m.

8. The comparison of distribution matrices can be performed by the use of a function W^i that maps X to $\mathbb{R}_+$, *i.e.* that aggregates the matrix into a single non-negative number, the Material Condition Index. In a multi-dimensional framework there are two ways of aggregating a distribution matrix. The first is to aggregate across all the units (as in the single dimensional case) and then across all the dimensions. Doing this, one obtains for all dimensions a series of scalar stacked in a m transposed vector. These m elements are then aggregated to obtain a single number. The second approach aggregates first across dimensions, thus generating for each unit an overall level of achievements that can then be stacked in a n vector; the elements of this vector are then aggregated into a single number. The first approach favors the dimensional perspective, while the second approach emphasizes the unit perspective. Following the terminology established by Kolm (1977), the first way can thus be seen as a *specific* aggregation procedure (called hereafter S-aggregation), while the second is an *individualistic* aggregation procedure (called hereafter I-aggregation). These two procedures can be formally described as follow:

$$W^S : \mathbb{R}_{++}^{n \times m} \to \mathbb{R}_{++} : X \to W^S(X) = W_m\left[W_n\left(x_{\cdot}^1\right),...,W_n\left(x_{\cdot}^m\right)\right]$$
$$W^I : \mathbb{R}_{++}^{n \times m} \to \mathbb{R}_{++} : X \to W^I(X) = W_n\left[W_m\left(x_1^{\cdot}\right),...,W_m\left(x_n^{\cdot}\right)\right]$$

where W_m and W_n are sub-aggregative functions across the dimensions or the units, respectively, while $W^i \ \forall i \in \{S, I\}$ is the function having the index as output.

9. Having described the general guidelines for the computation of an index in a multi-dimensional framework, we can now turn to the basic properties that the aggregation functions W^i should verify.

Properties of the aggregation function

10. The properties considered in what follows are all standard in the litterature on welfare comparisons and inequality; a satisfactory multidimensional measure should ideally be compliant with all of them to the largest extent possible. Box 1 below provides the analytical description of these properties, which can be outlined as follows[3]:

- The measure should not be overly sensitive to small changes in some of the entries of the distribution matrix (*continuity*).
- All terms of the distribution matrix should increase people's welfare (*monotonicity*).
- When all the terms of the distribution matrix have the same value, then the index should also be equal to this value (*normalization*).
- The rescaling of two distribution matrices by the same factor should not change their ranking according to the value of the index (*weak homotheticity*).
- The rescaling of two distribution matrices by different factors for each dimension should not change their ranking according to the value of the index (*strong homotheticity*).
- Information other than the one enclosed in the terms of the distribution matrix should be irrelevant for the evaluation of the well-being of individuals (*anonymity*).
- The index's values should be comparable across populations of different sizes (*cloning invariance*).
- If the material living standards of two sub-populations change in such a way that they rise for one group and are unchanged for the other, then the material living standards of the whole population should also rise (*subgroup consistency*)[4].
- If a focus is made on a sub-group of dimensions or on a sub-population instead of the whole sets, then the index obtained should be independant of the choice of the resulting complementary sub-set (*multidimensional essentiality*).
- If a transfer occurs from a richer to a poorer unit (with the initially poorer unit ending up with less achievement than the initially richer unit), then the value of the index should increase, as inequalities have been reduced (*multidimensional Pigou-Dalton smoothing*). The index satifying this property is then qualified as inequality adverse.
- A rearrangement of the achievements of two units, such that one gets the highest achievements in all dimensions and the other the lowest achievements (thus increasing the dimensions' correlations), should reduce the value of the index as the poorer unit cannot compensate the lower values resulting from the swap (*non-increasing comonotonic swaps*).

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3 See Chakravarty (1990) for a review of the fundations of the value judgments underlying each axioms.

4 This is a natural property, useful for analysis combining national and regional levels, but that is nonetheless not always satisfied by several common indices such as the Gini index (Hicks, 1997).

Box 1. Defining axioms for a multidimensional measure

Axiom 1 (CONTINUITY) $\forall n \in \mathbb{N}$, $\forall X, X^{'} \in \mathbf{X}$, the sets $\{W^i(X^{'}) > W^i(X)\}$ and $\{W^i(X) > W^i(X^{'})\}$ are open.

Axiom 2 (MONOTONICITY) $\forall n \in \mathbb{N}$, $\forall X, X^{'} \in X_n$, if $X \neq X^{'}$ and $X^{'} \geq X$ then $W^i(X^{'}) > W^i(X)$.

Axiom 3 (NORMALIZATION) $\forall n \in \mathbb{N}, \forall \alpha \in \mathbb{R}_{++}$, $\forall X \in X_n$ such that $X = \alpha \mathbf{1}_{n\times m}$ (where $\mathbf{1}_{n\times m}$ denote a $n \times m$ dimensional matrix where all entries are equal to one), then $W^i(X) = \alpha$.

Axiom 4 (WEAK HOMOTHETICITY) $\forall n \in \mathbb{N}, \forall \theta \in \mathbb{R}_{++}$, $\forall X, X^{'} \in X_n$, $W^i(X^{'}) > W^i(X) \Leftrightarrow W^i(\theta X^{'}) > W^i(\theta X)$.

Axiom 5 (STRONG HOMOTHETICITY) $\forall n \in \mathbb{N}$, $\forall X, X^{'} \in X_n$, $\forall \Lambda$ some positive diagonal matrices of size $n \times m$, $W^i(X^{'}) > W^i(X) \Leftrightarrow W^i(X^{'}\Lambda) > W^i(X\Lambda)$.

Axiom 6 (ANONYMITY) $\forall n \in \mathbb{N}$, $\forall X \in X_n$, $\forall \Pi$ some permutation matrix of size $n \times n$, $W^i(\Pi X) = W^i(X)$[5].

Axiom 7 (CLONING INVARIANCE) $\forall r \in \mathbb{N}$, $\forall X \in \mathbf{X}$, $W^i([X]_r) = W^i(X)$ where $[X]_r$ is the replication of order r of X (i.e. the n rows of X have been cloned r times).

Axiom 8 (SUBGROUP CONSISTENCY) $\forall n_1 \in \mathbb{N}$ and $\forall n_2 \in \mathbb{N}$ with $n_1 + n_2 = n$, $\forall (X_1; X_1^{'}) \in X_{n_1}$, $\forall (X_2; X_2^{'}) \in X_{n_2}$, $W^i(X_1^{'}) > W^i(X_1)$ and $W^i(X_2^{'}) = W^i(X_2) \Rightarrow W^i(X_1^{'}, X_2^{'}) > W^i(X_1, X_2)$.

Axiom 9 (MULTIDIMENSIONAL ESSENTIALITY) $\forall J \in \{n \in \mathbb{N}, m \in \mathbb{N}\}$, $\exists S \subset J$ a non-singleton set that is separable from its complement S^C.

Axiom 10 (MULTIDIMENSIONAL PIGOU-DALTON SMOOTHING) $\forall n \in \mathbb{N}$, $\forall X \in X_n$, $\forall \Omega$ some doubly stochastic matrix of size $n \times n$, $W^i(\Omega X) > W^i(X)$[6].

Axiom 11 (NON-INCREASING COMONOTONIC SWAPS) $\forall n \in \mathbb{N}, \forall X, X^{'} \in X_n$ with $X^{'}$ not a permutation of X and $\exists (n_k, n_l) \in n$ such that $x_k^{m'} = \max(x_k^m, x_l^m)$ $\forall m \in \mathbb{N}$, $x_l^{m'} = \min(x_k^m, x_l^m)$ $\forall m \in \mathbb{N}$, $x_n^{'} = x_n$ $\forall n \notin (n_k, n_l)$, then $W^i(X^{'}) < W^i(X)$.

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5. A permutation matrix is a square matrix with each row and column having one as an entry while all the others are zero.

6. A doubly stochastic matrix Ω of size $n \times n$ is a square matrix whose individual terms are non negative, that is not a permutation matrix, and such that $\mathbf{1}_n \Omega = \mathbf{1}_n$ and $\Omega \mathbf{1}_n^T = \mathbf{1}_n^T$ (where $\mathbf{1}_n^T$ denote the transposed of $\mathbf{1}_n$).

Characterization and properties of the Material Condition Index

11. Having described the properties that an aggregation function should verify, the remainder of this section identifies the function that leads to the Material Condition Index. As this function has a generalized mean strucure, this section starts by recalling the usefulness of generalized means for the assessment of living standards in the case of a single dimension.

What is the meaning of a generalized mean?

12. Let us return momentarily to the uni-dimensional case, and assume that $(x_1,..., x_n)$ is a distribution of income over n units of observations. The generalized means of curvature q $\mu_q(x_1,..., x_n)$ of this distribution is given by the following formula:

$$\mu_q(x_1,\ldots,x_n) = \left[\frac{1}{n}\sum_{i=1}^{n} x_i^q\right]^{\frac{1}{q}} \forall q \neq 0 \qquad (1)$$

$$= \prod_{i=1}^{n} x_i^{\frac{1}{n}} \text{ for q = 0}$$

13. The generalized mean reduces to the arithmetic mean $\mu(x_1,..., x_n)$ when $q = 1$, to the geometric mean when $q = 0$ and to the harmonic mean when $q = -1$. One can see that when q increases, greater emphasis is put on the upper tail of the distribution. Conversely, when q decreases, greater weight is placed on the lower tail. As a result we have:

$$\forall q > 1, \mu_q(x_1,..., x_n) > \mu(x_1,..., x_n)$$

and

$$\forall q < 1, \mu(x_1,..., x_n) > \mu_q(x_1,..., x_n)$$

14. The arithmetic case thus provides a natural dividing line for the choice of the curvature parameter: $\mu_q(x_1,..., x_n)$ is said to be inequality-adverse for $\forall q < 1$ (as a rise in income in the lower tail of the distribution will increase the generalized mean by more than a similar increase in the upper tail); equality-adverse for $\forall q > 1$ (as a rise in income in the upper tail of the distribution will increase the generalized mean by more than a similar rise in the lower tail) and inequality neutral for $q = 1$ (as a rise in any part of the income distribution will increase equally the generalized mean). The limiting cases explicitly express the concern for inequality that one selects according to the choice of the curvature parameter as for $q \to -\infty$, $\mu_q(x_1,..., x_n) \to \min(x_1,..., x_n)$ and for $q \to +\infty$, $\mu_q(x_1,..., x_n) \to \max(x_1,..., x_n)$: in otherwords, when the curvature parameter is $-\infty$, only a rise in the lowest income of the distribution will increase the generalized mean.

15. Because of these properties, generalized means are powerful tools for welfare analysis as they avoid the setting of arbitrary thresholds in the income distribution for the appraisal of inequality and poverty. On the contrary, generalized means consider the whole distribution, but place continuously greater weights on lower incomes as q diminishes. As a result, one can more and more mute the upper part of the distribution, while at the same time not completly ignoring incomes just above a threshold, which is the

case when using a poverty line (where units below the line receive full weight, while those above receive none).

16. The Material Condition Index is based on the notion of generalized means extended to a multi-dimensional framework, *i.e.* a nested generalized mean. The formulas below are given for I-aggregation:

$$W^I(X) = \left[\frac{1}{n} \sum_{i=1}^{n} \left[\left[\frac{1}{m} \sum_{j=1}^{m} (x_i^j)^r \right]^{\frac{1}{r}} \right]^q \right]^{\frac{1}{q}} \forall q < 1, \forall r < 1, q \neq 0, r \neq 0$$

$$= \prod_{i=1}^{n} \left[\prod_{j=1}^{m} (x_i^j)^{\frac{1}{m}} \right]^{\frac{1}{n}} \textit{for } q = 0 \text{ and } r = 0$$

$$= \left[\frac{1}{n} \sum_{i=1}^{n} \left[\prod_{j=1}^{m} (x_i^j)^{\frac{1}{m}} \right]^q \right]^{\frac{1}{q}} \forall q < 1, \; q \neq 0 \text{ and } r = 0$$

$$= \prod_{i=1}^{n} \left[\left[\frac{1}{m} \sum_{j=1}^{m} (x_i^j)^r \right]^{\frac{1}{r}} \right]^{\frac{1}{n}} \text{for } q = 0 \text{ and } \forall r < 1, \; r \neq 0$$

17. In the above expressions, q has the same interpretation than in the uni-dimensional case, expressing the concern for inter-individual inequality, while r penalises for the unbalancement in achievements between dimensions for each inviduals (what can be denominated as “intra-individual” inequality). This last notion will be discussed further latter in the paper.

Identification

18. We begin first with a theorem that identifies nested generalized means for I-aggregation functions.

Theorem 1: *An I-aggregation function* $W^I : \mathbb{R}_{++}^{n\times m} \to \mathbb{R}_{++}$ *is a nested generalized mean of curvature* $q<1$ *and* $r<1$ *if and only if* W^I *satisfies (A1), (A2), (A3), (A4), (A6), (A7), (A8), (A9) and (A10).*

Proof: See annex 1.

19. Theorem 1 states that the nested generalized mean is not only an aggregation structure that satisfies the nine standard properties described in Box 1 simultaneously, but also that these nine properties uniquely identify such structure. This theorem relies on weak homotheticity, a property useful when working with dimensions having the same unit of measurement, as is the case of the Material Condition Index. The strong version of homotheticity is more general, allowing dimensions of different nature, such as monetary and non-monetary variables. The theorem below reveals the binding character of this axiom on the choice of functionnal forms.

Theorem 2 *A nested generalized mean I-aggregation function* $W^I : \mathbb{R}_{++}^{n\times m} \to \mathbb{R}_{++}$ *verifies the strong version of homotheticity (A5) if and only if:*

$$W^I(X) = \prod_{i=1}^{n}\left[\prod_{j=1}^{m}\left(x_i^j\right)^{\frac{1}{m}}\right]^{\frac{1}{n}}$$

Proof: See annex 1.

20. The two theorems above are concerned with I-aggregation, *i.e.* aggregating first across units, in order to have an overall score for each of them, and then aggregating across dimensions. However, this procedure relies on a arbitrary choice of sequencing. A convenient property for a multi-dimensional index is ***path-independency***: this property implies that aggregating first across dimensions and then across units, or first across units and then across dimensions yield the same numerical results. This property would allow making a multi-dimensional assessment based on minimal data requirements. The following theorem characterizes the curvature restriction imposed on the general form of the MCI that is required by a path-independent nested generalized mean structure.

Theorem 3 *A nested generalized mean of curvatures* q *and* r *is path-independent if and only if* $q = r$.

Proof: See annex 1.

21. Finally, we saw from the first theorem that (A11) is neither necessary nor sufficient for the identification of a nested generalized mean structure. But this axiom is nonetheless important in a multidimensional framework as a change in the association between dimensions is also a source of inequality. The following theorem states that there is an implicit trade-off between the curvature restriction of the index and its sensitivity to an increase in the dimension's correlations.

Theorem 4 A path-independent nested generalized mean cannot satisfy (A11).

Proof: See annex 1.

22. The theorems presented so far provide theoretical justification for the MCI, and identify several implications for the use of a nested generalized means structure in a multidimensional framework. The following section provides a simple numerical illustration of these results and their interpretation.

Numerical example

23. For the sake of simplicity, let's assume that a joint distribution of household income (I), consumption (C) and wealth (W) across 5 units (displayed in line) is observed. Let's denote by X the corresponding matrix (numerical values are just used for illustration):

$$X=(I,C,W)=\begin{pmatrix} 3 & 1 & 2 \\ 6 & 2 & 5 \\ 7 & 3 & 10 \\ 7 & 5 & 14 \\ 10 & 5 & 24 \end{pmatrix}$$

24. The first step in the construction of the MCI is to normalize the entry of X in order to have a ratio-scale measure of the dimensions, with the lowest value set as 0% achievement level and the highest a 100% achievement. In this way, the comparability across the three monetary variables is guaranteed: 50% of the highest achievement in one variable is the same as 50% of the others (Alkire et al., 2010). The ratio scale transformation applied below is the one described in Anand *et al.* (1994): for each dimension, this transformation requires taking each value and subtracting from it the lowest achievement possible in the associated dimension (assuming it is zero), and then dividing the result by the difference between the maximum achievement observed and the minimum achievement possible in the distribution. The normalized matrix X^N is thus:

$$X^N=\begin{pmatrix} \frac{3-0}{10-0} & \frac{1-0}{5-0} & \frac{2-0}{24-0} \\ \frac{6-0}{10-0} & \frac{2-0}{5-0} & \frac{5-0}{24-0} \\ \frac{7-0}{10-0} & \frac{3-0}{5-0} & \frac{10-0}{24-0} \\ \frac{7-0}{10-0} & \frac{5-0}{5-0} & \frac{14-0}{24-0} \\ \frac{10-0}{10-0} & \frac{5-0}{5-0} & \frac{24-0}{24-0} \end{pmatrix}=\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix}$$

25. As outlined previously, the general idea behind a multidimensional measure is the same as in a one-dimensional world. For the latter, a vector is mapped into a single number (mean, variance, generalized mean...). For the former, a matrix is mapped into a single number. The crucial issue here is the order of aggregation. The first option aggregates first across each line, obtaining thus a global score for each individual, and then across the scores (I-aggregation). The second path starts from the columns, aggregating first across the dimensions, then aggregating each of the dimensions' scores into a single number (S-aggregation). In the case of aggregation *via* the arithmetic means at each step (or equivalently, in the case of a path-independent nested generalized mean of curvatures 1 and 1, with no concern for both forms of inequality), I-aggregation implies the following:

$$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \begin{pmatrix} 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \\ 1 \end{pmatrix} \rightarrow 0.6$$

while S-aggregation gives:

$$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ (0.6 & 0.7 & 0.5) \\ & \downarrow & \\ & 0.6 & \end{matrix}$$

26. From theorem 1, we know that inequality-adverse nested generalized means have a curvature strictly inferior to 1. Re-doing the previous computation for I-aggregation with a curvature of -2 (*i.e.* q and r are identical, and equal to -2) at each step yields to the following results:

$$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \begin{pmatrix} \left[\frac{1}{3}\left(0.3^{-2}+0.2^{-2}+0.1^{-2}\right)\right]^{\frac{1}{-2}} \\ \left[\frac{1}{3}\left(0.6^{-2}+0.4^{-2}+0.2^{-2}\right)\right]^{\frac{1}{-2}} \\ \left[\frac{1}{3}\left(0.7^{-2}+0.6^{-2}+0.4^{-2}\right)\right]^{\frac{1}{-2}} \\ \left[\frac{1}{3}\left(0.7^{-2}+1^{-2}+0.6^{-2}\right)\right]^{\frac{1}{-2}} \\ \left[\frac{1}{3}\left(1^{-2}+1^{-2}+1^{-2}\right)\right]^{\frac{1}{-2}} \end{pmatrix}$$

$$\begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \begin{pmatrix} 0.1 \\ 0.3 \\ 0.5 \\ 0.7 \\ 1 \end{pmatrix} \rightarrow \left[\frac{1}{5}\left(0.1^{-2}+0.3^{-2}+0.5^{-2}+0.7^{-2}+1^{-2}\right)\right]^{\frac{1}{-2}} = 0.3$$

27. As one can see, the final value of the index is half the value obtained for the case of a curvature of 1. This reduction is the result of discounting for the two forms of inequality mentioned above: the spread of each distribution (inter-individual inequality) and the inequality of achievement across dimensions for the same individual (intra-individual inequality). To get an intuitive understanding of this second concept, consider unit two of our normalized matrix X^N, with scores of 0.6 for income, 0.4 for consumption and 0.2 for wealth. This individual could suffer from this unbalanced material conditions (with higher income and lower wealth). Is it reasonnable to consider such pattern as a source of inequality? While this could be seen as paternalistic (*i.e.* correction to individual preferences are judged necessary according to a given normative standpoint), it can be justified on several grounds. Units achieving a relatively lower level of wealth in comparison to income and consumption are at greater risk of experiencing future economic hardship, facing low-consumption possibilities in the future if their income drops. Their material conditions could thus be seen as worse than having a bit less income but higher wealth (the penalization or the reward for this trade-off being given by the curvature parameter r). The same reasoning can be applied to income and consumption: units having relatively higher income but lower consumption face lower material conditions, as income is only an instrument to attain a sufficient level of consumption. One could argue that this view is also paternalistic, as people could have high incomes and low consumption as a result of their own preferences, in which case no judgment on such personal situation should be applied (Sen, 1999). But the penalization makes sense also from a different perspective, as the differences between high income and lower consumption point to a lack of redistribution. Redistributive schemes that increase people's consumption could improve overall material standards at the aggregate level. This failure is thus underlined by a lowering of the overall material performances measured.

28. Stated otherwise, all these interactions could be assessed through the following interpretation: concerns for intra-individual inequality reflect the view that, while there is some substitutability between the three dimensions of living standards, the degree of substitutability is not infinite and a balanced scheme is essential. So, in the same way as one penalizes inequality among individuals within each dimension, one has also to penalize inequality across dimensions for the same individuals.

29. The numerical example shown above used I-aggregation. According to theorem 3, S-aggregation with the same curvatures at each stage will yield to the same numerical result. The illustration below shows that this is effectively the case:

$$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\downarrow \ \downarrow \ \downarrow$$

$$\left(\left[\frac{1}{5} \begin{pmatrix} 0.3^{-2} \\ +0.6^{-2} \\ +0.7^{-2} \\ +0.7^{-2} \\ +1^{-2} \end{pmatrix} \right]^{\frac{1}{-2}} \quad \left[\frac{1}{5} \begin{pmatrix} 0.2^{-2} \\ +0.4^{-2} \\ +0.6^{-2} \\ +0.8^{-2} \\ +1^{-2} \end{pmatrix} \right]^{\frac{1}{-2}} \quad \left[\frac{1}{5} \begin{pmatrix} 0.2^{-2} \\ +0.4^{-2} \\ +0.6^{-2} \\ +0.8^{-2} \\ +1^{-2} \end{pmatrix} \right]^{\frac{1}{-2}} \right)$$

$$\downarrow$$

$$\begin{pmatrix} 0.5 & 0.4 & 0.2 \end{pmatrix} \rightarrow \left[\frac{1}{3} \left(0.5^{-2} + 0.4^{-2} + 0.2^{-2} \right) \right]^{\frac{1}{-2}} = 0.3$$

30. But if the index is computed using the more general framework presented in this paper, when curvatures differ, the value will be different, depending on the order of aggregation. To illustrate this, let’s assume that inequalities between dimensions are less penalized than inequality within each dimension, i.e. r is set at 0.8 while q remains equal to -2. In this case, I-aggregation yields to the following result:

$$\begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & 0.6 \\ 1 & 1 & 1 \end{pmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \begin{pmatrix} \left[\frac{1}{3} \left(0.3^{0.8} + 0.2^{0.8} + 0.1^{0.8} \right) \right]^{\frac{1}{0.8}} \\ \left[\frac{1}{3} \left(0.6^{0.8} + 0.4^{0.8} + 0.2^{0.8} \right) \right]^{\frac{1}{0.8}} \\ \left[\frac{1}{3} \left(0.7^{0.8} + 0.6^{0.8} + 0.4^{0.8} \right) \right]^{\frac{1}{0.8}} \\ \left[\frac{1}{3} \left(0.7^{0.8} + 1^{0.8} + 0.6^{0.8} \right) \right]^{\frac{1}{0.8}} \\ \left[\frac{1}{3} \left(1^{0.8} + 1^{0.8} + 1^{0.8} \right) \right]^{\frac{1}{0.8}} \end{pmatrix}$$

$$\begin{matrix}\rightarrow\\\rightarrow\\\rightarrow\\\rightarrow\\\rightarrow\end{matrix}\begin{pmatrix}0.2\\0.4\\0.6\\0.8\\1\end{pmatrix}\rightarrow\left[\frac{1}{5}\left(0.2^{-2}+0.4^{-2}+0.6^{-2}+0.8^{-2}+1^{-2}\right)\right]^{\frac{1}{-2}}=0.4$$

while S-aggregation now gives:

$$\begin{pmatrix}0.3 & 0.2 & 0.1\\0.6 & 0.4 & 0.2\\0.7 & 0.6 & 0.4\\0.7 & 1 & 0.6\\1 & 1 & 1\end{pmatrix}$$

$$\downarrow\ \downarrow\ \downarrow$$

$$\left(\left[\frac{1}{5}\begin{pmatrix}0.3^{-2}\\+0.6^{-2}\\+0.7^{-2}\\+0.7^{-2}\\+1^{-2}\end{pmatrix}\right]^{\frac{1}{-2}}\ \left[\frac{1}{5}\begin{pmatrix}0.2^{-2}\\+0.4^{-2}\\+0.6^{-2}\\+0.8^{-2}\\+1^{-2}\end{pmatrix}\right]^{\frac{1}{-2}}\ \left[\frac{1}{5}\begin{pmatrix}0.2^{-2}\\+0.4^{-2}\\+0.6^{-2}\\+0.8^{-2}\\+1^{-2}\end{pmatrix}\right]^{\frac{1}{-2}}\right)$$

$$\downarrow$$

$$\begin{pmatrix}0.5 & 0.4 & 0.2\end{pmatrix}\rightarrow\left[\frac{1}{3}\left(0.5^{0.8}+0.4^{0.8}+0.2^{0.8}\right)\right]^{\frac{1}{0.8}}=0.3$$

31. In this general approach with different curvatures, the two aggregation procedures provide two different pictures: S-aggregation yields to a result that is very close to the one obtained using the path-independency property, while I-aggregation yields to a value that is 10 percentage points higher. Thus, multidimensional measures are sensitive to the choice of aggregation, which could lead to different conclusions for a same matrix of achievements. This raises the question of which specification is the most suitable and meaningful for the measure of inviduals' material living standards.

32. At first glance, the use of path-independent measures could appear more convenient. Information on income, consumption and wealth is typically not available in the same dataset. Path-independency allows the measure to be calculated from different datasets, and thus allows the methodology to be applied to a wider set of applications. For example, using S-aggregation, one can compute a score for the dimensions based on different survey for each dimension, then aggregate these scores to obtain the multi-dimensional index. Path-independency ensures that the results will be comparable with other values computed from a joint survey using I-aggregation. This property, which is at the heart of the proposal by Foster *et al*. (2005) hence allows computing multi-dimensional measures without requiring a joint survey on the dimensions of interest. But this a particular case of the more general methodology presented here (where curvatures do differ), which comes with strong implications.

33. The drawback of this methodology is expressed by theorem 4, which states that a path-independent measure is insensitive to some unfair correlation rearrangements (coming from the lack of functional connection between axioms (A6) and (A11)). This is an important limitation of path-independent measures that could limit their use. An "unfair" rearrangement *(i.e.* such that an invidual increases his/her overall situation by exchanging some of their achievements with those of an other individual that become worse off) will modify the distribution of achievements accross the dimensions of living standards but will keep the overall level of unbalancement constant at the aggregate level (*i.e.* the correlations between each of the dimensions will remain the same on average but will change at the individual level). In other terms, the choice of path-independency is equivalent to ignoring the distributionnal change induced by unfair rearrangements: it will still reflect concerns for unbalancement in achievements for each individual, but only in a partial way by considering only the correlations at the aggregate level. This can be seen by considering the following modified version of the normalized matrix X^N:

$$\begin{pmatrix} 0.3 & 0.2 & \underline{0.6} \\ 0.6 & 0.4 & 0.2 \\ 0.7 & 0.6 & 0.4 \\ 0.7 & 1 & \underline{0.1} \\ 1 & 1 & 1 \end{pmatrix}$$

34. In the initial situation, individual 4 had higher achievements in all the dimensions than individual 1. Now, after a "fair" rearrangement that swaps the level of wealth between the two individuals (as shown by the underlined terms), individual 1 enjoys a better situation than initially, and overall inequality has decreased: as a result, the Material Condition Index should increase. But one can easily verify that, by using a path-independent measure, the index remains unchanged.

35. The economic meaning of path-independency is also problematic. In fact, imposing the same curvature for the two-aggregation steps means having the same concern for inter and intra-individual inequality. This an un-natural choice as concerns for the spread of the distribution within each dimension are probably stronger than concerns relative to the unbalancement of achievements for the same individual. In particular, path-independent measures will yield to the same numerical result when applied to the following distribution matrices $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \\ 5 & 5 & 5 \end{pmatrix}$ and $\begin{pmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 1 & 3 & 5 \end{pmatrix}$, despite the fact that the former exhibits more severe inequality at the individual level than the latter. The choice of path-independency relies thus on a trade-off. Its flexibility regarding data availability broadens its applicability, making it desirable from a statistical perspective. But the cost is a quasi-complete ignorance of the interaction between achievements, which is the main reason for interest in the multidimensional approach (that tries to measure the synergy between the different achievements that an individual is facing), in comparison to an uni-dimensional approach. So while being easier to compute, path-independent indices are conceptually less suitable than the more general framework presented in this paper, where curvatures can differ. The next section will investigate this issue from an empirical perspective.

36. Finally, we saw in the preceding section that the nested generalized mean structure satisfies a number of standard properties. One that is nonetheless questionnable in a multidimensional approach is weak homotheticity (A4). This axiom states that changes in the units of measurement do not modify the ranking of different matrices of achievements. In a single-dimensional framework, this is true by

definition, as only one unit of measurement isused. But in the multidimensional case, it is unlikely that all the dimensions will have the same scales. The suitable axiom here is strong homotheticity (A5): rankings should be unaffected by a unilateral change on one single dimension. Theorem 2 states than only zero curvatures are compliant with such rescaling. For example, in a three dimensional case with equal weights of 1/3, one can easily verify using the example above that if the aggregation is performed using the arithmetic means at each step, and if all units are doubled, this is equivalent to apply a weight of 1/2 to that dimension, lowering the weight of the two others and thus changing rankings. Conversely, the double multiplicative structure of nested generalized mean of curvature zero neutralizes the rankings perturbations (while still changing the index final values). So, dimensions of different scales in a composite index that uses a nested generalized mean will be inconsistent if the measure unfolds over a long time span or if significant shocks occur unilaterally on each dimension. This discussion becomes irrelevant if the three dimensions are measured with the same unit. For this reason, the proposal made by Foster *et al*. (2005) of a nested generalized mean structure, under its general formulation, is particularly relevant for the assessment of households' material conditions which depends on the joint distribution of income, consumption and wealth. It constitutes a natural theoretical extension of the Atkinson measure on income (Atkinson, 1970). The next section shows that this extension is also empirically useful by providing evidence of the importance of a joint appraisal of household income, consumption and wealth instead of income alone in the case of a nationally-representative data set for France.

Evidence using a French micro dataset

The data

37. This section provides an empirical illustration of the properties of the Material Condition Index, using a French survey for the year 1995 that records the joint distribution of income, consumption and wealth on approximately 10 000 households[7]. This survey ("Enquête Budget des Familles"), produced every five years by INSEE, the French national statistical institute, provides a global picture of households financial conditions by recording their expenditures, income (before and after tax) and wealth. The scope covered by the latter variable is large but, in order to avoid measurement error due to the concentration of certain assets at the top-end of the distribution (which are more likely to be mismeasured in survey-based data), the definition of wealth considered here is limited to financial assets owned by households. Annex 2 of this paper provides a set of descriptives statistics on the three variables included in the survey.

38. Income, consumption and wealth do not serve the same objective. Income is a command over current resources, wealth is a measure of command over future ressources, while consumption is more an achievement in itself. In this sense, the three variables do not affect individuals' well-being in the same way. This reasoning suggests that the accumulation of income and wealth beyond a certain level may not enhance material conditions in a full " euro-for- euro" way but rather through some diminishing returns: the higher the level of income and wealth will be, the lower the returns on material living standards. While several methodological choices are available to take into account these diminishing effects (Anand *et al.*, 2000) this paper retains the simplest one, namely applying a log transformation to income and wealth while leaving consumption at its current level.[8] A ratio scale transformation is then applied to the log of income, the log of wealth and the level of consumption. This ensures that the three dimensions of material conditions range between 0 and 1.

[7] The year 1995 has been specifically selected as it is the wave that displays the best coverage of income, consumption and wealth (according to the aggregate amount recorded by the National Accounts), in comparison to the other waves available.

[8] The assumption used here does not imply that consumption is exempted from diminishing returns but rather that diminishing returns are sharper for income and wealth than for consumption. Such degree of differentiation is however very hard to identify empirically, and relies on subjective appreciations.

Setting the curvature

39. One of the objective of this paper is to include inequality concerns in an overall assessment of material living standards of different population groups in a multidimensional setting. The choice of curvature parameters is thus crucial. A recurrent criticism of the uni-dimensional Atkinson measure is the arbitrary choice left to the user for the inequality aversion parameter. While this is sometimes viewed as arbitrary, this criticism is wrong for at least two reasons:

- The first is that standard inequality measures also implicitly set a cuvature. The Gini index, for example, is more sensitive to changes in the middle of the distribution than in its tail, thus being equivalent to an Atkinson index with a curvature close (but inferior) to one (cf. the S-Gini of Yitzhaki, 1983). Similarly, the Generalized Entropy class of index (Cowell et al., 1981), which includes the widely used Theil measure, can be mapped into an Atkinson index according to a given curvature parameter. Thus, the Atkinson index does not depart fundamentally from other indices. On the contrary, this index enhances the measurement of inequality with an explicit display of the level of inequality aversion that other commonly-used measures implicitly hide.

- The second reason is that the freedom of choice in setting the parameter can be easily resolved. For that, let us consider the following example: one can see that a generalized mean with a zero curvature is very sensitive to upper incomes by comparing its value on the two following distribution (1,2,10) and (1,2,100). When the curvature decreases sufficiently, the effect of high incomes become less important. The generalized mean of curvature -1 is 15/8 for the first distribution, and, no matter the level to which higher incomes will rise, the value is bounded to 16/8. A -1 curvature is pivotal for generalized means: curvatures inferior to the neutral case 1 but superior to -1 imply a weak concern for inequality, while -1 could be viewed as the expression of a ‘medium’ concern. In turn, curvatures inferior to -1 ensure that small changes in low incomes will have a much larger impact on the generalized mean value than very large changes occuring on middle and upper incomes (a strong aversion to inequality). This example highlights that a meaningful and clear-cut classification of curvatures can be operated in practice for the building of an inequality aversion line.

Results

40. Table 1 presents the standard practice for measuring material living standards by computing the generalized means of income (normalized through a ratio scale transformation) by household categories and for the whole population. Table 2 presents the Material Condition Index over the same categories, using a path-independent measure (q=r). The last two tables make use of the more general approach: Table 3 presents the results for a non path-independent measure using S-aggregation with a constant penalization for unbalancement between dimensions (r=0.5) while q varies; and Table 4 presents the same measure using I-aggregation.

41. A comparison of Table 1 and 2 shows that the Material Condition Index under path-independency provides a substantially different picture of households material well-being than for the case of income alone:

- First, for every category and for all levels of inequality aversion considered, achievement levels are systematically lower than those for income alone (*e.g.* 17 percentage points lower for the neutral case). This implies that income tends to provide an over-appreciation of average households’ material well-being. This is due to the fact that, on average, most households have levels of wealth lower than those of income. This result is magnified when a penalization for inequalities is introduced (*e.g.* for an inequality aversion of -2, the MCI for the entire population, at 0.29, is 40 percentage points lower than for the income generalized mean, at 0.69).

- Second, when moving from a weak to a stronger aversion to inequality, the decline in the Material Condition Index is larger than in the case of income alone. For the whole population, moving from the weak (0.5) to the medium (-1.0) value of the inequality-aversion parameter lowers the MCI by 12 percentage points (from 0.535 to 0.412), a decrease 6 times higher than in the case of income (2 percentage points, from 0.733 to 0.715). This indicates that the joint distribution of consumption and wealth is far more unequal than that of income.

- Third, this result is reinforced when considering the values of the MCI for various deciles of (income) living standards: the value of the MCI for the upper decile is twice as large than that of the bottom decile, both for the neutral (0.75 and 0.38 respectively) and for the weak aversion case (0.74 and 0.36). Conversely in the case of income, the value of the generalized mean for the upper decile is 60% higher, both for the neutral case (0.83 and 0.51 respectively) and for the weak aversion case (0.83 and 0.5).

- Fourth, the MCI also highlights sharper heterogeneity between and within each of the sub-categories considered. This can be seen by moving from the neutral to the strong (-2) inequality-aversion cases for each household category: the decline in the MCI is always higher than in the case of income alone. For example, the MCI of single households is almost five times higher in the neutral case than in the strong case (0.485 instead of 0.104 respectively), while it is only 1.1 times higher when considering household income only (the same applies to farmers and, to a lesser extent, to other categories).

- Finally, breakdown by deciles of living standards reveals also some differences that could not have been identified using income alone: by moving along the inequality aversion line, the income generalized means remains almost unchanged (except for the first decile), showing a strong homogeneity within deciles. Conversely, the MCI displays some significant reductions, meaning that the structure of wealth and consumption in each deciles of living standards is not as homogeneous as the one of income.

Table 1. Income generalized means, 1995

	Income Generalized mean (1995)					
	Neutral	Weak		Medium	Strong	
	1	0.5	0	-1	-2	-3
Deciles of living standards						
1	0.514	0.503	0.492	0.470	0.450	0.432
2	0.693	0.693	0.693	0.693	0.693	0.693
3	0.717	0.717	0.717	0.717	0.717	0.717
4	0.735	0.735	0.735	0.735	0.735	0.735
5	0.749	0.749	0.749	0.749	0.749	0.749
6	0.762	0.762	0.762	0.762	0.762	0.762
7	0.775	0.775	0.774	0.774	0.774	0.774
8	0.787	0.787	0.787	0.787	0.787	0.787
9	0.802	0.802	0.802	0.802	0.802	0.802
10	0.836	0.836	0.836	0.836	0.835	0.835
Socio-economic status						
Farmers	0.696	0.687	0.677	0.650	0.617	0.580
Self-employed	0.749	0.742	0.733	0.710	0.678	0.639
Professional	0.750	0.747	0.744	0.736	0.723	0.704
Executives	0.791	0.788	0.784	0.773	0.756	0.729
Workers	0.742	0.739	0.736	0.727	0.714	0.695
Unemployed	0.711	0.706	0.700	0.686	0.666	0.639
Areas						
Rural	0.730	0.726	0.721	0.707	0.688	0.661
Urban	0.736	0.732	0.727	0.715	0.698	0.673
Suburbs of Paris	0.758	0.754	0.749	0.736	0.716	0.688
Paris	0.741	0.735	0.728	0.710	0.687	0.657
Household types						
Singles	0.692	0.689	0.684	0.673	0.658	0.637
Couples without children	0.743	0.739	0.734	0.721	0.702	0.675
Couples with one child	0.759	0.755	0.751	0.738	0.719	0.692
Couples with two children	0.768	0.764	0.760	0.749	0.731	0.705
Couples with three children and more	0.769	0.765	0.761	0.749	0.731	0.704
Single parents	0.719	0.715	0.711	0.699	0.682	0.658
Total	**0.737**	**0.733**	**0.728**	**0.715**	**0.697**	**0.671**

Source: Computations by the author

Table 2. Material Condition index under path-independency, 1995

	Material Condition Index (1995)					
	Neutral	Weak		Medium	Strong	
	1	0.5	0	-1	-2	-3
Deciles of living standards						
1	0.388	0.360	0.327	0.245	0.152	0.083
2	0.457	0.420	0.377	0.285	0.210	0.159
3	0.491	0.459	0.421	0.336	0.249	0.171
4	0.520	0.493	0.462	0.391	0.319	0.257
5	0.544	0.520	0.494	0.435	0.371	0.306
6	0.573	0.552	0.530	0.477	0.415	0.344
7	0.598	0.582	0.563	0.519	0.466	0.399
8	0.628	0.613	0.596	0.558	0.516	0.471
9	0.671	0.658	0.643	0.606	0.556	0.489
10	0.755	0.745	0.732	0.703	0.665	0.619
Socio-economic status						
Farmers	0.554	0.528	0.496	0.402	0.237	0.113
Self-employed	0.610	0.585	0.556	0.483	0.393	0.299
Professional	0.571	0.549	0.523	0.457	0.373	0.279
Executives	0.687	0.671	0.653	0.605	0.541	0.461
Workers	0.538	0.514	0.488	0.423	0.345	0.265
Unemployed	0.528	0.493	0.450	0.342	0.222	0.122
Areas						
Rural	0.553	0.524	0.487	0.388	0.262	0.159
Urban	0.558	0.530	0.497	0.411	0.292	0.154
Suburbs of Paris	0.598	0.574	0.546	0.474	0.386	0.298
Paris	0.587	0.557	0.520	0.418	0.279	0.165
Household types						
Singles	0.485	0.446	0.399	0.289	0.184	0.104
Couples without children	0.576	0.550	0.519	0.446	0.363	0.287
Couples with one child	0.598	0.578	0.556	0.505	0.448	0.393
Couples with two children	0.617	0.599	0.579	0.533	0.480	0.416
Couples with three children and more	0.609	0.591	0.571	0.527	0.479	0.426
Single parents	0.517	0.491	0.463	0.399	0.335	0.278
Total	**0.563**	**0.535**	**0.501**	**0.412**	**0.290**	**0.162**

Source: Computations by the author

Table 3. Material Condition Index under S-aggregation, 1995

Unbalancement between dimensions is penalized with r=0.5 in each case

	Material Condition Index (1995)					
	Neutral	Weak		Medium	Strong	
	1	0.5	0	-1	-2	-3
Deciles of living standards						
1	0.378	0.360	0.343	0.312	0.282	0.253
2	0.429	0.420	0.412	0.396	0.381	0.366
3	0.468	0.459	0.450	0.431	0.410	0.385
4	0.502	0.493	0.484	0.466	0.447	0.428
5	0.529	0.520	0.511	0.493	0.474	0.453
6	0.562	0.552	0.542	0.522	0.499	0.474
7	0.590	0.582	0.573	0.553	0.531	0.505
8	0.622	0.613	0.603	0.584	0.564	0.544
9	0.668	0.658	0.648	0.625	0.597	0.565
10	0.754	0.745	0.734	0.710	0.683	0.653
Socio-economic status						
Farmers	0.546	0.528	0.508	0.460	0.392	0.334
Self-employed	0.606	0.585	0.563	0.515	0.467	0.421
Professional	0.563	0.549	0.534	0.502	0.467	0.430
Executives	0.685	0.671	0.656	0.619	0.577	0.530
Workers	0.527	0.514	0.501	0.473	0.443	0.412
Unemployed	0.511	0.493	0.475	0.437	0.397	0.352
Areas						
Rural	0.542	0.524	0.504	0.461	0.415	0.371
Urban	0.548	0.530	0.512	0.473	0.429	0.372
Suburbs of Paris	0.592	0.574	0.555	0.514	0.473	0.434
Paris	0.580	0.557	0.531	0.477	0.421	0.373
Household types						
Singles	0.461	0.446	0.432	0.403	0.372	0.336
Couples without children	0.566	0.550	0.534	0.502	0.468	0.435
Couples with one child	0.594	0.578	0.565	0.536	0.506	0.476
Couples with two children	0.613	0.599	0.587	0.558	0.526	0.490
Couples with three children and more	0.605	0.591	0.577	0.549	0.518	0.485
Single parents	0.502	0.491	0.477	0.455	0.433	0.412
Total	**0.553**	**0.535**	**0.515**	**0.474**	**0.428**	**0.375**

Source: Computations by the author

Table 4. Material Condition Index under I-aggregation, 1995

Unbalancement between dimensions is penalized with r=0.5 in each case

	Material Condition Index (1995)					
	Neutral	Weak		Medium	Strong	
	1	0.5	0	-1	-2	-3
Deciles of living standards						
1	0.363	0.360	0.356	0.349	0.343	0.336
2	0.423	0.420	0.418	0.413	0.409	0.405
3	0.461	0.459	0.456	0.451	0.447	0.442
4	0.495	0.493	0.491	0.486	0.482	0.477
5	0.523	0.520	0.518	0.513	0.508	0.503
6	0.555	0.552	0.550	0.544	0.539	0.534
7	0.584	0.582	0.579	0.575	0.571	0.566
8	0.615	0.613	0.610	0.605	0.599	0.594
9	0.661	0.658	0.655	0.650	0.644	0.638
10	0.748	0.745	0.741	0.734	0.727	0.720
Socio-economic status						
Farmers	0.536	0.528	0.520	0.501	0.481	0.459
Self-employed	0.596	0.585	0.574	0.552	0.529	0.507
Professional	0.555	0.549	0.542	0.529	0.515	0.500
Executives	0.678	0.671	0.665	0.650	0.634	0.617
Workers	0.520	0.514	0.509	0.498	0.487	0.475
Unemployed	0.501	0.493	0.485	0.469	0.453	0.437
Areas						
Rural	0.531	0.524	0.515	0.499	0.482	0.464
Urban	0.538	0.530	0.522	0.506	0.490	0.473
Suburbs of Paris	0.583	0.574	0.565	0.548	0.529	0.510
Paris	0.568	0.557	0.545	0.522	0.499	0.477
Household types						
Singles	0.452	0.446	0.440	0.428	0.416	0.404
Couples without children	0.557	0.550	0.543	0.529	0.514	0.498
Couples with one child	0.586	0.578	0.573	0.559	0.544	0.530
Couples with two children	0.607	0.599	0.594	0.580	0.566	0.551
Couples with three children and more	0.598	0.591	0.584	0.571	0.558	0.545
Single parents	0.495	0.491	0.484	0.474	0.464	0.454
Total	**0.543**	**0.535**	**0.526**	**0.509**	**0.492**	**0.475**

Source: Computations by the author

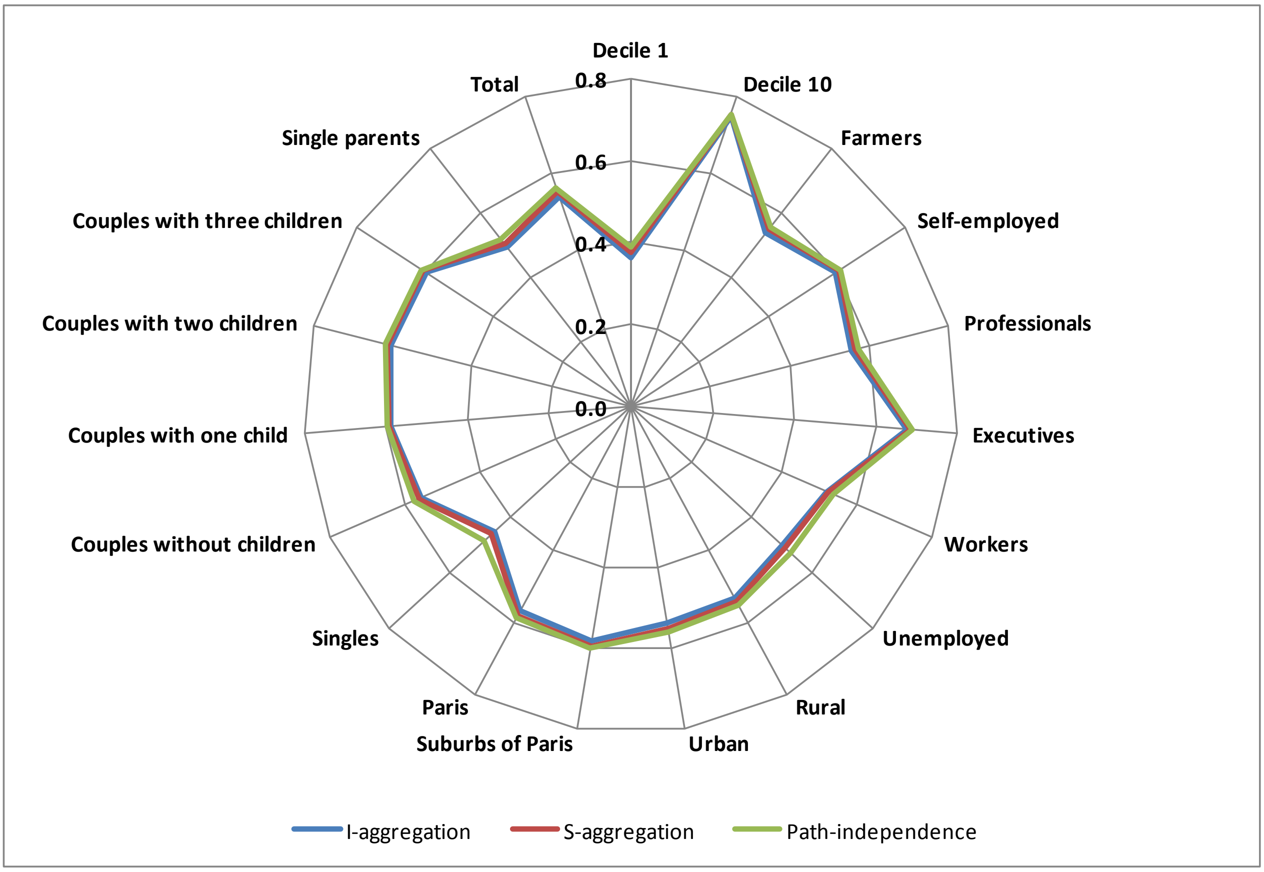
42. What happens when relaxing the path-independence property and moving to the general case? In Table 3 and 4, the parameter r, expressing the penalization imposed for unbalancement in the achievements of each individual is set equal to 0.5, while the parameter for aversion to inter individual inequalities q varies between 1 and -3. The two tables differ in terms of I-aggregation and S-aggregation used when constructing the multidimensional index (*i.e.* aggregation first across individuals and then among dimensions in Table 3; and aggregation first among dimensions and then across individuals in Table 4).

43. When moving to the general case, the numerical results are substantially altered, although the relative positions of each of the households' category considered remain the same. In particular:

- Whatever the order of aggregation selected, lower levels of achievements are displayed than for the income generalized mean, both overall and for each of the sub-categories.

- When no penalization is applied for inter individual inequalities (q=1), the MCI displays almost identical results whatever the order of aggregation (Figure 1).

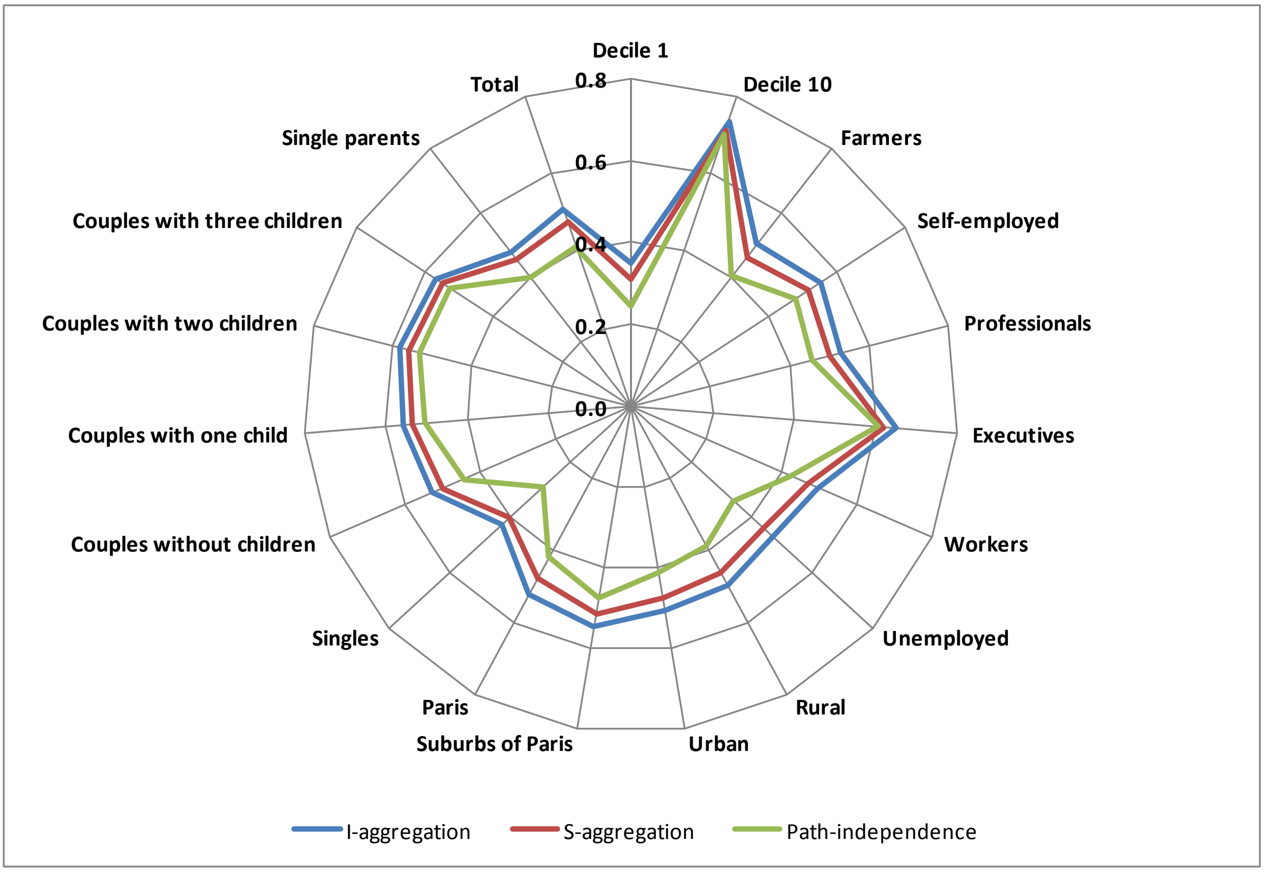
Figure 1. Comparison of the three specifications under the neutral aversion case



Source: Computations by the author

- However, large differences emerge when moving along the inequality aversion line; under the medium case (q=-1), path-independent measures display the lowest values for every categories considered, I-aggregation displays the highest values, while measures using S-aggregation are in-between the two other cases (Figure 2).

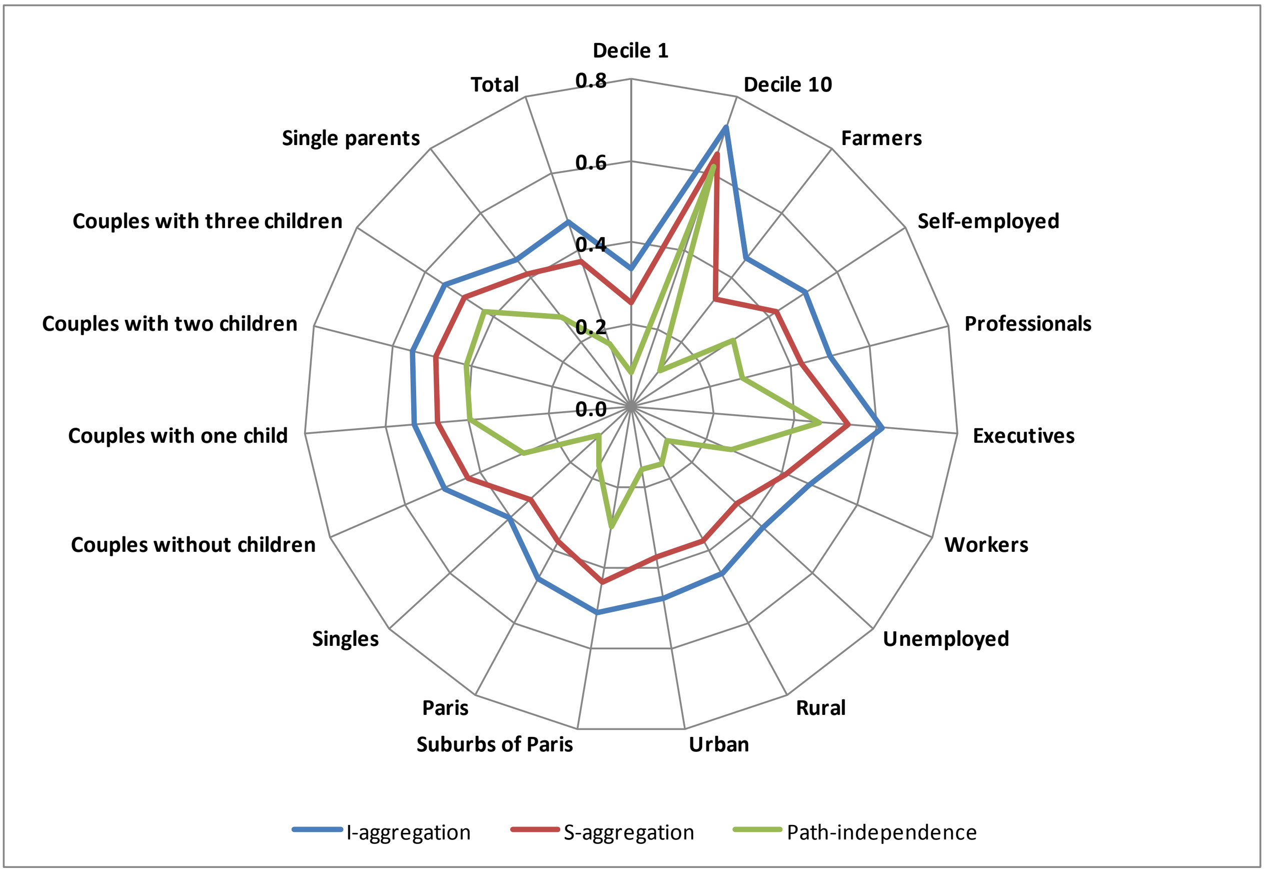
Figure 2. Comparison of the three specifications under the medium aversion case



Source: Computations by the author

- Such pattern is further reinforced under the assumption of a stronger aversion to inequality (q=-3), where the differences between the three types of measures are more clearly delineated (Figure 3).

Figure 3. Comparison of the three specifications under the strong aversion case



Source: Computations by the author

44. These results have their roots in the use of the information contained in the joint distribution of income, consumption and wealth. I-aggregation establishes first a score for each individual that expresses the overall level of material conditions they are facing; it results in a distribution of scores that is then aggregated in order to get the final index's value. But, in the dataset used here, this distribution is less spread out than when each of the dimensions is considered separately. As a result, the I-aggregation index is less penalized in comparison to S-aggregation; further, the particular choice of path-independent measures, which restricts the penalization to be the same both for inter individual inequalities and for unbalancement between dimensions, understates the material living standards of individuals to an even larger extent. While these conclusions depend on the dataset used, they could not have been unveiled using aggregation procedures that do not consider explicitly the features of the joint distribution.

Conclusion

45. This paper has shown the usefulness of the nested Atkinson measure for illustrating the joint distribution of income, consumption and wealth. The relevance of the generalized means for analysis of income inequality has been well known since Atkinson (1970). The nested generalized mean structure proposed by Foster et al. (2005) provides a natural multidimensional extension of the Atkinson framework. This paper has described this measure on the basis of a set of basic properties that the index statisfies and shows that this is the *unique* multidimensional measure to do so. Proof of this result provides an axiomatic justification for focusing on multidimensional statistics of this form.

46. The picture of material living standards that emerges from such multidimensional approach shows that consideration of the joint distribution of income, consumption and wealth, rather than of income alone, provides a different measure of households’ material conditions, as outlined by the empirical applications. The joint appraisal of the three variables simultaneously thus matters, but so does the specific form of the MCI that is used (and the set of restrictions that they impose). It is hoped that this approach will enhance our understanding of households' material living conditions by complementing traditional approaches relying on similar fundations and will represent a useful contribution for future research in this area.

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ANNEX 1: PROOFS

Proof of theorem 1: i) Suppose that $W^I : \mathbb{R}_{++}^{n\times m} \to \mathbb{R}_{++} : X \to W^I(X) = W_n[W_m(x_1^{\cdot}),..., W_m(x_n^{\cdot})]$ satisfies the nine properties. For this direction of verification, the proof's strategy is the following: we first prove that $W_n(.)$ is a generalized mean by showing that the nine properties lead to the conditions stated in the theorem of Foster *et al.* (2008) that characterizes a generalized mean in the uni-dimensional case for aggregation across units, and then that $W_m(.)$ is also a generalized mean according to theorem 2 of Blackorby et al. (1982) for the aggregation across dimensions for one unit.

Note first that as $W^I(.)$ verifies (A10), $W^I(.)$ is Schur-concave (cf. Kolm (1977)). By (A2), both $W_n(.)$ and $W_m(.)$ are increasing in each of their arguments. Thus, by the properties of compositions involving Schur-concave functions (cf. Marshall *et al.* (1979)) $W_n(.)$ is itself Schur-concave. As Schur-concavity implies symmetry, $W_n(.)$ is a symmetric function.

Let now for $\forall n \in \mathbb{N},\ \forall X \in X_n,\ \forall \eta \in \mathbb{R}_{++}$ assume that $W^I(X) = \eta$, and define $X^* \in X_n$ such that $x_n^{m^*} = \eta \quad \forall n \in \mathbb{N},\ \forall m \in \mathbb{N}$. (A3) implies that $W^I(X) = W^I(X^*)$. $\forall \lambda \in \mathbb{R}_{++}$ we have by (A4) $W^I(\lambda X) = W^I(\lambda X^*)$ and by (A3) $W^I(\lambda X^*) = \lambda\eta$, so $W^I(\lambda X) = \lambda W^I(X)$. $W^I(.)$ is thus homogeneous of degree one et also is $W_n(.)$. As $W^I(.)$ verifies (A8), (A1), and (A7), so does $W_n(.)$. Let assume now that $W_m(x_1^{\cdot}) = \overline{W} > 0$. By (A3), we have $W^I(\overline{W},\ldots,\overline{W}) = W_n(\overline{W},\ldots,\overline{W}) = \overline{W}$, and so W_n satisfies (A3). We have demonstrated here that $W_n(.)$ is symmetric, continuous, cloning invariant, subgroup consistent, homogeneous of degree one and normalized, which are the conditions of the theorem in Foster et al. (2008). Thus, $\exists q \in \mathbb{R}$ such as $W_n(.)$ is a generalized mean of curvature $q < 1$ (as a generalized mean is Shur-concave if its curvature is strictly inferior to 1, cf. Blackorby et al. (1982)):

$$W_n[W_m(x_1^{\cdot}),\ldots,W_m(x_n^{\cdot})] = \left[\frac{1}{n}\sum_{i=1}^{n}[W_m(x_i^{\cdot})]^q\right]^{\frac{1}{q}} \forall q < 1, q \neq 0$$

$$= \prod_{i=1}^{n}\left[W_m(x_i^{\cdot})\right]^{\frac{1}{n}} \textit{ for } q = 0$$

We now turn to $W_m(.)$. First, one note that for $n = 1$ no two-stage aggregation procedure is required, having thus in this case $W^I(.) = W_m(.)$. This in turn implies that $W_m(.)$ is continuous, homogeneous of degree one, monotonic and Schur-concave thus symmetric.

Let's apply now (A9) in the case of an I-aggregation, assuming without loss of generality that the strictly essential subset of the choosen dimension is $\{1,2\}$. Thus, $\forall i \in \{1,..., n\}$ we can write $W_m(.)$ in the following form:

$$W_m\left(x_i^{\cdot}\right) = W\left[W^1\left(x_i^1, x_i^2\right), x_i^3, ..., x_i^m\right]$$

with W increasing in $W^1\left(x_i^1, x_i^2\right)$. By symmetry of $W_m(.)$ we can also write it as:

$$W_m\left(x_i^{\cdot}\right) = W\left[W^1\left(x_i^1, x_i^3\right), x_i^2, ..., x_i^m\right]$$

and so $\{1,3\}$ is also a strictly essential subset. $\{1,2\}$ and $\{1,3\}$ are strictly essential and overlap. By Gorman's overlapping theorem (cf. Gorman (1968)), $W_m\left(x_i^{\cdot}\right)$ can be written the following way:

$$W_m\left(x_i^{\cdot}\right) = W\left[W^1\left(x_i^1\right) + W^2\left(x_i^2\right) + W^3\left(x_i^3\right), x_i^4, ..., x_i^m\right]$$

Repeating the above reasoning with every strictly essential subset of two elements and using symmetry, one finds that $W_m\left(x_i^{\cdot}\right)$ is additive separable:

$$W_m\left(x_i^{\cdot}\right) = W^*\left[\sum_{j=1}^{m} \phi\left(x_i^m\right)\right]$$

with $W^*(.)$ and $\phi(.)$ increasing by (A2).

We have identified here the conditions of theorem 2 of Blackorby et al. (1982) on $W_m(.)$ (as homogeneity of degree one implies ratio-full scale comparability, and increasingness of $W_m(.)$ by increasingness of $W^*(.)$ and $\phi(.)$ induce strong Pareto). Therefore, $\forall i \in \{1,..., n\}$ $W_m(.)$ is a generalized mean of curvature $r < 1$:

$$W_m\left(x_i^{\cdot}\right) = \left[\frac{1}{m} \sum_{j=1}^{m} \left(x_i^j\right)^r\right]^{\frac{1}{r}} \forall r < 1, r \neq 0$$

$$= \prod_{j=1}^{m} \left(x_i^j\right)^{\frac{1}{m}} \textit{ for } r = 0$$

Finally, $W^I(X)$ is a nested Atkinson measure of curvatures $q < 1$ and $r < 1$:

$$W^I(X) = \left[\frac{1}{n} \sum_{i=1}^{n} \left[\left[\frac{1}{m} \sum_{j=1}^{m} \left(x_i^j\right)^r\right]^{\frac{1}{r}}\right]^q\right]^{\frac{1}{q}} \forall q < 1, \forall r < 1, q \neq 0, r \neq 0$$

$$= \prod_{i=1}^{n} \left[\prod_{j=1}^{m} \left(x_i^j\right)^{\frac{1}{m}}\right]^{\frac{1}{n}} \textit{ for } q = 0 \text{ and } r = 0$$

$$= \left[\frac{1}{n} \sum_{i=1}^{n} \left[\prod_{j=1}^{m} \left(x_i^j \right)^{\frac{1}{m}} \right]^q \right]^{\frac{1}{q}} \forall q < 1,\ q \neq 0 \text{ and } r = 0$$

$$= \prod_{i=1}^{n} \left[\left[\frac{1}{m} \sum_{j=1}^{m} \left(x_i^j \right)^r \right]^{\frac{1}{r}} \right]^{\frac{1}{n}} \text{for } q = 0 \text{ and } \forall r < 1,\ r \neq 0$$

The fact that an I-aggregation function with a nested general means structure satisfies (A1), (A2), (A3), (A4), (A6), (A7), (A8), (A9) and (A10) is straightforward. We leave this direction of verification to the reader's inspection.

Proof of theorem 2: The proof follows directly from theorem 4 of Tsui et al. (1997) which states that any continuous welfare ordering verifying (A5) and (A2) is of the Cobb-Douglas form *i.e.* a generalized mean of curvature zero.

Proof of theorem 3: We prove this result for $\forall 0 < q < 1$ and $\forall 0 < r < 1$ and leave the other cases to the reader as the proofs are identicals. Path-independency means that $\forall n \in \mathbb{N},\ \forall X \in X_n$ I-aggregation and S-aggregation provide the same results. Suppose that as $W^I(X)$, $W^S(X)$ has also a nested general mean structure of curvatures r and q. Path-independency requires then:

$$W^I(X) = W^S(X)$$

$$\Leftrightarrow \left[\frac{1}{n} \sum_{i=1}^{n} \left[\left[\frac{1}{m} \sum_{j=1}^{m} \left(x_i^j \right)^r \right]^{\frac{1}{r}} \right]^q \right]^{\frac{1}{q}} = \left[\frac{1}{m} \sum_{j=1}^{m} \left[\left[\frac{1}{n} \sum_{i=1}^{n} \left(x_i^j \right)^q \right]^{\frac{1}{q}} \right]^r \right]^{\frac{1}{r}}$$

Let $k = \frac{r}{q}$ and $\frac{1}{m}\left(x_i^j\right)^r = Z_i^j$. If $W^I(X) > W^S(X)$ then:

$$\left[\frac{1}{n} \sum_{i=1}^{n} \left[\sum_{j=1}^{m} Z_i^j \right]^{\frac{1}{k}} \right]^k > \sum_{j=1}^{m} \left[\frac{1}{n} \sum_{i=1}^{n} \left(Z_i^j \right)^{\frac{1}{k}} \right]^k$$

This last expression reduces to Minkowski's inequality (cf. Kolmogorov *et al.* (1970) and Kannappan (2009)) and is true for $k > 1$. Thus, $W^I(X) > W^S(X)$ for $r > q$, $W^I(X) < W^S(X)$ for $r < q$ and $W^I(X) = W^S(X)$ for $r = q$.

Proof of theorem 4: The proof is straightforward and needs no formal statements. Path independency allows to choose freely between I-aggregation and S-aggregation, but this liberty induces the neutralization of the effect coming from a non-increasing comonotonic swap by the anonymity axiom (a desirable property than can hardly be overpassed). Let's aggregate here according to the latter and proceed to a non-increasing comonotonic swap for two units $(n_k, n_l) \in n$ with k having after more in all dimensions. S-aggregation first aggregates by columns, thus taking one by one the rearranged vectors. But according to (A6), the result of this first stage is unchanged compared to the situation before the swap as anonymity ensures equal treatment for each unit of observation. For each column, the swapping is thus simply a permutation of their terms which yields the same results as before. The final result after aggregating across dimensions is thus unchanged, and so in particular is not modified after an increase in dimension correlations. As S-aggregation yields the same results as I-aggregation in the case of path independency, this completes the proof.

ANNEX 2: DESCRIPTIVE STATISTICS

Table 5. Monetary variables (in euro)

	1995				2000				2005			
	Mean	Min	Max	S.E	Mean	Min	Max	S.E	Mean	Min	Max	S.E
Income	24,369	100	538,340	21,120	27,067	644	436,119	20,130	26,952	100	572,929	20,718
Wealth	13,638	100	760,820	36,465	18,375	100	707,485	45,217	24,201	100	900,883	136,259
Consumption	22,783	100	234,373	19,477	27,007	1,048	237,021	18,746	32,596	1,081	761,025	32,899

Source: Computations by the author

Table 6. Socio-demographic status of respondents

	1995	2000	2005
Socio-economic status			
Farmers	2%	2%	2%
Self-employed	6%	5%	5%
Professional	25%	27%	31%
Executives	10%	10%	12%
Workers	20%	19%	17%
Unemployed	36%	37%	33%
Areas			
Rural	25%	26%	25%
Urban	59%	61%	59%
Suburbs of Paris	12%	10%	*na*
Paris	4%	3%	16%
Household types			
Singles	25%	26%	*na*
Couples without children	26%	29%	*na*
Couples with one child	15%	13%	*na*
Couples with two children	15%	15%	*na*
Couples with three children and more	8%	7%	*na*
Single parents	10%	10%	*na*

Source: Computations by the author