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GENERAL EQUILIBRIUM MODELLING OF TRADE AND THE ENVIRONMENT

by

John Beghin, Sébastien Dessus, David Roland-Holst
and Dominique van der Mensbrugghe

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RÉSUMÉ

L'intérêt porté aux conséquences de l'activité économique sur l'environnement s'est considérablement accru, aussi bien dans les pays Membres de l'OCDE que dans les pays non membres. La recherche dans ce domaine est encore balbutiante, et les données nécessaires à l'analyse empirique encore trop rares. Ce document décrit les spécifications du modèle utilisé pour les six études de cas entreprises au Centre de Développement de l'OCDE, afin d'étudier d'une part les liens entre la croissance et la pollution et, d'autre part, les liens entre la pollution et les politiques commerciales.

Le modèle tend à décrire plusieurs phénomènes clés de la relation entre environnement et activité économique. Ainsi, le modèle *a)* relie les émissions de pollution à la consommation de produits polluants (et non à la production) ; *b)* inclut les émissions provenant de la demande finale ; *c)* intègre une possibilité de substitution entre intrants polluants et non polluants (comme le capital ou le travail) ; *d)* prend en compte les effets dynamiques de l'accumulation du capital, de la croissance de la population, de la productivité et du progrès technologique, des générations de capital (au travers d'une spécification *putty/semi-putty*) ; *e)* mesure les conséquences des taxes visant à limiter la pollution. Le modèle décrit par ailleurs des phénomènes qui ne sont pas spécifiquement utiles à l'analyse des problèmes environnementaux, mais qui peuvent être aussi utiles à la formulation de politiques économiques concernant le marché du travail ou les ménages. Ce modèle comporte aussi des lacunes dans la description des liens entre activité économique et environnement. Dans sa forme actuelle, ce modèle ne calcule que les *coûts* économiques d'une limitation des émissions. Il ne calcule pas les *bénéfices* d'une telle limitation, qui peuvent se révéler importants. En outre, ce modèle ne décrit pas l'incorporation de technologies visant explicitement à réduire les émissions, qui pourrait être décidée par les producteurs. Les résultats obtenus tendent donc à surestimer les coûts de la limitation des émissions, si cette dernière est aussi rendue possible par l'adoption de technologie plus « propres ». La dernière lacune vient du fait que le modèle ne décrit que la pollution de type industriel, sans tenir compte d'autres types de dégradation de l'environnement, tels que la déforestation, l'érosion et l'appauvrissement des sols ou le rejet des déchets notamment.

La première section présente brièvement les principales caractéristiques du modèle. La deuxième section décrit en détail et les différents blocs du modèle. Une liste des principales différences entre pays concernant les données et les spécifications est proposée dans la troisième section, tandis que la dernière section dresse quelques conclusions.

SUMMARY

The environmental impacts of economic activity have become an increasingly urgent concern in both OECD Member countries, as well as in non-Member countries. Research in this area is still in its infancy, and the data required to buttress analytical studies is still sparse. This paper describes the base model specification for a series of six country case studies undertaken at the OECD Development Centre to analyse the links between growth and emissions, and emissions and trade instruments.

The model attempts to capture some of the key features relating to environmental emissions. These features include: *a)* linking emissions to the consumption of polluting inputs (as opposed to output); *b)* including emissions generated by final demand consumption; *c)* integrating substitutability between polluting and non-polluting inputs (including capital and labour); *d)* capturing important dynamic effects such as capital accumulation, population growth, productivity and technological improvements, and vintage capital (through a putty/semi-putty specification); and *e)* the impact of emission taxes to limit the level of pollution. On top of these important elements for studying environmental linkages, the model includes other structural features which may be of interest to policy makers, such as detailed labour markets and household specification which is conducive for analysing the incidence of economic policies. While the model is rich in structure, it also lacks some elements for a more complete analysis of environmental linkages. In its current form, the model is only useful for calculating the economic *costs* of limiting emissions, without the concomitant, but certainly important, evaluation of the *benefits*. The second major lacuna is the lack of abatement technology which is a relevant decision variable for producers. The results of the analysis may therefore tend to overstate the costs of controlling emissions if more cost-effective alternatives exist in the form of abatement equipment or “cleaner” capital. The third deficiency is that the study focuses on “industry-based” type pollution, and ignores other significant environmental issues such as deforestation, soil degradation and erosion, solid waste and its disposal, and other serious potential problems.

The first section of the paper provides a brief overview of some of the key features of the model. This is followed by a complete description of each block of the model. The third section provides a list of the differences of the data and model specification across country implementation. This is followed by a few concluding remarks.

PREFACE

The links between the environment and trade have given rise to significant debate, which reached a peak with the signing of the Uruguay Round Agreements. However until recently, there has been little quantitative analysis to shed light on the importance of these links, or on the main mechanisms through which changes in trade regimes impact on the environment.

In an attempt to understand these mechanisms, and as a component of its research programme on "Sustainable Development: Environment, Resource Use, Technology, and Trade", the Development Centre has undertaken detailed economic analysis of six developing economies around the Pacific Rim: three in Latin America — Chile, Costa Rica, and Mexico, and three in Asia Pacific — the People's Republic of China, Indonesia, and Viet Nam. A complete data base has been developed for each of these six countries including detailed industrial definition (between 22 and 93 sectors), and structural features (multiple labour markets, households, and trading partners).

This Technical Paper provides a complete description of the computable general equilibrium (CGE) model which underlies the six country case studies. The CGE model has been used to analyse: *a)* the links between growth and the environment; *b)* the links between trade policies and the environment; and *c)* efficient economic policies which can readily be implemented in the context of a developing economy. This model can also assist in addressing many subsidiary issues such as the impact of trade and environmental policies on household distribution, and the role of technological progress.

Quantifying the response of both economic and environmental variables to policy changes, such as trade or environmental measures, is a necessary pre-condition to the design of sustainable and coherent reforms.

Jean Bonvin
President, Development Centre
September, 1996

I. INTRODUCTION

1. Introduction

This paper provides the complete technical specification of a computable general equilibrium model for the six country case studies of the OECD Development Centre's programme on *Sustainable Development: Environment, Resource Use, Technology and Trade*. The six countries are Mexico, Costa Rica and Chile in Latin America, and China, Indonesia, and Viet Nam in the Asian Pacific region. A detailed data base has been developed for each one of the countries with significant structural detail on production, labour markets, income distribution, and trading patterns. In order to quantify the environmental impacts of changes in trade regimes, as well as to assess the trade-offs between growth and the environment, we have developed a computable general equilibrium model. The model incorporates features which are important in the context of the environment, notably a careful specification of the energy sector, and a detailed description of emissions.

The next section provides a brief overview of the main features of the model. While each country has its own specific features, the core of the models is the same. (Model differences are detailed in Chapter 3). Chapter 2 provides a block-by-block description of the model, motivating the structural specification of each block of the model. Chapter 3 describes some of the implementation details, particularly with respect to the country-specific details. Chapter 4 provides some concluding remarks.

2. Overview of the Model

Production

All sectors are assumed to operate under constant returns to scale and cost optimisation. Production technology is modelled by a nesting of constant-elasticity-of-substitution (CES) functions. See Figure 1 for a schematic diagram of the nesting. The implementation of the model allows for all permissible special cases of the CES function, notably Leontief and Cobb-Douglas.

In each period, the supply of **primary** factors — capital, land, and labour — is usually predetermined.[1] The model includes adjustment rigidities. An important feature is the distinction between old and new capital goods. In addition, capital is assumed to be partially mobile, reflecting differences in the marketability of capital goods across sectors.[2]

Once the optimal combination of inputs is determined, sectoral output prices are calculated assuming competitive supply (zero-profit) conditions in all markets.

Consumption and Closure Rule

All income generated by economic activity is assumed to be distributed to consumers. Each representative consumer allocates optimally his/her disposable income among the different commodities and saving. The consumption/saving decision is completely static: saving is treated as a "good" and its amount is determined simultaneously with the demand for the other commodities, the price of saving being set arbitrarily equal to the average price of consumer goods.[3]

The government collects income taxes, indirect taxes on intermediate inputs, outputs and consumer expenditures. The default closure of the model assumes that the government deficit/saving is exogenously specified.[4] The indirect tax schedule will shift to accommodate any changes in the balance between government revenues and government expenditures.

The current account surplus (deficit) is fixed in nominal terms. The counterpart of this imbalance is a net outflow(inflow) of capital, which is subtracted (added to) the domestic flow of saving. In each period, the model equates gross investment to net saving (equal to the sum of saving by households, the net budget position of the government and foreign capital inflows). This particular closure rule implies that investment is driven by saving.

Foreign Trade

Goods are assumed to be differentiated by region of origin. In other words, goods classified in the same sector are different according to whether they are produced domestically or imported. This assumption is frequently known as the *Armington* assumption. The degree of substitutability, as well as the import penetration shares are allowed to vary across commodities and across agents. The model assumes a single Armington agent. This strong assumption implies that the propensity to import and the degree of substitutability between domestic and imported goods is uniform across economic agents. This assumption reduces tremendously the dimensionality of the model. In many cases this assumption is imposed by the data. A symmetric assumption is made on the export side where domestic producers are assumed to differentiate the domestic market and the export market. This is implemented using a *Constant-Elasticity-of-Transformation* (CET) production possibility frontier.

Dynamic Features and Calibration

The current version of the prototype has a simple recursive dynamic structure as agents are assumed to be myopic and to base their decisions on static expectations about prices and quantities. Dynamics in the model originate in three sources: i) accumulation of productive capital and labour growth; ii) shifts in production technology; and iii) the putty/semi-putty specification of technology.

a. Capital accumulation

In the aggregate, the basic capital accumulation function equates the current capital stock to the depreciated stock inherited from the previous period plus gross investment. However, at the sectoral level, the specific accumulation functions may differ because the demand for (old and new) capital can be less than the depreciated stock of old capital. In this case, the sector contracts over time by releasing old capital goods. Consequently, in each period, the new capital vintage available to expanding industries is equal to the sum of disinvested capital in contracting industries plus total saving generated by the economy, consistent with the closure rule of the model.

b. The putty/semi-putty specification

The substitution possibilities among production factors are assumed to be higher with the new than the old capital vintages — technology has a putty/semi-putty specification. Hence, when a shock to relative prices occurs (e.g. the imposition of an emissions tax), the demands for production factors adjust gradually to the long-run optimum because the substitution effects are delayed over time. The adjustment path depends on the values of the short-run elasticities of substitution and the replacement rate of capital. As the latter determines the pace at which new vintages are installed, the larger is the volume of new investment, the greater the possibility to achieve the long-run total amount of substitution among production factors.

c. Dynamic calibration

The model is calibrated on exogenous growth rates of population, labour force, and GDP. In the so-called Business-as-Usual (BaU) scenario, the dynamics are calibrated in each region by imposing the assumption of a balanced growth path. This implies that the ratio between labour and capital (in efficiency units) is held constant over time.[5] When alternative scenarios around the baseline are simulated, the technical efficiency parameter is held constant, and the growth of capital is endogenously determined by the saving/investment relation.

In the equations which follow, the following indices will be used extensively. Note that the time index generally be dropped from the equations.

i Production sectors. ***j*** is an alias for ***i***.

nf Represents the non-fuel commodities.

e Represents fuel commodities.

l Represents the labour types.

v Represents the capital vintages.

h Represents the households.

g Represents the government expenditure categories.

f Represents the final demand expenditure categories (including *g* as a subset).

r Represents trading partners

p Represents different types of effluents

t Time index.

II. MODEL DESCRIPTION

1. Production

Production is based on a nested structure of *Constant Elasticity of Substitution* (CES) functions (see Figure 1). Each sector produces a gross output[6], *XP*, which given the assumption of constant returns to scale is undetermined by the producer. It will be determined by equilibrium conditions. The producer therefore minimises costs subject to a production function which is of the CES type. At the first level, the producer chooses a mix of a value added aggregate, *VA*[7], and an intermediate demand aggregate, *ND*.[8] In mathematical terms, this leads to the following formulation:

$$\min PVA_i VA_i + PN_i ND_i$$

s.t.

$$XP_i = \left[a_{va,i} VA_i^{\rho_i^p} + a_{nd,i} ND_i^{\rho_i^p} \right]^{1/\rho_i^p}$$

where *PVA* is the aggregate price of value added, *PN*, is the price of the intermediate aggregate, a_{va} and a_{nd} are the CES share parameters, and ρ is the CES exponent[9]. The exponent is related to the CES elasticity, via the following relationship:

$$\rho_i^p = \frac{\sigma_i^p - 1}{\sigma_i^p} \Leftrightarrow \sigma_i^p = \frac{1}{1 - \rho_i^p} \quad \text{and} \quad \sigma_i^p \geq 0$$

Note that in the model, the share parameters incorporate the substitution elasticity using the following relationships:

$$\alpha_{va,i} = \left(a_{va,i}\right)^{\sigma_i^p} \quad and \quad \alpha_{nd,i} = \left(a_{nd,i}\right)^{\sigma_i^p}$$

Solving the minimisation problem above, yields Equations (1.1.1) and (1.1.3) in Table 1.1. Because of the assumption of vintage capital, we are allowing the substitution elasticities to differ according to the vintage of the capital. Depending on the available data, and due to the importance of energy in terms of pollution, we separate energy demand from the rest of intermediate demand, and incorporate the demand for energy directly in the value added nest. Hence, the equations below are not specified in terms of a value added bundle, but a value added plus energy bundle. Equation (1.1.1) determines the volume of aggregate intermediate non-energy demand, by vintage, *ND*. Equation (1.1.2) determines the total demand for intermediate non-energy aggregate inputs (summed over vintages), *ND*. Equation (1.1.3) determines the level of the composite bundle of value added demand and energy, *KEL*. The CES dual price of *ND*, and *KEL*, *PXv*, is defined by Equation (1.1.4). Equation (1.1.5) determines the aggregate unit cost, *PX*, exclusive of an output subsidy/tax[10]. Finally, we allow the possibility of an output subsidy or tax, generating a wedge between the producer price and the output price, *PP*, yielding Equation (1.1.6). The production tax is multiplied by an adjustment factor which normally is fixed at unit value. However, it is possible to endogenise the average level of the production tax to achieve a pre-determined fiscal target.

Table 1.1: **Top Level Production Equations**

$$ND_j^v = \alpha_{nd,j}^v \left[\frac{PX_j^v}{PN_j}\right]^{\sigma_j^{p,v}} XPv_j^v \qquad (1.1.1)$$

$$ND_j = \sum_v NDv_j^v \qquad (1.1.2)$$

$$KEL_j^v = \alpha_{kel,j}^v \left[\frac{PXv_j^v}{PKEL_j^v}\right]^{\sigma_j^{p,v}} XPv_j^v \qquad (1.1.3)$$

$$PXv_j^v = \left[\alpha_{nd,j}^v PN_j^{1-\sigma_j^{p,v}} + \alpha_{va,j}^v PKEL_j^{v\,1-\sigma_j^{p,v}}\right]^{1/(1-\sigma_j^{p,v})} \qquad (1.1.4)$$

$$PX_j XP_j = \sum_v PXv_j^v XPv_j^v \qquad (1.1.5)$$

$$PP_j XP_j = PX_j\left(1+\delta^p \tau_j^p - \varphi_j^p\right) XP_j \qquad (1.1.6)$$

The next level of the CES nest concerns on the one side aggregate intermediate demand, *ND*, and on the other side, the *KEL* bundle. The split of *ND* into intermediate demand is assumed to follow the Leontief specification, in other words a substitution elasticity of 0. (We also assume that the share coefficients are independent of the vintage.) The demand for non-fuel intermediate goods is determined by Equation (1.2.1). The intermediate demand coefficients are given by $a_{nf,j}$. The price of aggregate intermediate demand is given by adding up the unit price of intermediate demand. This is specified in Equation (1.2.2) in Table 1.2. Demand for each good is specified as a demand for the Armington composite (described in more detail below), an aggregation of a domestic good and an import good which are imperfect substitutes. Therefore, while there is no substitution of one intermediate good for another, there will be substitution between domestic demand and import demand depending on the relative prices. The price of the Armington good is given by *PA.*

At the same level, the *KEL* bundle is split between labour and a capital-energy bundle, *KE*. It is assumed here as well, that the substitution possibilities between labour and the *KE* bundle depend on the vintage of the capital. The optimisation problem is similar to above, i.e. cost minimisation subject to a CES aggregation function. If *AW* is the aggregate sectoral wage rate, and *PKE* is the price of the *KE* bundle, aggregate labour demand, *AL* and demand for the *KE* bundle, are given by Equations (1.2.3) and (1.2.4). $\alpha_{l,i}$ and $\alpha_{ke,i}$ are the CES share parameters, and σ^v is the CES elasticity of substitution. The price of *KEL* bundle, *PKEL*, is determined by Equation (1.2.5), which is the CES dual price.

Table 1.2: **Second Level CES Production Equations**

$$XAp_{nf,j} = \sum_v a_{nf,j} ND_j^v \qquad (1.2.1)$$

$$PN_j = \sum_{nf} a_{nf,j} PA_{nf} \qquad (1.2.2)$$

$$AL_j^d = \sum_v \alpha_{l,j}^v \left[\frac{PKEL_j^v}{AW_j}\right]^{\sigma_j^{v,v}} KEL_j^v \qquad (1.2.3)$$

$$KE_j^v = \alpha_{ke,j}^v \left[\frac{PKEL_j^v}{PKE_j^v}\right]^{\sigma_j^{v,v}} KEL_j^v \qquad (1.2.4)$$

$$PKEL_j^v = \left[\alpha_{l,j}^v \left(AW_j\right)^{1-\sigma_j^{v,v}} + \alpha_{ke,j}^v \left(PKE_j^v\right)^{1-\sigma_j^{v,v}}\right]^{1/(1-\sigma_j^{v,v})} \qquad (1.2.5)$$

The combined labour bundle is split into labour demand by type of labour, each with a specific wage rate, *W*.[11] (Though labour markets are assumed to clear for each skill category, we allow for differential wage rates across sectors reflecting potential different institutional arrangements). Equation (1.3.1) determines labour demand by skill type in each sector, using a CES aggregation function. We allow for changes in labour efficiency which can be specified by both skill type and by sector. The producer decision can also be influenced by a wage tax, which is represented by the variable τ^l. The dual price, or the average sectoral wage, *AW*, is defined by Equation (1.3.2).

Table 1.3: **Labour Demand**

$$L_{l,j}^{d} = \frac{\alpha_{lj}^{l}}{\lambda_{lj}} \left[\frac{\lambda_{lj} AW_j}{\Phi_{lj} W_l \left(1+\tau_l^l\right)} \right]^{\sigma_j^l} AL_j^d \tag{1.3.1}$$

$$AW_j = \left[\sum_l \alpha_{lj}^{l} \left(\frac{\Phi_{lj} W_l \left(1+\tau_l^l\right)}{\lambda_{lj}} \right)^{1-\sigma_j^l} \right]^{1/(1-\sigma_j^l)} \tag{1.3.2}$$

The next table describes the dis-aggregation of the capital-energy bundle, *KE*, into its energy and capital-land components. Equation (1.4.1) determines the demand for aggregate energy. Equation (1.4.2) determines the demand for the capital-land bundle by vintage, *KT*, where *PKT* is the price of the capital-land bundle. Equation (1.4.3) defines the dual price of the *KE* bundle.

Table 1.4: **Capital-Land Bundle and Energy Bundle Demand**

$$Ev_j^v = \alpha_{e,j}^v \left[\frac{PKE_j^v}{PEv_j^v} \right]^{\sigma_j^{k,v}} KE_j^v \tag{1.4.1}$$

$$KTv_j^{d,v} = \alpha_{kt,j}^v \left[\frac{PKE_j^v}{PKT_j^v} \right]^{\sigma_j^{k,v}} KE_j^v \tag{1.4.2}$$

$$PKE_j^v = \left[\alpha_{e,j}^v \left(PEv_j^v\right)^{1-\sigma_j^{k,v}} + \alpha_{k,j}^v \left(PKT_j^v\right)^{1-\sigma_j^{k,v}} \right]^{1/(1-\sigma_j^{k,v})} \tag{1.4.3}$$

The next level in the nest determines the demand for the capital and land factors. Equation (1.5.1) defines land demand by sector and vintage, *Tv*, where *PT* is the price of land. Similarly, Equation (1.5.2) determines demand for capital by sector and vintage, *Kv*, where *R* is the rental rate of capital. Note that the rental rate is both sector and vintage specific. Both equations incorporate technology shifters (which will be explained in the section on dynamics).[12] Equations (1.5.4) and (1.5.5) determine respectively aggregate sectoral land demand and capital demand.

Table 1.5: **Capital and Land Demand**

$$Tv_j^{d,v} = \frac{\alpha_{t,j}^v}{\lambda_{t,j}^v}\left[\frac{\lambda_{t,j}^v PKT_j^v}{PT_j}\right]^{\sigma_j^{s,v}} KT_j^v \qquad (1.5.1)$$

$$Kv_j^{d,v} = \frac{\alpha_{k,j}^v}{\lambda_{k,j}^v}\left[\frac{\lambda_{k,j}^v PKT_j^v}{R_j^v}\right]^{\sigma_j^{s,v}} KT_j^v \qquad (1.5.2)$$

$$PKT_j^v = \left[\alpha_{t,j}^v\left(\frac{PT_j}{\lambda_{t,j}^v}\right)^{1-\sigma_j^{s,v}} + \alpha_{k,j}^v\left(\frac{R_j^v}{\lambda_{k,j}^v}\right)^{1-\sigma_j^{s,v}}\right]^{1/\left(1-\sigma_j^{s,v}\right)} \qquad (1.5.3)$$

$$T_j^d = \sum_v Tv_j^{d,v} \qquad (1.5.4)$$

$$K_j^d = \sum_v Kv_j^{d,v} \qquad (1.5.5)$$

The energy bundle determined by Equation (1.4.1) is further dis-aggregated by energy-type. The number of fuel types will depend on the available data. We let the index *e* range over the number of fuel types (eventually the dimension of *e* could even be 1). Equation (1.6.1) determines the demand for the different types of fuels. The λ factor allows for energy efficiency improvement over time which can be sector specific, as well as vintage specific. Equation (1.6.2) determines the CES dual price, *PEv*, of the energy bundle.

Table 1.6: **Decomposition of the Energy Bundle**

$$XAp_{e,j} = \sum_v \frac{\alpha^f_{e,j,v}}{\lambda^e_{jv}} \left[\frac{\lambda^e_{jv} PEv^v_j}{PA_e} \right]^{\sigma^f_{jv}} Ev^v_j \quad (1.6.1)$$

$$PEv^v_j = \left[\sum_e \alpha^f_{e,j,v} \left(\frac{PA_e}{\lambda^e_{jv}} \right)^{1-\sigma^f_{jv}} \right]^{1/(1-\sigma^f_{jv})} \quad (1.6.2)$$

2. Income Distribution

Production generates income, both wage and non-wage, which is distributed in some form to three main institutions: households, government, and financial institutions (both domestic and foreign).

Equation (2.1.1) determines gross operating surplus, *KY*. It is the sum across all vintages and all sectors of capital remuneration, and it incorporates factor payments from abroad. Equation (2.1.2) defines company income, *CY*, it is equal to a share of gross operating surplus (the rest being distributed to households and to foreigners). Equation (2.1.3) determines corporate taxes, Tax^c. The base tax rate is given by the parameter κ^c. However, corporate taxes can be endogenised (in order to meet a fiscal target, for example), in which case the adjustment parameter, δ^c, becomes endogenous. Equation (2.1.4) defines retained earnings, i.e. corporate saving. Corporate saving is equal to a residual share of after-tax company income, net of transfers to the rest of the world.[13] The remaining amount of net company income is distributed to households.

Table 2.1: **Corporate Earnings Equations**

$$KY = \sum_v \sum_i R^v_i Kv^{d,v}_i + ER \sum_r FP^k_r \quad (2.1.1)$$

$$CY = \chi^k KY \quad (2.1.2)$$

$$Tax^c = \delta^c \kappa^c CY \quad (2.1.3)$$

$$Sav^p_c = \left(1 - \sum_h \phi^c_h \right) \left(1 - \delta^c \kappa^c \right) CY - ER \sum_r TR^r_c \quad (2.1.4)$$

Household income derives from two main sources, capital and labour income. Additionally, households receive transfers from the government and from abroad. Equation (2.2.1) defines total labour income, *YL* as the product of total labour demand and the wage rate. There are two adjustments. One comes from wages earned abroad, the other concerns wages remitted to foreign labour. In the latter case, a fixed share of total domestic labour income is assumed to be distributed to foreign labour, while in the former case, foreign wage income is assumed to be constant (in dollar terms).

Labour income is distributed to the households. Equation (2.2.2) defines total household income, *YH*. It is the sum of labour income, distributed capital income and net company income, income from land, and transfers from the government, TR_g^h and from abroad, TR_r^h. Capital, company, and land income are distributed to households using fixed shares. The adjustment factor δ^{HTr} on government transfers can be used as a fiscal instrument in order to achieve a specified target, similar to the adjustment factors on other taxes in the model. Household direct tax, Tax^h, is given by Equation (2.2.3), where κ^h is the tax rate. The adjustment factor δ^{HTx} can be endogenous if the government saving/deficit is exogenous. In this case, the household tax schedules shifts in or out to achieve the net government balance. Otherwise, the household tax schedule is exogenous, and the factor stays at its initial value of 1. Finally Equation (2.2.4) defines household disposable income, *YD*. Disposable income is equal to total household income less taxes and transfer payments to the rest of the world.

Table 2.2: **Household Income Equations**

$$YL_l = \chi_l^l \sum_i \Phi_{lj} W_l L_{l,j}^d + ER \sum_r FW_r^l \qquad (2.2.1)$$

$$\begin{aligned} YH_h \;=\; & \sum_l \Xi_{hl} YL_l + \phi_h^k \chi^h KY + \phi_h^c \left(1 - \delta^c \kappa^c\right) CY + \phi_h^t \sum_j PT_j T_j^d \\ + \; & P\delta^{HTr}\; TR_{g,h}^h + ER \sum_r TR_r^h \end{aligned} \qquad (2.2.2)$$

$$Tax_h^H = \delta^h \kappa_h^h YH_h \qquad (2.2.3)$$

$$YD_h = YH_h - Tax_h^H - ER \sum_r TR_h^r \qquad (2.2.4)$$

3. Household Consumption and Savings

Household disposable income is allocated to goods, services, labour, and savings using the *Extended Linear Expenditure System* (ELES) specification.[14] The consumption function of each household follows the same specification, but income elasticities are household-specific. The consumer problem can be set up as follows:

$$\max \ U = \sum_{i=1}^{n} \mu_i \ln\left(C_i - \theta_i\right) + \mu_s \ln\left(\frac{S}{cpi}\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} PC_i C_i + S = YD$$

$$\text{and} \quad \sum_{i=1}^{n} \mu_i + \mu_s = 1$$

where U is the utility function, C_i is consumption by commodity, S is household saving, PC is the vector of consumer prices, and YD is disposable income. μ and θ are parameters which will be given an interpretation below. S can be thought of as demand for a future bundle of consumer goods. For reasons of simplification, it is assumed that the saving bundle is evaluated using the consumer price index, *cpi*. Lluch provides a more detailed theoretical analysis of how savings enters the utility maximisation problem.

Solving the above optimisation problem leads to the following demand functions:

$$C_i = \theta_i + \frac{\mu_i Y^*}{PC_i}$$

$$S = \mu_s Y^* = YD - \sum_{i=1}^{n} PC_i C_i$$

$$Y^* = YD - \sum_{j=1}^{n} PC_j \theta_j$$

Consumption is the sum of two parts, θ, which is often called the *subsistence minima* or *floor consumption*, and a fraction of Y^*, which is often called the *supernumerary income*. Y^* is equal to disposable income less total expenditures on the subsistence minima.

Table 3.1 presents the equations of the consumer demand system. Equation (3.1.1) defines the consumer price vector (for goods and services), *PC*, it is the Armington price incorporating household specific indirect taxes and subsidies. Equation (3.1.2) defines supernumerary income, that is disposable income less total expenditures on the subsistence minima. (The subsistence minima are adjusted each time period by the growth rate in population). Consumer demand for goods and services is given by Equation (3.1.3).[15] Household savings is determined as a residual and is given in Equation (3.1.4). Aggregate household saving is determined by Equation (3.1.5). Equation (3.1.6) defines the consumer price index.

Table 3.1: **Household Consumption and Savings Equations**

$$PC_{ih} = PA_i(1+\tau_{ih}^h)(1-\varphi_{ih}^h) \quad (3.1.1)$$

$$Y_h^* = YD_h - Pop_h \sum_i PC_{ih}\theta_{ih} \quad (3.1.2)$$

$$XAc_{ih} = Pop_h\ \theta_{ih} + \mu_{ih} Y_h^* / PC_{ih} \quad (3.1.3)$$

$$HSav_h^p = YD_h - \sum_i PC_{ih}\, XAc_{ih} \quad (3.1.4)$$

$$S_H = \sum_h HSav_h^p \quad (3.1.5)$$

$$cpi_h = \frac{\sum_i PC_{ih}\, XAc_{ih}}{\sum_i PC_{ih,0}\, XAc_{ih}} \quad (3.1.6)$$

4. Other Final Demands

All other final demand accounts (except stock changes) are integrated into a single demand matrix component. In the most general version of the model, the final demand components are government current expenditures, government capital expenditures, private capital expenditures, trade and transport margins for domestic sales, and trade and transport margins for imports. All the final demand vectors are assumed to have fixed expenditure shares. The closure of the final demand accounts will be discussed below.

Equation (4.1.1) determines the composition of final demand for each of the final demand components. The demand for goods are determined as constant shares of the volume of total final demand, *TFD*. The index *f* covers government current and capital expenditures, private capital expenditure, and both domestic and import trade margin expenditures. Equation (4.1.2) determines the value of final demand expenditures, *TFDV*. Equation (4.1.3) determines the price of final demand expenditures inclusive of taxes and subsidies, *PFD*. Equation (4.1.4) determines the aggregate final demand price deflator for each type of final demand account, *PTFD*. Trade and transport margins will be discussed in more detail below. Equations (4.1.5) and (4.1.6) determine the revenue side of the margins.

Table 4.1: **Final Demand Expenditure Equations**

$$XAFD_i^f = afd_i^f TFD_f \qquad (4.1.1)$$

$$TFDV_f = \sum_i PFD_i^f XAFD_i^f \qquad (4.1.2)$$

$$PFD_i^f = PAFD_i^f \left(1+\tau_i^f\right)\left(1-\varphi_i^f\right) \qquad (4.1.3)$$

$$PTFD_f = \sum_i afd_i^f PFD_i^f \qquad (4.1.4)$$

$$TFDV_{Margd} = \sum_i \xi_i^d PD_i XD_i \qquad (4.1.5)$$

$$TFDV_{Margm} = \sum_i \xi_i^m PM_i XM_i \qquad (4.1.6)$$

Government current expenditures include expenditures on goods and services. Government aggregate expenditures on goods and services are fixed in real terms. Total nominal government expenditures, *GExp*, is determined by Equation (4.2.1) in Table 4.2. There are several exogenous elements which enter this equation including transfers to households, TR_g^h. Note the potential adjustment factor attached to the household transfer variable. Also note that all domestic transfers are typically held fixed and are multiplied by a price index in order to ensure the homogeneity of the model. Equation (4.2.2) defines the government expenditure deflator, *PG*. Finally, Equation (4.2.3) is simply an identity which equates aggregate real government expenditures to the variable *TFD* (for the accounts indexed by *g*).

Table 4.2: **Current Government Expenditure Equations**

$$
\begin{aligned}
GExp \;=\; & \sum_g \sum_i PA_i XAFD_i^g + \sum_g \left(FDIT_g - FDSubs_g\right) \\
+ \;& P\sum_h \left(\delta^{HTr} TR_{g,h}^h\right) + ER\sum_r TR_g^r
\end{aligned}
\qquad (4.2.1)
$$

$$PG_g\ TG_g = \sum_i PFD_i^g XAFD_i^g \qquad (4.2.2)$$

$$TFD_g = TG_g \qquad (4.2.3)$$

5. Government Revenues and Saving

Government derives most of its revenues from direct corporate and household taxes, and indirect taxes. Subsidies are also provided which enter as negative revenues. Equations (5.1.1)-(5.1.4) in Table 5.1 list all the different indirect taxes paid by production activities, household consumption, final demand expenditures, and exports, respectively, *PITx, SITx*, *HITx*, *FDITx*, and *EITx*. Equation (5.1.5) describes the sum of all indirect taxes.

Table 5.1: **Indirect Tax Equations**

$$PITx = \sum_i \left(\delta^p \tau_i^p - \varphi_i^p\right) PX_i XP_i \qquad (5.1.1)$$

$$HITx = \sum_h \sum_i PA_i \tau_{ih}^h XAc_{ih} \qquad (5.1.2)$$

$$FDITx_f = \sum_i PA_i \tau_i^f XAFD_i^f \qquad (5.1.3)$$

$$EITx = \sum_r \sum_i PEr_{ir} \tau_{ir}^E ESr_{ir} \qquad (5.1.4)$$

$$TIndTax = PITx + HITx + \sum_f FDITx_f + EITx \qquad (5.1.5)$$

Equations (5.2.1)-(5.2.3) in Table 5.2 define the level of subsidies for household consumption, other final demand expenditures, and exports, respectively, *HSubs*, *FDSubs*, and *ESubs*. Total subsidies is given by Equation (5.2.4).

Table 5.2: **Subsidy Equations**

$$HSubs = \sum_h \sum_i PA_i (1+\tau_{ih}^h)\varphi_{ih}^h XAc_{ih} \qquad (5.2.1)$$

$$FDSubs_f = \sum_i PA_i \, (1+\tau_i^f) \, \varphi_i^f \, XAFD_i^f \qquad (5.2.2)$$

$$ESubs = \sum_r \sum_i PEr_{ir} \, (1+\tau_{ir}^E) \, \varphi_{ir}^E \, ESr_{ir} \qquad (5.2.3)$$

$$TSubs = HSubs + \sum_f FDSubs_f + ESubs \qquad (5.2.4)$$

Table 5.3 defines fiscal closure for the government. Equation (5.3.1) describes total income from import tariffs, where *WPM* are world prices, τ^m are tariffs, and *XMr* represents import volumes. All the relevant import variables are doubly indexed since they represent variables by sector and region of origin. The exchange rate is used to convert world prices (e.g. in dollars) into local currency. There is an additional adjustment factor δ^{Tar} which allows the aggregate tariff rate to vary endogenously. Equation (5.3.2) identifies miscellaneous government revenues, these are all revenues less household direct taxes. Equation (5.3.3) provides total current government nominal revenues, *GRev*. Equation (5.3.4) and (5.3.5) define respectively the nominal and real level of government saving. Two government closure rules are implemented. Under the default rule, government saving is held fixed (typically at its base value), and one of the taxes (or government transfers to households) is allowed to adjust (uniformly) to achieve the government fiscal target. Under the second closure rule, all tax levels and transfers are fixed, and real government saving is endogenous. This latter rule can have significant consequences on the level of investment since investment is savings driven.

Table 5.3: **Government Revenues and Closure Equations**

$$YTrade = ER\,\delta^{Tar}\sum_{r}\sum_{i} WPM_{ri}\tau_{ri}^{m} XMr_{ri} \qquad (5.3.1)$$

$$\begin{aligned} MiscRev &= Tax^{c} + TIndTax - TSubs + YTrade + ER\sum_{r} TR_{r}^{g} \\ &+ \sum_{l}\sum_{i}\tau_{l}^{l} W_{l}\Phi_{li} L_{l,j}^{d} + \sum_{i}\left[\tau_{i}^{vd} PD_{i} XD_{i} + \tau_{i}^{vm} PM_{i} XM\ \right] \end{aligned} \qquad (5.3.2)$$

$$GRev = MiscRev + \sum_{h} Tax_{h}^{H} \qquad (5.3.3)$$

$$S_{G} = GRev - GExp \qquad (5.3.4)$$

$$RSg = S_{G} / P \qquad (5.3.5)$$

6. Trade, Domestic Supply and Demand

Similar to many trade CGE models, we have assumed that imported goods are not perfect substitutes for goods produced domestically.[16] The degree of substitution will depend on the level of disaggregation of the commodities. For example, wheat is more substitutable as a commodity than grains, which in turn are more substitutable than a commodity called primary agricultural products. The Armington assumption reflects two stylised facts. Trade data shows the existence two-way trade which is consistent with the Armington assumption. As well, and related, the Armington assumption leads to a model where perfect specialisation, which is rarely observed, is avoided.

In this version of the model, we have adapted the CES functional specification for the Armington assumption. This has some undesirable properties which have been explored in more detail elsewhere[17], but alternative formulations have proven to be deficient as well. The adoption of the *Constant Elasticity of Transformation* (CET) specification for exports alleviates to some extent the deficiencies of the Armington CES specification. We also assume that there is only one domestic Armington agent, this is sometimes known as *border-level* Armington specification. It is parsimonious in both data requirements and computational resources.

To allow for the existence of multiple trading partners, the model adopts a two-level CES nesting to represent the Armington specification (see Figure 2). At the top level, agents choose an optimal combination of the domestic good and an import aggregate which is determined by a set of relative prices and the degree of substitutability. Let *XA* represent aggregate demand for an Armington composite, with the associated Armington price of *PA*. Each agent then minimises the cost of obtaining the Armington composite, subject to an aggregation function. This can be formulated by:

$$\begin{aligned} \min \quad & PD\ XD + PM\ XM \\ \text{s.t.} \quad & XA = \left[a_d\, XD^{\rho} + a_m\, XM^{\rho}\right]^{1/\rho} \end{aligned}$$

where *XD* is demand for the domestic good, *PD* is the price of obtaining the domestic good, *XM* is demand for the aggregate imported good, *PM* is the aggregate import price, *a* are the CES share parameters, and ρ is the CES exponent. ρ is related to the CES substitution elasticity via the following relation:

$$\rho = \frac{\sigma - 1}{\sigma} \Leftrightarrow \sigma = \frac{1}{1-\rho}$$

At the second level of the nest, agents choose the optimal choice of imports across regions, again as a function of the relative import prices and the degree of substitution across regions. Note that the import prices are region specific, as are the tariff rates. The second level nest also uses a CES aggregation function. The CES formulation implies that the substitution of imports between any two pairs of importing partners is identical. Table 6.1 lists the solution of the optimisation problem described above. Equation (6.1.1) determines domestic demand for the Armington aggregate across all agents of the economy, *XA*. Equations (6.1.2) and (6.1.3) determine respectively, the optimal demand for the domestic component of the Armington aggregate, *XD*, and aggregate import demand, *XM*. Equation (6.1.4) defines the price of the Armington bundle, *PA*, which is the CES dual price. Both the domestic price of domestic goods, and the price of the aggregate import bundle are adjusted to incorporate a value added tax and trade and transportation margins. Both the tax and margin are assumed to differ between domestic and import goods.

Table 6.1: **Top-level Armington Equations**

$$XA_i = \sum_j XAp_{ij} + \sum_h XAc_{ih} + \sum_f XAFD_f \qquad (6.1.1)$$

$$XD_i = \beta_i^d \left(\frac{PA_i}{PD_i\left(1+\tau_i^{vd}+\xi_i^d\right)} \right)^{\sigma_i^m} XA_i \qquad (6.1.2)$$

$$XM_i = \beta_i^m \left(\frac{PA_i}{PM_i\left(1+\tau_i^{vm}+\xi_i^m\right)} \right)^{\sigma_i^m} XA_i \qquad (6.1.3)$$

$$PA_i = \left[\beta_i^d \left(PD_i\left(1+\tau_i^{vd}+\xi_i^d\right)\right)^{1-\sigma_i^m} + \beta_i^m \left(PM_i\left(1+\tau_i^{vm}+\xi_i^m\right)\right)^{1-\sigma_i^m} \right]^{1/(1-\sigma_i^m)} \qquad (6.1.4)$$

The equations in Table 6.2 describe the decomposition of the aggregate import bundle, *XM* into its components, i.e. imports by region of origin. Each demand component will be a function of the price of the exporting partner, as well as partner-specific tariff rates. Equation (6.2.1) determines import volume by sector and region of origin, *XMr*, where *PMr* is the partner specific import price, in domestic currency and inclusive of tariffs. Equation (6.2.2) defines the price of the aggregate import bundle, *PM*, which is the CES dual price. Finally, Equation (6.2.3) defines the domestic import price, *PMr*, which is equal to the import price of the trading partner, converted into local currency, and inclusive of the partner-specific tariff rate.

Table 6.2: **Second-level Armington Equations**

$$XMr_{ri} = \beta^r_{ri}\left(\frac{PM_i}{PMr_{ri}}\right)^{\sigma^w_i} XM_i \qquad (6.2.1)$$

$$PM_i = \left[\sum_r \beta^r_{ri}\left(PMr_{ri}\right)^{1-\sigma^w_i}\right]^{1/\left(1-\sigma^w_i\right)} \qquad (6.2.2)$$

$$PMr_{ri} = ER\ WPM_{ri}\left(1+\tau^m_{ri}\right) \qquad (6.2.3)$$

Treatment of domestic production is symmetric to the treatment of domestic demand. Domestic producers are assumed to perceive the domestic market as different from the export market. The reason is similar: a high level of aggregation. Further, export markets might be more difficult to penetrate, forcing perhaps different quality standards than those applicable for the domestic market. This formulation assumes a production possibilities frontier where each producer maximises sales, subject to being on the frontier, and influenced by relative prices. The optimisation problem is formulated somewhat differently since the object of the local producer is to maximise sales, not to minimise costs. We therefore have:

$$\begin{aligned} \max \quad & PD\ XD + PE\ ES \\ s.t. \quad & XP = \left[\gamma_d XD^{\lambda} + \gamma_e ES^{\lambda}\right]^{1/\lambda} \end{aligned}$$

where *XD* is aggregate domestic sales of domestic production, *ES* is foreign sales of domestic production (exports), with a producer export price of *PE*, *XP* is aggregate domestic production with a producer price of *PP*, γ are the CET share parameters, and l is the CET exponent. The CET exponent is related to the CET substitution elasticity, Λ via the following relation:

$$\lambda = \frac{\Lambda + 1}{\Lambda} \Leftrightarrow \Lambda = \frac{1}{\lambda - 1}$$

Analogous to the Armington specification, producer supply decisions are assumed to be undertaken it two steps (see Figure 3). First, producers choose the optimal combination of domestic supply and aggregate export supply. Then, an additional step which optimises export supply across trading partners. The top-level producer supply decisions, in reduced form, are given by Equations (6.3.1) and (6.3.2), where the share parameters are α^t and the CET substitution elasticity is σ^t.[18] Equation (6.3.3) is the CET dual price function, which determines sectoral domestic output. If the CET elasticity is infinite, producers perceive no differentiation across markets, in which case both domestic and export goods are sold at the uniform producer price, *PP*, and output is simply the sum of domestic supply and export supply. (The formulas reflect an adjustment for stock building. Domestic change in stocks are priced at the aggregate producer price, *PP*, and imported stock changes are priced at the aggregate import price. Trade margins and the value added tax are not applied to stock changes). Sectoral stock building is modelled as a fixed share of a volume of stock building, *StB*. This formulation implies that stock building is simply subtracted (added) from (to) total current output, *XP*.)

Table 6.3: **Top-level CET Equations**

$$\begin{cases} XD_i = \alpha^t_{d,i}\left(\dfrac{PD_i}{PP_i}\right)^{\sigma^t_i}\left(XP_i - \alpha^{d,st}_i StB\right) & \text{if} \quad \sigma^t_i < \infty \\ PD_i = PP_i & \text{if} \quad \sigma^t_i = \infty \end{cases} \qquad (6.3.1)$$

$$\begin{cases} ES_i = \alpha^t_{e,i}\left(\dfrac{PE_i}{PP_i}\right)^{\sigma^t_i}\left(XP_i - \alpha^{d,st}_i StB\right) & \text{if} \quad \sigma^t_i < \infty \\ PE_i = PP_i & \text{if} \quad \sigma^t_i = \infty \end{cases} \qquad (6.3.2)$$

$$\begin{cases} PP_i = \left[\alpha^t_{d,i} PD_i^{1+\sigma^t_i} + \alpha^t_{e,i} PE_i^{1+\sigma^t_i}\right]^{1/(1+\sigma^t_i)} & \text{if} \quad \sigma^t_i < \infty \\ XP_i = XD_i + ES_i + \alpha^{d,st}_i StB & \text{if} \quad \sigma^t_i = \infty \end{cases} \qquad (6.3.3)$$

The second-level CET nest determines the optimal supply of exports to individual trading partners, *ESr*. Equation (6.4.1) defines export supply by region of destination. Equation (6.4.2) determines the aggregate export price, *PE*.

Table 6.4: **Second-level CET Equations**

$$ESr_{ir} = \alpha^r_{ir}\left(\frac{PEr_{ir}}{PE_i}\right)^{\sigma^z_i} ES_i \qquad (6.4.1)$$

$$PE_i = \left[\sum_r \alpha^r_{ir}\left(PEr_{ir}\right)^{1+\sigma^z_i}\right]^{1/\left(1+\sigma^z_i\right)} \qquad (6.4.2)$$

Table 6.5 presents the equations which determine export demand by the regional trading partners, and the export market equilibrium condition. Equation (6.5.1) defines export demand by trading partner, *ED*. If the exporting country has some market power, it will face a downward sloping demand curve. This is implemented using a constant elasticity function, with the elasticity given by σ^e. Export demand will also be influenced by the price of competing exports. This is reflected in the variable *WPINDEX*, which is exogenous since it is assumed the domestic economy does not influence export prices of its trading partners. (Changes in the *WPINDEX* could show the impacts of exogenous changes in the terms-of-trade). Under the small-country assumption the export demand elasticity is infinity, and the exporting country faces a flat demand curve, i.e. the export price is fixed (in dollar terms). Equation (6.5.2) converts the domestic export producer price into the domestic export price inclusive of taxes and subsidies (however, it is still in local currency). Equation (6.5.3) defines the export market equilibrium, i.e. the equality between domestic export supply and foreign demand.

Table 6.5: **Export Demand and Market Equilibrium**

$$\begin{cases} ED_{ir} = \alpha^e_{ir}\left(\dfrac{ER\ WPINDEX_{ir}}{WPE_{ir}}\right)^{\sigma^e_{ir}} & \text{if} \quad \sigma^e_{ir} < \infty \\ WPE_{ir} = ER\ WPINDEX_{ir} & \text{if} \quad \sigma^e_{ir} = \infty \end{cases} \qquad (6.5.1)$$

$$WPE_{ir} = (1+\tau^E_{ir})(1-\varphi^E_{ir})PEr_{ir} \qquad (6.5.2)$$

$$ESr_{ir} = ED_{ir} \qquad (6.5.3)$$

7. Equilibrium Conditions

The first factor market equilibrium condition concerns labour. Labour demand, by skill type is generated by production decisions. In terms of supply, the model implements a simple labour supply curve, where labour supply is a function of the real wage. Equation (7.1.1) defines the labour supply curve. If the supply elasticity is less than infinity, labour supply is a function of the equilibrium real wage rate. In the extreme case where the elasticity is zero, labour is fully employed and fixed. If the elasticity is infinite, the real wage is fixed and there is no constraint on labour supply. This may be an appropriate assumption in cases where the level of unemployment is relatively high. Equation (7.1.2) determines equilibrium on the labour market. If the labour supply curve is not flat, it determines the equilibrium wage rate. If the labour supply curve is flat, it sets labour supply identically equal to aggregate labour demand. Labour by skill type is assumed to be perfectly mobile across sectors, therefore Equation (7.1.2) determines the uniform wage by skill type. Because the model allows for wages to vary across sectors, the uniform wage is actually the aggregate wage which varies uniformly across sectors for each skill type, and the relative wages across sectors are held fixed at their base levels.

Table 7.1: **Equilibrium Conditions for the Labour Market**

$$\begin{cases} L_l^s = a_l \left(\dfrac{W_l}{P} \right)^{\omega_l} & \text{if} \quad \omega_l < \infty \\ W_l = P\,\overline{W_l} & \text{if} \quad \omega_l = \infty \end{cases} \qquad (7.1.1)$$

$$\sum_i L_{l,i}^d = L_l^s \qquad (7.1.2)$$

Land demand, similar to demand for labour and capital, is generated by the production sector. Land supply is modelled using the CET specification. If the elasticity is infinite, land is perfectly mobile across sectors. If the elasticity is zero, land is fixed and sector-specific. Between these two extreme values, land is partially mobile and sectoral supply will reflect the relative rate-of-return of land across sectors. Equations (7.2.1)-(7.2.3) reflect either situation (finite or infinite). In the case of a finite CET elasticity, Equation (7.2.1) determines the aggregate price of land, *PLand*, which is the CET dual price. *TLand* is aggregate land supply which is exogenous. Equation (7.2.2) determines sectoral supply of land, T^s, and Equation (7.2.3) is the equilibrium condition which determines the sector-specific land price, *PT*. In the case of infinite elasticity, Equation (7.2.1) determines the aggregate (uniform) price of land through an equilibrium condition which equates total land supply, *TLand*, to aggregate land demand. Equation (7.2.2) trivially sets the sectoral land price equal to the economy-wide land price, and Equation (7.2.3) equates sectoral supply to sectoral demand.

Table 7.2: **Land Supply and Market Equilibrium**

$$\begin{cases} PLand = \left[\sum_i \alpha_i^{ts} PT_i^{1+\omega^s} \right]^{1/(1+\omega^s)} & \text{if} \quad \omega^s < \infty \\ TLand = \sum_i T_i^d & \text{if} \quad \omega^s = \infty \end{cases} \tag{7.2.1}$$

$$\begin{cases} T_i^s = \alpha_i^{ts} \left(\dfrac{PT_i}{PLand} \right)^{\omega^s} TLand & \text{if} \quad \omega^s < \infty \\ PT_i = PLand & \text{if} \quad \omega^s = \infty \end{cases} \tag{7.2.2}$$

$$T_i^s = T_i^d \tag{7.2.3}$$

---

8. Determination of Vintage Output and Capital Market Equilibrium

The model is set up to run in either comparative static mode or recursive dynamic mode. Capital market equilibrium is different in the two cases, and each will be described separately.

Comparative Static Capital Market Equilibrium

In comparative static mode, there is no distinction made between old and new capital. Each sector determines demand for a single aggregate capital good. On the supply side, the model implements a CET supply allocation function (similar to land above). There is a single "capitalist" who owns all the capital in the economy, and supplies it to the different sectors based on each sector's rate of return. Capital mobility across sectors is determined by the "capitalist's" CET substitution elasticity. The substitution elasticity is allowed to vary from 0 to infinity. If the elasticity is 0, there is no capital mobility. This is an adequate description of a short term scenario. In the polar case, the substitution elasticity is infinite and there is perfect capital mobility. An intermediate value would allow for partial capital mobility.

The equations in Table 8.1 determine the equilibrium conditions for the capital market in comparative static mode. Equation (8.1.1) determines the aggregate rental rate. If there is partial capital mobility, the aggregate rental rate is the CET dual price of the sector specific rates of return. If there is perfect capital mobility, the aggregate rental rate is determined by an equilibrium condition which equates aggregate capital demand to total capital supply. Equation (8.1.2) determines either sectoral capital supply, or the sectoral rental rate. If capital is partially mobile, sectoral capital supply is determined by the CET first order condition, i.e. sectoral capital supply is a function of each sectors relative rate of return. If capital is perfectly mobile, the equivalent condition identically sets the sectoral rate of return to the economy-wide rate of return. Finally, Equation (8.1.3) determines the sectoral rate of return in the case of partial capital mobility. Under perfect capital mobility, it trivially equates capital supply to capital demand.

Table 8.1: **Capital Market Equilibrium in Comparative Static**

$$\begin{cases} TR = \left[\sum_i \alpha_i^s R_i^{1+\Lambda}\right]^{1/(1+\Lambda)} & \text{if} \quad \Lambda < \infty \\ TK^s = \sum_i K_i^d & \text{if} \quad \Lambda = \infty \end{cases} \tag{8.1.1}$$

$$\begin{cases} K_i^s = \alpha_i^s \left(\frac{R_i}{TR}\right)^{\Lambda} TK^s & \text{if} \quad \Lambda < \infty \\ R_i = TR & \text{if} \quad \Lambda = \infty \end{cases} \tag{8.1.2}$$

$$K_i^s = K_i^d \tag{8.1.3}$$

Recursive-Dynamic Capital Market Equilibrium

Sectoral output is essentially determined by aggregate demand for domestic output, see Equation (6.3.3). (In the simplest case, with no market differentiation, output is equal to the sum of domestic demand for domestic output, plus export demand, i.e. *XP = XD+ED*.) The producer decides the optimal way to divide production of total output across vintages. At first, the producer will use all the capital installed at the beginning of the capital, this is the depreciated installed capital from the previous period. If demand exceeds what can be produced with the *old* capital, the producer will demand new capital. If demand is lower than the output which can be produced with the *old* capital, the producer will disinvest some of the installed capital.

Equation (8.2.1) provides the capital/output ratio for old capital, χ (note that $Kv^{d,Old}$ reflects the optimal capital demand for old capital by the producer). Once the capital/output ratio is determined, it is easy to determine the optimal output using old capital. Equation (8.2.2) determines this quantity, XPv^{Old}, where an upper bound is given by total output. If the producer owns too much old capital, i.e. the desired output exceeds total demand, the producer will disinvest the difference between the initial capital stock and the capital stock which will produce the desired demand. Equation (8.2.3) determines output produced with new capital as a residual.

Table 8.2: **Determination of Vintage Output**

$$\chi_i^{Old} = \frac{Kv_i^{d,Old}}{XPv_i^{Old}} \qquad (8.2.1)$$

$$XPv_i^{Old} = \min\left(K_{i,0}^s / \chi_i^{Old}, XP_i\right) \qquad (8.2.2)$$

$$XPv_i^{New} = XP_i - XPv_i^{Old} \qquad (8.2.3)$$

If a sector is in decline, i.e. it has too much installed capital given its demand, it will disinvest. The capital supply curve is a simple constant elasticity function of the relative rental rates. The higher the rental rate on old capital, the higher the supply of old capital. The formula which is used is:

$$K_i^{s,Old} = K_{i,o}^s \left[\frac{R_{i,t}^{Old} / R_{i,t}^{New}}{R_{i,t-1}^{Old} / R_{i,t-1}^{New}}\right]^{\eta_i^k}$$

where η^k is the dis-investment elasticity. Another way to think of this is to subtract the two capital numbers, i.e.

$$K_{i,0}^s - K_i^{s,Old} = K_{i,o}^s \left[1 - \left(\frac{R_{i,t}^{Old} / R_{i,t}^{New}}{R_{i,t-1}^{Old} / R_{i,t-1}^{New}}\right)^{\eta_i^k}\right]$$

This represents the supply of disinvested capital, which increases as the relative rental rate of old capital decreases. At the limit, when the rental rates are equalised, there is no disinvested capital. At equilibrium, demand for old capital (in each declining sector), must equal supply of old capital. We can therefore invert the first equation to determine the rental rate on old capital, assuming the sector is in decline and supply equals demand. Equation (8.3.1) determines the relative rental rate on old capital for sectors in decline, i.e. it is the ratio of the old rental rate to the new rental rate. It is bounded above by 1, because the rental rate on old capital in declining sectors is not allowed to exceed the rental rate on new capital.

Equation (8.3.2) determines the rental rate on mobile capital. Mobile capital is the sum of new capital, disinvested capital, and installed capital in expanding sectors. It is not necessary to subtract immobile capital from each side of the capital equilibrium condition, i.e. the rental rate on mobile capital can be determined from the aggregate capital equilibrium condition. Equation (8.3.3) is an identity setting the rental rate on new capital equal to the rental rate on mobile capital. Equation (8.3.4) determines the rental rate of old capital. If a sector is disinvesting, the rental rate on old capital is essentially determined by Equation (8.3.1). If a sector is expanding, than *RR* is equal to 1, and therefore the rental rate on old capital in expanding sectors will be equal to the rental rate of new capital.

Table 8.3: **Capital Market Equilibrium**

$$RR_{i,t} = \min\left(1,\ RR_{i,t-1}\left(\frac{Kv_i^{d,v}}{K_{i,0}^s}\right)^{1/\eta_i^k}\right) \qquad (8.3.1)$$

$$\sum_i K_i^d = K^s \qquad (8.3.2)$$

$$R_i^{New} = TR \qquad (8.3.3)$$

$$R_i^{Old} = TR\ RR_i \qquad (8.3.4)$$

---

9. Macro Closure

Government closure was discussed above, current government savings are determined either endogenously with fixed tax rates, or exogenously, with one of the tax adjustment factors endogenous.

Equation (9.1.1) is the ubiquitous savings equals investment equation. In Equation (9.1.1), $TFDV_{zp}$ is the value of private investment expenditures, whose value must equal total resources allocated to the private investment sector: retained corporate earning, Sav_c^p, total household savings, S_h, government savings, S_g, the sum across regions of foreign capital flows, S_f, and net of stock building expenditures.

The last closure rule concerns the balance of payments. First, we make the small country assumption for imports, i.e. local consumption of imports will not affect the border price of imports, *WPM*. Equation (9.1.2) is the overall balance of payments equation. The value of imports, at world (border) prices, must equal the value of exports, at border prices (i.e. inclusive of export taxes and subsidies) plus net transfers and factor payments, and plus net capital inflows. The balance of payments constraint is dropped from the model due to Walras' Law

The final equations of the model, Equations (9.1.3)-(9.1.5) are used to calculate the domestic price index which is used to inflate real domestic transfers. Note that real GDP is measured in efficiency units. The numéraire of the model is the exchange rate.

Table 9.1: **Closure Equations**

$$TFDV_{zp} = Sav_c^p + S_H + ER\sum_r S_{fr} + S_G - \sum_i \left(\alpha_i^{d,st} PP_i + \alpha_i^{m,st} PM_i\right) StB \qquad (9.1.1)$$

$$\begin{aligned} ER\sum_r\sum_i WPM_{ri}\,XMr_{ri} \;=\; & \sum_r\sum_i WPE_{ir}\,ESr_{ir} + ER\sum_r S_{fr} \\ - \; & \left(1-\chi_l^l\right)\sum_i \Phi_{li} W_l L_{l,i}^d + ER\sum_l\sum_r FW_r^l \\ - \; & \chi_c^r KY + ER\sum_r FP_r^c - ER\sum_r TR_c^r \\ - \; & ER\sum_r\sum_r\left(TR_h^r - TR_r^h\right) - ER\sum_r\left(TR_g^r - TR_r^g\right) \end{aligned} \qquad (9.1.2)$$

$$GDPVA = \sum_l W_l \sum_i \Phi_{li} L_{li}^d + \sum_i\sum_v PT_i Tv_i^{d,v} + \sum_i\sum_v R_i^v Kv_i^{d,v} \qquad (9.1.3)$$

$$RGDP = \sum_l W_{l,0} \sum_i \Phi_{li} \lambda_{li}^l L_{li}^d + \sum_i\sum_v PT_{i,0} \lambda_{t,i}^v Tv_i^{d,v} + \sum_i\sum_v R_{i,0}^v \lambda_{k,i}^v Kv_i^{d,v} \qquad (9.1.4)$$

$$P = \frac{GDPVA}{RGDP} \qquad (9.1.5)$$

10. Dynamics

Pre-Determined Variables

The first table presents the variables which are pre-determined, i.e. they do not depend on any contemporaneous endogenous variables. Equation (10.1.1) determines the labour supply shift factor which is equal to the previous period's labour supply shift factor multiplied by an exogenously specified labour supply growth rate. (All dynamic equations reflect the fact that the time steps may not be of equal size. The growth rates are always given as per cent per annum increases.) Equation (10.1.2) provides a similar equation for population. The population and labour growth rates are allowed to differ. Government (real) expenditures and the transfers between government and households grow at the rate of growth of GDP. This latter growth rate is exogenously specified (for the BaU scenario). Equations (10.1.3)-(10.1.4) provide the relevant formulas. Users can input there own exogenous assumptions about these variables. Equation (10.1.5) determines the amount of installed capital at the beginning of the period. If a sector is expanding, this will equal the amount of old capital in the sector at the end of the period. If a sector is declining, the amount of old capital at the end of the period will be less than the initial installed capital. The depreciation rate is exogenous.

Table 10.1: **Pre-Determined Variables**

$$a_{l,t} = \left(1+\gamma^{l}\right)^{n} a_{l,t-n} \quad (10.1.1)$$

$$Pop_{t} = \left(1+\gamma^{p}\right)^{n} Pop_{t-n} \quad (10.1.2)$$

$$TG_{t} = \left(1+\gamma^{y}\right)^{n} TG_{t-n} \quad (10.1.3)$$

$$TR^{h}_{g,t} = \left(1+\gamma^{y}\right)^{n} TR^{h}_{g,t-n} \quad (10.1.4)$$

$$K^{s}_{i,0,t} = \left(1-\delta\right)^{n} K^{d}_{i,t-n} \quad (10.1.5)$$

Capital Stock

The motion equation for the aggregate capital stock is given by the following 1-step formula:

$$K_t = (1-\delta)K_{t-1} + I_{t-1}$$

where K is the aggregate capital stock, δ is the annual rate of depreciation, I_{t-1} is the level of real investment in the previous period. Using mathematical induction, we can deduce the multi-period transition equation:

$$\begin{aligned} K_t &= (1-\delta)\left[(1-\delta)K_{t-2} + I_{t-2}\right] + I_{t-1} \\ &\vdots \\ K_t &= (1-\delta)^n K_{t-n} + \sum_{j=1}^{n}(1-\delta)^{j-1} I_{t-j} \end{aligned}$$

If the step size if greater than 1, the model does not calculate the intermediate values for the path of real investment. The investment path is estimated using a simple linear growth model, i.e.

$$I_j = (1+\gamma^i) I_{j-1}$$

where

$$\gamma^i = \left(\frac{I_t}{I_{t-n}} \right)^{1/n} - 1$$

Note that the formula for the investment growth depends on the contemporaneous level of real investment. This explains why the current capital stock is not pre-determined. If real investment increases (e.g. because foreign transfers increase), this will have some effect on the current capital stock via its influence on the estimated growth rate of real investment. Inserting the formula for the estimated real investment stream in the capital stock equation, we derive:

$$K_t = (1-\delta)^n K_{t-n} + \sum_{j=1}^{n} (1-\delta)^{j-1} (1+\gamma^i)^{n-j} I_{t-n}$$

Further derivation, yields Equation (10.2.1) for the aggregate capital stock. Equation (10.2.2) defines the annualised growth rate of real investment which is used to calculate the aggregate capital stock. Equation (10.2.3) determines the level of normalised capital. There are two indices of capital stock. The first index is the normalised level of capital stock. This index is called normalised because it is the level of capital stock in each sector which yields a rental rate of 1. The second index is the actual level of the capital stock, given in base year prices. The latter variable is only used in two equations. It is used to determine the depreciation allowance, and it is used to update the level of the capital stock in Equation (10.2.1) (because it is in the same units as the level of real investment).[19]

Table 10.2: **Aggregate Capital Stock**

$$K_t = (1-\delta)^n K_{t-n} + \frac{(1+\gamma^i)^n - (1-\delta)^n}{\gamma^i + \delta} I_{t-n} \quad (10.2.1)$$

$$\gamma^i = \left(\frac{I_t}{I_{t-n}}\right)^{1/n} - 1 \quad (10.2.2)$$

$$K_t^s = \frac{K_{t-n}^s}{K_{t-n}} K_t \quad (10.2.3)$$

Productivity

Productivity enters the value added bundle — labour, land, and capital — as separate efficiency parameters for the three factors, differentiated by sector and by vintage. In the current version of the model, and for lack of better information, the labour efficiency factor (and the energy efficiency factor) are exogenous. In defining the reference simulation, the growth path of real GDP is pre-specified, and a single economy-wide efficiency factor for land and capital is determined endogenously. In subsequent simulations, i.e. with dynamic policy shocks, the capital and land efficiency factors are exogenous, and the growth rate of real GDP is endogenous.

Equation (10.3.1) defines the growth rate of real GDP. In defining the reference simulation, both lagged real GDP and the growth rate γ^y are exogenous, therefore the equation is used to determine the common efficiency factor for land and capital. In subsequent simulations, Equation (10.3.1) determines γ^y, i.e. the growth rate of real GDP. Equations (10.3.2) and (10.3.3) determine respectively the efficiency factors for capital and land. Both are set to the economy-wide efficiency parameter determined by Equation (10.3.1), however, the model allows for a partition of sectors, where the index *i'* indexes a subset of all the sectors. It is assumed that the sectors not indexed by *i'* have no efficiency improvement in land-capital. Equation (10.3.4) determines the common capital-labour efficiency growth factor, which is stored in a file for subsequent simulations. There are alternative methods for specifying and implementing the reference scenario.

Table 10.3: **Capital-Land Efficiency**

$$RGDP_t = \left(1+\gamma^y\right)^n RGDP_{t-n} \qquad (10.3.1)$$

$$\lambda^v_{k,i'} = \lambda_{kt} \qquad (10.3.2)$$

$$\lambda^v_{t,i'} = \lambda_{kt} \qquad (10.3.3)$$

$$\lambda_{kt,t} = \left(1+\gamma^{kt}_t\right)^n \lambda_{kt,t-n} \qquad (10.3.4)$$

Vintage Re-Calibration

At the beginning of each new period, the parameters of the production structure need to be modified to reflect the changing composition of capital. As a new period begins, what was new capital gets added to old capital, i.e. the new *Old* capital has a different composition from the previous *Old* capital. A simple rule is used to re-calibrate the production structure: the parameters are calibrated such that they can re-produce the previous period's output using the aggregate capital of the previous period, but with the *Old* elasticities. (The parameters of the *New* production structure are not modified.) The relevant formulas are not re-produced here but can be found in the GAMS code.

11. Emissions

Emissions data at a country and detailed level have rarely been collated. An extensive data set exists for the United States which includes thirteen types of emissions, see Table 11.1.[20]

The emission data for the United States has been collated for a set of over 400 industrial sectors. Generally, the emission data has been directly associated with the volume of output. This has several consequences. First, the only way to reduce emissions, with a given (abatement) technology, is to reduce output. This is often an unpleasant message for developing country policy makers. The second consequence is that it ignores important sources of pollution outside the production side of the economy, namely household consumption. In an attempt to ameliorate this situation, the pollution data of the United States has been regressed on a small subset of inputs of the US input output table. Using econometric estimates, we have shown that the level of emissions can be explained by a very small subset of inputs.[21] This allows producers to substitute away from polluting inputs, and to use the same pollution coefficients for final demand consumption.

Since the emission factors are originally calculated from a US data base, they are appropriately scaled so as to be consistent with the definition of output and inputs of the designated country. The following example shows how this is done in practice. Assume, in a specific sector, that output in 1990 has the value $1 billion, and that the estimated amount of lead emitted from that sector is 13,550 pounds. If we normalise the output price to 1 in 1990, the emission factor has units 1.355×10^{-5} pounds per (1990) USD, or 13.55 pounds per million (1990) USD. If output, in the same sector is 300 billion pesos (in Mexico in 1988), the dollar equivalent is $131.5 million (1988 USD). Abstracting from inflation, this leads to lead emissions of 1,782 pounds. The emission factor for lead in Mexico (in this sector) would then be 5.94 pounds per billion 1988 pesos.

Table 11.1: **Emission Types**

Air Pollutants		
1.	Suspended particulates	PART
2.	Sulfur dioxide (SO_2)	SO2
3.	Nitrogen dioxide (NO_2)	NO2
4.	Volatile organic compounds	VOC
5.	Carbon monoxide (CO)	CO
6.	Toxic air index	TOXAIR
7.	Biological air index	BIOAIR
Water Pollutants		
8.	Biochemical oxygen demand	BOD
9.	Total suspended solids	TSS
10.	Toxic water index	TOXWAT
11.	Biological water index	BIOWAT
Land Pollutants		
12.	Toxic land index	TOXSOL
13.	Biological land index	BIOSOL

Equation (11.2.1) defines the total level of emissions for each pollutant *p*. The bulk of the pollution is assigned to the direct consumption of goods, which is the second term in the expression. The level of pollution associated with the consumption of each good is constant (across a row of the SAM), i.e. there is no difference in the amount of pollution emitted per unit of consumption whether it is generated in production or in final demand consumption. The first term in Equation (11.2.1) represents what we call process pollution. It is the residual amount of pollution in production which is not explained by the consumption of inputs. In the estimation procedure, a process dummy proved to be significant in certain sectors. If an emission tax (or taxes) are exogenous, they are specified in physical units, i.e. dollars per pound (or tonne). Equation (11.2.2) converts this into a nominal amount.

The equations in Table 11.3 re-produce the corresponding equations in the text if a pollution tax is imposed. The tax can be generated in one of two ways. It can either be specified exogenously (in which case it is multiplied by a price index to preserve the homogeneity of the model), or it can be generated endogenously be specifying a constraint on the level of emission. In the latter case, Equation (11.2.1) is used to define the pollution level constraint, and the tax which is generated by the constraint is the shadow price of Equation (11.2.1), and Equation (11.2.2) is not active.

Table 11.2: **Emission Levels**

$$E_p = \sum_i \upsilon_i^p XP_i + \sum_i \pi_i^p \left(\sum_j XAp_{ij} + \sum_h XAc_{ih} + \sum_f XAFD_f^i \right) \qquad (11.2.1)$$

$$\tau_{Poll} = P\, \bar{\tau}_{Poll} \qquad (11.2.2)$$

The tax is implemented as an excise tax, i.e. it is implemented as a tax per unit of emission in the local currency. For example, in the United States it would be the equivalent of \$x per tonne of emission. It is converted to a price wedge on the consumption of the commodity (as opposed to a tax on the emission), using the commodity specific emission coefficient. For example, in Equation (1.1.5'), the tax adds an additional price wedge between the unit cost of production exclusive of the pollution tax, and the final unit cost of production. Let production equal 100 (million dollars for example), and let the amount of pollution be equal to 1 tonne of emission per 10 million dollars of output. Then the total emission in this case is 10 tonnes. If the tax is equal to \$25 per ton of emission, the total tax bill for this sector is \$250. In the formula below, ν is equal to 0.1 (tonnes per million dollars of output), *XP* is equal to 100 (million dollars), and t_p is equal to \$25. The consumption based pollution tax is added to the Armington price, see Equation (6.1.4'). However, the Armington decomposition occurs using basic prices, therefore, the taxes are removed from the Armington price in the decomposition formulae, see Equations (6.1.2') and (6.1.3'). Equation (5.3.3') determines the modification to the government revenue equation.

Table 11.3: **Emission Price Wedges**

$$PX_j XP_j = \sum_v PXv_j^v XPv_j^v + \sum_p \upsilon_j^p XP_j \tau_{Poll} \qquad (1.1.5')$$

$$PA_i = \left[\beta_i^d PD_i^{1-\sigma_i^m} + \beta_i^m PM_i^{1-\sigma_i^m}\right]^{1/(1-\sigma_i^m)} + \sum_p \pi_i^p \tau_{Poll} \qquad (6.1.4')$$

$$XD_i = \beta_i^d \left(\frac{PA_i - \sum_p \pi_i^p \tau_{Poll}}{PD_i} \right)^{\sigma_i^m} XA_i \qquad (6.1.2')$$

$$XM_i = \beta_i^m \left(\frac{PA_i - \sum_p \pi_i^p \tau_{Poll}}{PM_i} \right)^{\sigma_i^m} XA_i \qquad (6.1.3')$$

$$GRev = MiscRev + Tax^h + \sum_p \tau_{Poll} E_p \qquad (5.3.3')$$

III. COUNTRY SPECIFIC DETAILS

This section describes the characteristics of each of the countries' Social Accounting Matrices (SAM) used to calibrate the model. Table III.1 reports the number of sectors (and the corresponding number of products) available in each of the country's SAM, the number of households and labour types, the number of partner regions, the number of capital and land types. When two numbers are reported in the same cell, the first denotes the number in the original SAM, and, the second, the number in the model. For instance, the original Chilean SAM contains 74 sectors of production, but the model is run with 72 sectors, after aggregation. Table III.1 also reports the year for which the SAM is constructed, its currency and unit, the unit used in the model, and the exchange rate of the country.

Table III.1: **SAM Dimensions**

	Chile	China	Costa Rica	Indonesia	Mexico	Viet Nam
Sectors	74/72	64/64	40/38	22/22	93/93	50/50
Households	5/5	10/10	10/10	10/10	20/20	1/1
Labour	20/20	16/16	16/16	16/16	8/8	1/1
Partner region	26/5	1/1	1/1	1/1	1/1	1/1
Capital	1/1	1/1	1/1	1/1	3/1	1/1
Land	non available	non available	non available	7/7	non available	non available
Year of the Base SAM	1992	1987	1991	1990	1989	1989
Currency	peso	yuan	colone	rupiah	peso	dong
Unit in the Base SAM	10^6	10^4	10^6	10^9	10^6	10^9
Unit in the model	10^6	10^8	10^9	10^{12}	10^9	10^{15}
Exchange rate (national currency/US$)*	3.6259	3.7221	122.43	1842.8	2.4615	4300

* *Source* : IMF, series *rf*.

The level of structural detail in each SAM is country-specific. Appendix 2 presents a complete generic SAM. Table III.2 provides a description of the available accounts for each one of the individual countries (all the other flows in the SAM are present for all countries). Availability is indicated with a cross.

Table III.2: **Flow of Funds Availability by Country**

	Chile	China	Costa Rica	Indonesia	Mexico	Viet Nam
Enterprise direct taxes	x	x	x	x	x	.
Income distributed to households	x	x	.	x	x	x
Enterprise saving	.	x	x	x	x	x
Export taxes	.	.	x	.	.	.
Payments from foreign labour	x	.	.	.	.	.
Foreign labour income	x	.	.	.		.
Capital income to government	.	.	.	x	.	.
Government transfers to households	.	x	.	.	.	.
Household direct taxes	x	.	x	x	x	x
Labour tax	x	.	.	.	.	.
Trade and transport margins	x	.	.	x	.	.
Capital income distribution matrix	.	.	x	x	.	x
Fixed Capital consumption	x	.	.	.	.	.
Stock change	x	x	x	.	.	.
Intra-enterprise transfers	.	.	.	x	.	.
Intra-government transfers	.	.	.	x	.	.
Intra-household transfers	.	.	.	x	.	.
Corporate transfers to ROW	.	.	x	x	x	.
Foreign capital income	x	.	x	x	.	.
Government transfers to ROW	x	.	x	x	.	.
Household transfers to ROW	x	.	x	.	.	.
Capital income from ROW	x	.	x	x	x	.
Transfers from ROW to enterprises	.	.	.	x	.	.
Transfers from ROW to government	x	.	x	x	.	.
Transfers from ROW to households	x	x	x	x	x	.

IV. CONCLUDING REMARKS

This Technical Paper presents the detailed specification of the prototype CGE model used for assessing the links between economic activity and the environment. Quantifying the response of both economic and environmental variables to policy changes, such as trade or environmental measures, is a necessary condition to the design of coherent reforms. Three main aspects of the CGE model presented in this paper account for its specificity with respect to previous analyses:

First, it embodies a high level of disaggregation for pollutants, products, sectors, and types of households. This model has been used to simulate the impacts of abatement policies targeted to specific air emissions, measuring, at the same time, the effect on related water and soil pollutants. Trade policy reform, and the induced resource reallocation, do not have uniform outcomes across sectors. The expansion or contraction of specific activities have differentiated environmental consequences. The product disaggregation of the model highlights certain environmental outcomes of trade policy. Moreover, income distribution issues arising from environmental and trade policies, and the question of the redistribution of environmental tax receipts, are briefly discussed and can be further investigated due to the detailed classification of households.

Second, this model explicitly includes dynamic features, allowing the introduction of exogenous factors such as productivity shifts and demographic changes, that affect the growth and pollution trajectory. The modelling of a vintage structure for capital also captures important dynamic effects, such as the relation between capital accumulation and the adjustment capacity of the economy to environmental regulation. It is possible to assess to what extent new investment favours the substitution from polluting factors to non-polluting factors. Therefore, negative outcomes of growth in terms of pollution arising from scale effects can be compared to positive ones, to determine the aggregate impact.

Third, most economy-wide studies on growth and environment linkages rely on effluent intensities associated with output, and do not take into account substitution between non-polluting and polluting factors. Abating pollution is then achieved principally by reducing output in pollution intensive sectors, with a significant cost in terms of growth. By contrast, in our model pollution emissions are linked to polluting input use, rather than output. Technical adjustment by substituting non-polluting factors for polluting factors may therefore be assessed. Moreover, the model includes emissions generated by final consumption, and thus describes the abatement of emissions from both the production and the final consumption sides.

This model has been used to assess environmental and economic linkages in a diverse group of countries. While it represents a progress in the tools used to designed optimal policy interventions, there is still a wide scope for improving the methodology. First and foremost is the need to assess not only the economic costs of abatement, but also the economic and non economic benefits. Further research is necessary in the valuation of a clean environment for households, and in the identification of the potentially important feed backs between environmental damage and the economy (e.g. soil degradation and harm to human capital). Finally, a proper assessment of abatement technology, embodied in new capital, would provide a more complete set of policy options for policy makers.

NOTES

1. Capital supply is to some extent influenced by the current period's level of investment.

2. For simplicity, it is assumed that old capital goods supplied in second-hand markets and new capital goods are homogeneous. This formulation makes it possible to introduce downward rigidities in the adjustment of capital without increasing excessively the number of equilibrium prices to be determined by the model (see Fullerton, 1983).

3. The demand system is a version of the Extended Linear Expenditure System (ELES) which was first developed by Lluch (1973). The formulation of the ELES in this model is based on atemporal maximisation — see Howe (1975). In this formulation, the marginal propensity to save out of supernumerary income is constant and independent of the rate of reproduction of capital.

4. In the reference simulation, the real government fiscal balance converges (linearly) towards 0 by the final period of the simulation.

5. This involves computing in each period a measure of Harrod-neutral technical progress in the capital-labour bundle as a residual. This is a standard calibration procedure in dynamic CGE modelling — see Ballard *et al.* (1985).

6. Gross output is divided into two parts, one part produced with *old* capital, and the residual amount produced with *new* capital.

7. The value added bundle also contains demand for energy, see below.

8. Some models of this type assume a top level Leontief, i.e. a substitution elasticity of zero, in which case there is no substitution possibility between intermediate demand and value added. The GAMS implementation of the model can handle all of the special cases of the CES, i.e. Leontief and Cobb-Douglas.

9. The CES is described in greater detail in Appendix 1.

10. The unit cost equation will be affected by production-specific emission taxes. Emission taxes are discussed in section 11.

1[1]. The current model specification includes only a single level nest for dis-aggregating the aggregate labour bundle. In other words, the substitution across any pair of labour skills is uniform.

12. Only the Indonesian model includes land as a specific factor of production. All the other country models incorporate the land specification, if the data were to be developed from the existing SAMs.

13. In the reference simulation, both the private corporate saving rate and the household saving rate are adjusted (upwards), under the assumption that domestic saving, as a share of GDP, will increase in the future. The adjustments are based on rules of thumb, but could be made explicit in the model.

14. For references, see Lluch (1973) or Deaton and Muellbauer (1980).

15. As noted earlier, the *m* parameters are adjusted in the reference simulation in order to increase the level of domestic saving.

16. This is known as the Armington assumption — see Armington (1969).

17. See for example Robinson et. al. (1992).

18. Note the difference between the Armington CES and the CET. First, the relation between the exponent and the substitution elasticity is different. Second, the ratio of the prices and the share parameter in the reduced forms are inverted. This is logical since the goal of the producer is to maximise revenues. For example, an increase in the price of exports, relative to the composite aggregate price, will lead to an increase in export supply.

19. The following numerical example may clarify issue. Assume the value of the capital stock is 100. Assume, as well, that capital remuneration is 10. Capital remuneration is simply *rK* where *r* is the rental rate and *K* the demand for capital. In this example, *rK* is equal to 10, which implies a rental rate of 0.1. The model assumes a normalisation rule such that the rental rate is 1, and normalises the capital data to be consistent with the normalisation rule. In other words, the normalised capital demand is 10, and it is really an index of capital volume. The non-normalised level of capital is used only in the accumulation function and in determining the value of the depreciation allowance. All other capital stock equations use the normalised value of capital.

20. See Martin *et al.* (1991).

21. See Dessus *et al.* (1994).

APPENDIX 1 – THE CES/CET FUNCTIONS

Because of the frequent use of the constant elasticity of substitution (CES) function, this appendix will develop some of the properties of the CES, including some of its special cases. The CES function can be formulated as a cost minimisation problem, subject to a technology constraint:

$$\min \sum_i P_i X_i$$
$$\text{subject to}$$
$$V = \left[\sum_i a_i (\lambda_i X_i)^\rho \right]^{1/\rho}$$

where *V* is the aggregate volume (of production, for example), *X* are the individual components ("inputs") of the production function, *P* are the corresponding prices, and *a* and λ are technological parameters. *a* are most often called the share parameters. λ are technology shifters. The parameter ρ is the CES exponent, which is related to the CES elasticity of substitution, which will be defined below.

From the first-order conditions we can derive demand for the inputs, assuming *V* and the prices are fixed:

(1) $$X_i = \alpha_i \lambda_i^{\sigma-1} \left(\frac{P}{P_i} \right)^\sigma V$$

where we define the following relationships:

$$\rho = \frac{\sigma - 1}{\sigma} \quad \Leftrightarrow \quad \sigma = \frac{1}{1-\rho} \quad \text{and} \quad \sigma \ge 0$$

$$\alpha_i = a_i^\sigma$$

and

(2) $$P = \left[\sum_i \alpha_i \lambda_i^{\sigma-1} P_i^{1-\sigma} \right]^{1/(1-\sigma)}$$

P is called the CES dual price, it is the aggregate price of the CES components. The parameter σ, is called the substitution elasticity. This term comes from the following relationship which is easy to derive from Equation (1):

$$\frac{\partial\left(X_i / X_j\right)}{\partial\left(P_i / P_j\right)} \frac{\left(P_i / P_j\right)}{\left(X_i / X_j\right)} = -\sigma$$

In other words, the elasticity of substitution between two inputs, with respect to their relative prices, is constant. (Note, we are assuming that the substitution elasticity is a positive number). For example, if the price of input *i* increases by 10 per cent with respect to input *j*, the ratio of input *i* to input *j* will decrease by (around) σ times 10 per cent.

The Leontief and Cobb-Douglas functions are special cases of the CES function. In the case of the Leontief function, the substitution elasticity is zero, in other words, there is no substitution between inputs, no matter what the input prices are. Equations (1) and (2) become:

(1') $$X_i = \frac{\alpha_i V}{\lambda_i}$$

(2') $$P = \sum_i \alpha_i \frac{P_i}{\lambda_i}$$

The aggregate price is the weighted sum of the input prices. The Cobb-Douglas function is for the special case when σ is equal to one. It should be clear from Equation (2) that this case needs special handling. The following equations provide the relevant equations for the Cobb-Douglas:

(1'') $$X_i = \alpha_i \frac{P}{P_i} V$$

(2'') $$P = A^{-1} \prod_i \left(\frac{P_i}{\alpha_i \lambda_i}\right)^{\alpha_i}$$

where the production function is given by:

$$V = A \prod_i \left(\lambda_i X_i\right)^{\alpha_i}$$

and

$$\sum_i \alpha_i = 1$$

Note that in Equation (1'') the value share is constant, and does not depend directly on technology change.

Calibration

Typically, the base data set along with a given substitution elasticity are used to calibrate the CES share parameters. Equation (1) can be inverted to yield:

$$\alpha_i = \frac{X_i}{V}\left(\frac{P_i}{P}\right)^{\sigma}$$

assuming the technology shifters have unit value in the base year. Moreover, the base year prices are often normalised to 1, simplifying the above expression to a true value share. Let's take the Armington assumption for example. Assume aggregate imports are 20, domestic demand for domestic production is 80, and prices are normalised to 1. The Armington aggregate volume is 100, and the respective share parameters are 0.2 and 0.8. (Note that the model always uses the share parameters represented by α, not the share parameters represented by *a*. This saves on compute time since the *a* parameters never appear explicitly in any equation, whereas *a* raised to the power of the substitution elasticity, i.e. α, occurs frequently.)

With less detail, the following describes the relevant formulas for the CET function which is similar to the CES specification.

$$\begin{aligned} &\max && \sum_i P_i X_i \\ &s.t. && V = \left[\sum_i g_i X_i^{\lambda}\right]^{1/\lambda} \end{aligned}$$

where *V* is the aggregate volume (e.g. aggregate supply), *X* are the relevant components (sector-specific supply), *P* are the corresponding prices, *g* are the CET share parameters, and λ is the CET exponent. The CET exponent is related to the CET substitution elasticity, Λ via the following relation:

$$\lambda = \frac{\Lambda + 1}{\Lambda} \Leftrightarrow \Lambda = \frac{1}{\lambda - 1}$$

Solution of this maximisation problem leads to the following first order conditions

$$X_i = \gamma_i \left(\frac{P_i}{P} \right)^{\Lambda} V$$

$$P = \left[\sum_i \gamma_i P_i^{1+\Lambda} \right]^{1/(1+\Lambda)}$$

where the γ parameters are related to the primal share parameters, *g*, by the following formula:

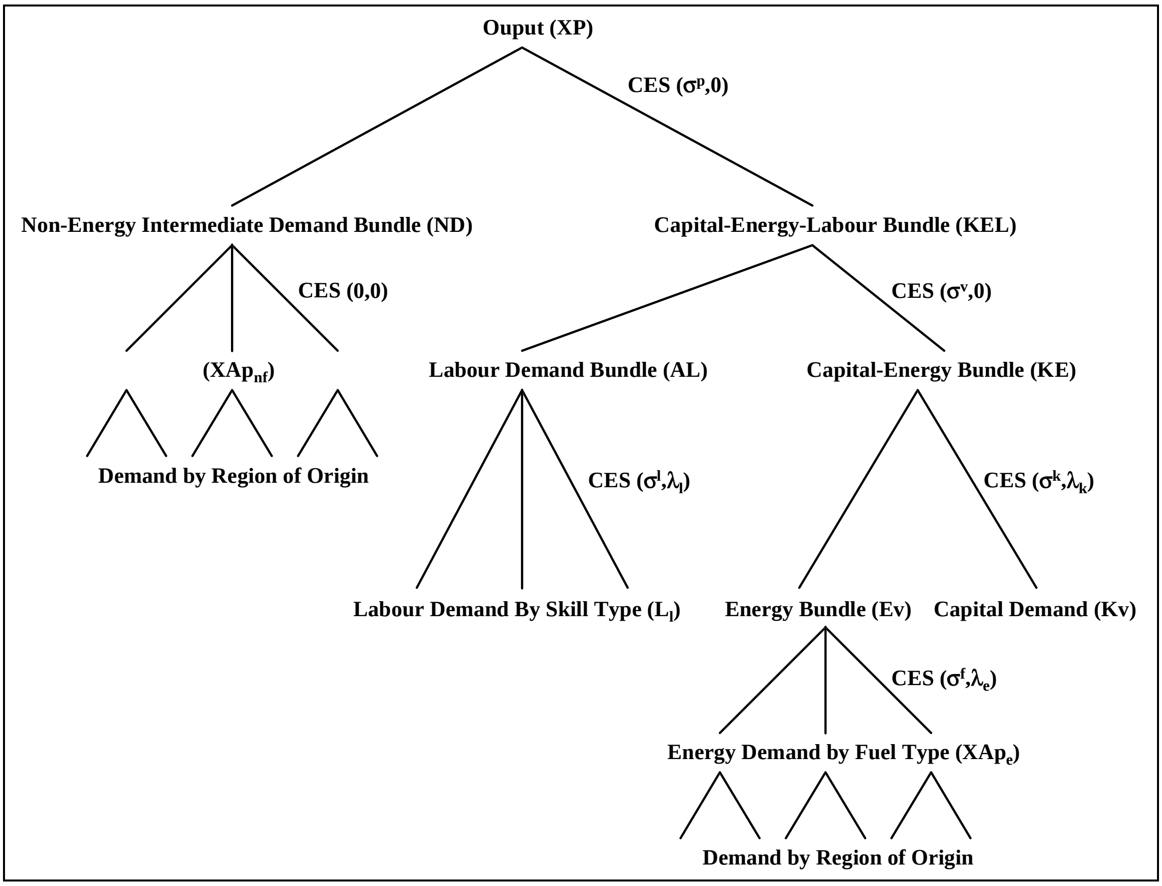
$$\gamma_i = g_i^{-\Lambda} \Leftrightarrow g_i = \left(\frac{1}{\gamma_i} \right)$$

APPENDIX 2 – The Generic Social Accounting Matrix

	Commodities	*Activities*	*Trade & Transport margins*	*Enterprise*	*Labour*	*Capital*	*Households*	*Government*	*Capital account*	*Stock Change*	*Export tax*	*Tariffs*	*Rest of the World*
Commodities		Intermediate consumption	Domestic margin expenditures				Households consumption	Government consumption	Fixed capital formation	Stock change			Exports
Activities	Output												
Trade & Transport margins	Domestic margins												
Enterprise				Intra-enterprise transfers		retained enterprise income							Transfers from ROW to enterprises
Labour		Compensation of employees											Payments from foreign labour
Capital		Operating surplus											Capital income from ROW
Households				Income distributed to households	Labour income	Distributed profits	Intra-household transfers	Government transfers to households					Transfers from ROW to households
Government		Indirect taxes		Enterprise direct taxes	Labour tax	Capital income to government	Households direct taxes	Intra-government transfers			Export taxes	Import tariffs	Transfers from ROW to government
Capital account		Fixed capital consumption		Enterprise savings			Households savings	Government savings					Net foreign savings
Stock Change									Stock change				
Export tax		Export taxes											
Tariffs	Import tariffs												
Rest of the World	Imports			Corporate transfers to ROW	Foreign labour income	Foreign capital income	Household transfers to ROW	Government transfers to ROW					

FIGURES

Figure 1: **Production Nesting**



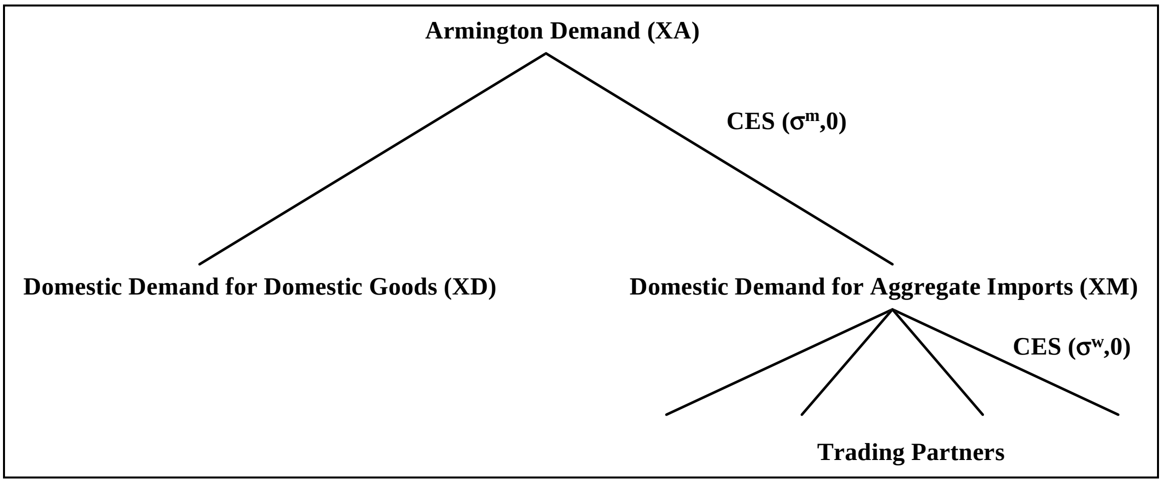
Notes:

1. Each nest represents a different CES bundle. The first argument in the CES function represents the substitution of elasticity. The elasticity may take the value zero. Because of the putty/semi-putty specification, the nesting is replicated for each type of capital, i.e. *old* and *new*. The values of the substitution elasticity will generally differ depending on the capital vintage, with typically lower elasticities for *old* capital. The second argument in the CES function is an efficiency factor. In the case of the *KE* bundle, it is only applied on the demand for capital. In the case of the decomposition of labour and energy, it is applied to all components.

2. Intermediate demand, both energy and non-energy, is further decomposed by region of origin according to the *Armington* specification. However, the *Armington* function is specified at the border and is not industry specific.

3. The decomposition of the intermediate demand bundle, the labour bundle, and the energy bundle will be specific to the level of aggregation of the model. The diagram represents only schematically the decomposition and is not meant to imply that there are three components in the CES aggregation.

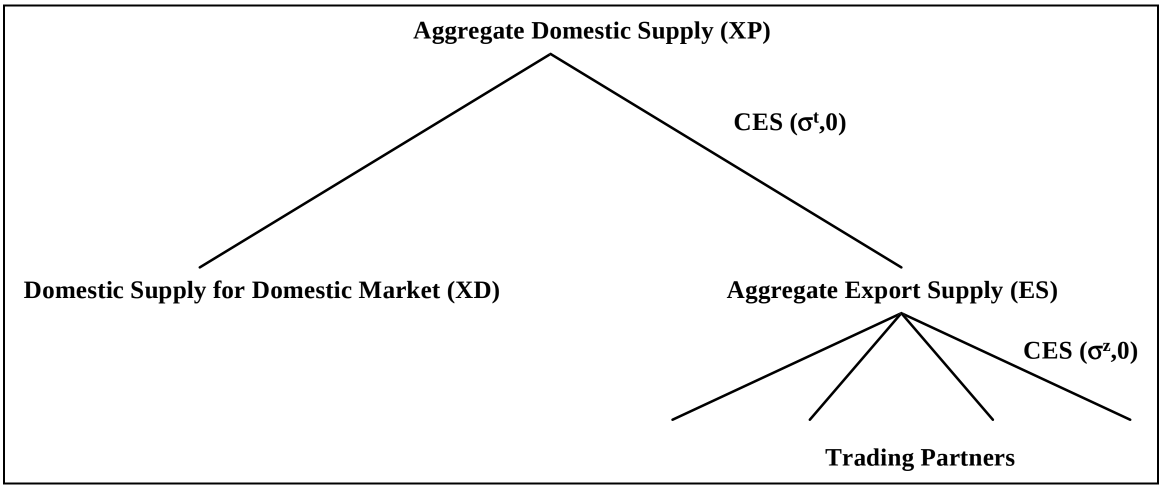
Figure 2: **Armington Nesting**



Note(s):

1. Import demand is modelled as a nested CES structure. Agents first choose the optimal level of demand for the so-called Armington good (*XA*). In a second stage, agents decompose the Armington aggregate good into demand for the domestically produced commodity (*XD*), and an aggregate import bundle (*XM*). At the third and final stage, agents choose the optimal quantities of imports from each trading partner. Import prices and tariffs are specific to each of the trading partners.

Figure 3: **Output Supply (CET) Nesting**



Note(s):

1. The market for domestic output is modelled as a nested CET structure. In a first stage, output (*XP*) is determined by equilibrium conditions. In a second stage, producers choose the optimal mix of goods supplied to the domestic market (*XD*), and an aggregate export supply (*ES*). At the third and final stage, producers choose the optimal mix of exports to each of the individual trading partners. The export price of each trading partner is region-specific. Under the small-country assumption, the export price is fixed (in foreign currency terms), otherwise, each trading partner has a downward sloping demand curve, and the export price is determined endogenously through an equilibrium condition.

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