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# Alternative Solution Methods in Applied General Equilibrium Analysis

**Richard G. Harris**

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No. 53: ALTERNATIVE SOLUTION METHODS IN APPLIED GENERAL EQUILIBRIUM ANALYSIS

by

Richard G. Harris

April 1988

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This paper discusses the relative strengths and weaknesses of alternative solution methods in applied general equilibrium (AGE) analysis. The particular focus is not on the technical properties of solution algorithms, but instead deals with the general issue of using linearised approximation methods, more popularly referred to as the Johansen class of AGE models, relative to models which are solved in level form via solution of a system of non-linear equations. Particular attention is given to the practical aspects of AGE empirical work with special emphasis on problems encountered in multi-country modelling.

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Ce texte décrit les forces et faiblesses relatives de différentes méthodes de solution utilisées dans l'analyse appliquée d'équilibre général. L'objet essentiel de ce document ne porte pas sur les propriétés techniques des algorithmes de solution, mais plutôt sur la question générale de l'utilisation de méthodes d'approximation linéaire, populairement appelées modèles de type Johansen, par rapport au choix de méthodes visant à solutionner le modèle en niveau à l'aide d'un système d'équations non-linéaires. Une attention particulière est portée aux aspects pratiques de l'analyse empirique effectuée à l'aide de tels modèles et notamment aux problèmes rencontrés dans le cadre d'une modélisation comportant plusieurs pays.

**ALTERNATIVE SOLUTION METHODS IN APPLIED GENERAL EQUILIBRIUM ANALYSIS**

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Richard G. Harris

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The author is Professor at Queen's University, Kingston, Canada. He wrote this paper as Consultant to the Economics and Statistics Department. Thanks are due to Jean-Marc Burniaux, François Delorme and John Martin for comments on an earlier draft.

**ALTERNATIVE SOLUTION METHODS IN APPLIED GENERAL EQUILIBRIUM ANALYSIS**

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# ALTERNATIVE SOLUTION METHODS IN APPLIED GENERAL EQUILIBRIUM ANALYSIS

## INTRODUCTION

This paper discusses the relative strengths and weaknesses of alternative solution methods in applied general equilibrium (AGE) analysis. The particular focus is not on the technical properties of solution algorithms (1), but instead deals with the general issue of using linearised approximation methods, more popularly referred to as the Johansen class of AGE models, relative to models which are solved in level form via solution of a system of non-linear equations. Particular attention will be focused on the practical aspects of AGE empirical work with special emphasis on problems encountered in international multi-country modelling. For a comprehensive introduction to AGE modelling, the reader is referred to Shoven and Whalley (1984) and Dervis et al. (1982).

The paper proceeds as follows. In Section 2 the basic Johansen model will be introduced and compared with AGE models developed in full non-linear form. Section 3 will address the strengths and weaknesses of using the Johansen method, versus using a full non-linear model. Section 4 will discuss the same issues in using the full non-linear model. Finally, Section 5 will briefly discuss the issues regarding "closure" assumptions.

## I. LINEARISED VERSUS NON-LINEARISED MODELS

All AGE analysis starts with a basic economic model hopefully suited to the practical problem under consideration. By far the most popular and most traditional of this class of models is the flexible-price Walrasian model in which all markets clear and some version of Walras law is satisfied. Government policy is usually introduced exogenously as a set of tax or tariff distortions in ad valorem form. Strict market clearing is quite often relaxed in a number of applications. This is usually introduced through some type of price rigidity allowing excess demand or supply in some market. The labour market, or the international capital market is often a candidate market in typical applications (2). A related issue is what is often referred to as the macroeconomic closure of the model (see Section 5).

In any case the end result, whatever particular theoretical structure is involved, is a set of equations involving a set of endogenous and exogenous variables. It will be convenient to be a bit formal at this point to illustrate what is involved. Suppose we denote endogenous variables by x, and exogenous variables by $\alpha$. If x is of dimension n, a fully specified model must involve n functionally independent equations. Denote these equations by:

$$F_i(x_1, x_2, .., x_n; \alpha_1, ..., \alpha_m) = 0 \quad i = 1, ..., n \qquad [1]$$

In a typical Walrasian model the $F_i$ would be supply-demand equations, and the $x_i$ prices. Denote an equilibrium solution to these equations as

$x^*(\alpha)$; note that any solution will depend upon the set of exogenous variables ($\alpha_i$). In approaching the problem of finding solutions $x^*(\alpha)$ to an applied general equilibrium model, a variety of approaches have been taken. One of the first methods was proposed by Johansen (1960) and involves taking a linearisation of F around a known solution $x^0=x^*(\alpha^0)$. Let $\Delta x$ denote a "small change in x from $x^0$ and likewise $\Delta\alpha$ a small change in $\alpha$ from $\alpha^0$. If $J(x^0,\alpha^0)$ is the matrix of derivatives of equations [1] with respect to changes in x, and $B(x^0,\alpha^0)$ is the matrix of derivatives of [1] with respect to changes in $\alpha$, we have the following set of linear equations:

$$J(x^0,\alpha^0)\Delta x + B(x^0,\alpha^0)\Delta\alpha = \underline{0}_n \quad (3) \qquad [2]$$

Johansen converts [2] into a log-linear form (4)

$$J^*\hat{x} = -B^*\hat{\alpha} \qquad [3]$$

Equation [3] is an extremely convenient form to work with once the coefficient matrices $J^*$ and $B^*$ have been calculated. Both $\alpha$ and x can be quite large in dimension, but using linear methods solutions for the rate of growth of x, given growth rates in the exogenous variables $\alpha$, can be obtained very easily and quickly on a modern computer. In implementing the Johansen method there are essentially three steps:

1. Calculating the coefficient matrices $J^*$ and $B^*$ from the underlying theoretical structure as embedded in the $F_i$.

2. Obtaining an initial solution $x^0$ and $\alpha^0$.

3. Choosing an appropriate policy simulation via choice of $\hat{\alpha}$ and then solving [3] for the solution $\hat{x}$.

In a large-scale model none of these steps is particularly easy, but in some competitive models the structure of the $J^*$ and $B^*$ matrices is particularly convenient and familiar to economists from the ^-calculus commonly used in international trade theory (5).

Strictly speaking, the Johansen method is not a method by which the model embedded in equations [1] is "solved"; rather it is a technique for carrying out a comparative statics exercise on the model. For most policy analysis this is all that is needed, since one is principally interested in asking how a given endogenous variable will respond to a change in some exogenous variable which in many cases is a policy variable.

A great deal of AGE analysis approaches the issue of comparative statics by attacking the solution of [1] more directly. Particular functional forms for the $F_i$ are chosen and solutions $x^*(\alpha)$ are computed to some degree of accuracy for particular choices of $\alpha$. The methods by which this solution is carried out are varied, but typically involve numerical algorithms of the fixed-point type, or search algorithms which use some type of Newton method. For expositional purposes I shall refer to all such methods as an AGE model solved in "level form"; this refers to the fact that the solution method actually gives an (approximate) value for the level of the equilibrium variables in question.

Comparative statics are carried out by solving the model twice, for two different values of $\alpha$ and then comparing the results. One immediate consequence of this methodology is that the predicted changes in endogenous variables are "exact" for whatever change in $\alpha$. This is to be compared to the Johansen method which provides predictive changes in x for only small changes in $\alpha$. It then becomes a matter of judgement as to what constitutes a small change in $\alpha$.

An AGE analysis done in level form involves the following steps:

1. Specification of the appropriate theoretical model as reflected in the choice of the $F_i$ functions.

2. Choosing values for the specific parameters in the $F_i$ through a calibration or estimation procedure.

3. Solution of the system [1] for alternative values of $\alpha$.

Unfortunately, step 2 is much more involved than might seem apparent. Not only must one pick an entire functional form for each $F_i$, but it must be done in such a manner that the results that the model produces appear reasonable for the problem at hand. One way in which this is done is a procedure which has come to be known as "benchmarking" the model. The idea of "benchmarking" is quite simple. In a particular application there is an observed historical data set, call it $D = (x^0, \alpha^0)$, which the investigator would like to think of as a natural reference point. The $F_i$ must be chosen or "calibrated" in such a way that the solution of the model has the property that $x^0 = x^*(\alpha^0)$, i.e. the equilibrium solution of the appropriately calibrated model coincides with the observed data set D. Benchmarking has the virtue that it not only is a procedure by which reasonable results are forced out of the model, but the calibration procedure itself tends to identify exactly many of the free parameters embedded in the functional forms $F_i$. Since these procedures are exact and are not accompanied by statistical confidence intervals, the entire procedure is quite controversial. More will be said on this in Section 4.

In any particular model the economic structure often allows a great reduction in the dimensionality of the solution problem. This has been used to great advantage in a large number of AGE exercises. The most common simplification comes from using an assumption of constant returns to scale in production. It is well known that, with constant costs, a condition of industry equilibrium is that price equal average cost. If this condition holds for each sector or industry, the set of equations in [1] can be reduced from one of supply equal demand for each commodity, to supply equals demand in the factor markets only. Thus, if, for example, there are 100 commodities, but only 4 factors of production, a model of apparent dimension 104, can be reduced relatively simply to one of only 4 dimensions (6). In some cases choice of particular functional forms allows a great simplification in the solution of [1] by analytical methods.

An alternative method, used by some to solve AGE models, is an optimisation approach based upon an idea of Negishi (1960), and expanded upon by Dixon (1975). The basic idea is that a Walrasian competitive equilibrium can be reproduced as the solution to the maximization of a linear social welfare function, defined as the weighted sum of utilities of the consumer groups in the model, where the weights on the welfare function must be endogenously chosen. Algorithms using this idea are based on optimisation methods together with an iterative procedure on the welfare weights. It has been argued that computationally this procedure is more efficient when the number of consumer groups is small relative to the number of goods in the model. It is important to note that the optimisation problem at each step is inherently highly non-linear, involving a large number of variables.

Use of this procedure in AGE models has not been very extensive. The most well-known model using this method is the GEM model of Ginsburgh and Waelbroeck (1981). Given the limited experience with such methods it is difficult to make very precise statements about the merits or otherwise of this approach. I am skeptical, however, as to the likelihood that this approach will be adopted to any great extent by AGE analysts for the following reasons:

1. One significant problem is that the equivalence between market equilibrium and pseudo-welfare optimisation problems has only been established for a limited set of theoretical models. Walrasian models with specific tax or tariff distortions are the one class of models for which positive results are available. Outside of this very limited class, however, the answer is at best unknown, and in a number of models with incomplete markets or imperfect competition it is well known that there is no equivalence between market equilibrium and an optimisation solution. Because of this the method seems to have rather limited applicability.

2. The connection between parameter choice and benchmarking is not clear in these models. In principle, the benchmark data set can be produced by an appropriate choice of functional forms on utility and production, plus an appropriate set of weights in the objective function. The method by which benchmarking should proceed is not entirely clear. Ginsburgh and Waelbroeck seem to use a fairly eclectic procedure in generating model parameters, and the relationship of these parameters to the benchmark data set is not clear.

3. The computational efficiency of this method relative to either the Johansen method or the levels method is evident. For pure Walrasian models with single consumers, such as the "real business cycle" models of neoclassical intertemporal equilibrium, the method would clearly be useful. For models with even a modest number of consumers, given the combined use of optimisation and iterative techniques, it is not clear that the number of function evaluations would be reduced (7). Furthermore, solving large-scale optimisation problems efficiently is still a fairly demanding task. Using linear programming methods, as in the GEM model, involves the additional step of approximating a non-linear function by a set of linear functions. This additional step in a large-scale model substantially increases the coding requirements put on the model builder, with the consequent increase in work in checking and debugging code. Solving large scale non-linear optimisation problems is for the most part not much different than solving a set of simultaneous non-linear equations corresponding to the first-order conditions of the maximization problem. The computational superiority of these methods is at best speculative.

In summary, the optimisation approach is limited in applicability, and with the exception of a few models with very special structure, seems to offer little in the way of significant computational savings over the other major methods. For these reasons I shall concentrate my comments upon the other methods of AGE analysis.

## II. THE JOHANSEN METHOD

The Johansen method is one of great simplicity, and because of the inherent linearity in its methodology, it is also subject to considerable savings on computer costs. There is essentially no self-imposed constraint on model size due to computational constraints. By far the most active users of the Johansen method are the team working on the ORANI model developed by Peter Dixon et al. (1982) in Australia. The ORANI model is both large and has been used quite extensively for policy purposes within Australia. Its success has spawned a number of imitators, and it is worth considering in some detail what the relative virtues of this type of methodology are. Unfortunately, some of this material may seem to the non-specialist fairly technical, but I can only assure the reader that the points made are of considerable practical significance to a large model building research project.

1. The Johansen method seems greatly simplified relative to the levels method in that all that is required is that the modeller specify the coefficient matrices $J^*$ and $B^*$, rather than the exact functional form for each of the $F_i$. The coefficients in $J^*$ and $B^*$, however, typically involve terms involving product or factor shares and price and income elasticities. In each case these can be specified directly, or, as is more often the practice, a particular functional-form is chosen, such as the nested-CES class from which an elasticity is derived. One advantage of choosing a restrictive functional form from which the elasticities are derived, is that the number of independent functional form parameters from which elasticities are derived, is far less than the number of elasticities themselves. In large models this is of considerable practical importance. It is worth noting, however, that if for some reason it is desirable to use as unrestricted a functional form as possible, then elasticities derived from the class of flexible functional forms can be readily used within a Johansen model, and this is not always the case within a levels model (8).

2. Coding and model flexibility: One of the principal virtues of the Johansen method is that coding different model structures is often quite simple, in that coding for one type of model is directly applicable for another model type. For example often a short-run version of a longer-run model is used for simulations where the time frame is of central importance. Thus, in the long-run model the capital stocks by industry might be thought of as endogenous variables, while in the short-run model they can be thought of as fixed or exogenous variables. In some cases this change can be accommodated within the general framework simply by moving what was previously an endogenous variable to the left-hand side of the linear system [3] and treating it as an exogenous variable. This

ability to switch model structures is a great cost saving, given that coding and code debugging are probably one of the most costly aspects of AGE empirical analysis. Furthermore, the larger the model, the greater are these cost savings.

3. Transparency of results: Users of the Johansen method will consistently maintain, and with justification, that the method provides for fairly quick understanding of where the results come from. Against charges that all AGE models are giant "black boxes", this is certainly a worthwhile feature for any model to have. The reason the results are "transparent" stems from the linear nature of the equations between growth in endogenous variables and growth in exogenous variables. In many cases both the $J^*$ and $B^*$ matrices have a great many zeroes, so that the source of change in some endogenous variable is fairly readily traceable to some particular exogenous variable, or some key elasticity. It is important, however, not to make too much of this feature. After all the great importance of doing AGE analysis lies in the interactions between markets. Experience tells us that in high dimensional models, even those of the purest theoretical form, it is often extremely difficult to give <u>a priori</u> predictions as to the qualitative comparative- static properties of the model (9).

The deficiencies of the Johansen method have received little attention in professional journals, given that the use of this type of model is far less common than AGE models in level form. To some these deficiencies are obvious, and have to do with the approximate form of the analysis. For the uninitiated some elementary exposition is probably worthwhile.

The Johansen method is simply a linear approximation to a non-linear set of equations. The solution it gives will depend upon just how "non-linear" the true model is, and the step size of the underlying exogenous variables. To illustrate this, consider Figure 1 in which the curve $f(x,\alpha)$ is the true relationship between the exogenous and the endogenous variable. The slope of the straight line tangent to the curve $f(.)$ at $(x^0,\alpha^0)$ is the graphical equivalent of the linearised coefficient matrices used in the Johansen method. For a given change $\Delta\alpha$, the predicted change in x from $x^0$, is $\Delta x$. As drawn, the predicted change in x is greater than the actual change. The error of the predicted change in x from the true change in x will depend upon how non-linear the true model is, and upon how large the contemplated change in $\alpha$ is. In higher dimensions the basic intuition drawn from this picture is still valid, but the cross effects between variables complicate things. If the approximation error is large, one way to proceed is to move the exogenous variables $\alpha$ only part way toward their ultimate values, predict a new value for x, and then re-evaluate the coefficient matrices $J^*$ and $B^*$. Having done this the $\alpha$'s are moved a little further toward their ultimate values and so on. Iteration in this way will eventually lead to a more accurate prediction of the change in x induced by a large change in $\alpha$ (10). This way of proceeding in fact mimics Jacobian methods for solving the full non-linear system [1]. So in any case one is back to actually using a full non-linear method, and the virtues of simple linearity have now been lost.

In practice it is difficult to make predictions as to when linear methods are likely to work well and when they are not likely to work. In most of the applications of ORANI, for example, that I have seen, the changes in policy variables have been quite modest on a percentage basis. From my own experience I would guess that if distortions exceed 50 per cent on an ad valorem basis, you are likely to start running into quite severe problems with the linearised version of a typical model.

In the literature there is one reported set of results on comparing linearised Johansen results with a full non-linear calculation. Dixon et al. (1984) report the effect of cutting a 42 per cent tariff on one good, and find the error of using only the linearised results on a number of variables to average about 40 per cent, although the largest error was 143 per cent and the smallest 12.5 per cent (11). These results suggest that linearised methods are quite suspect for policy changes of the order described in the above tariff-cutting exercise. Problems of a more substantive nature are likely to occur if the policy experiments involve elimination or imposition of quantitative restrictions, as is often the case in the examination of international trade or agricultural policy. In the first place it is not atypical for quantitative restrictions to produce tariff-equivalent distortions of greater than 100 per cent. Secondly, the shifts in income which accompany these changes in quantitative restrictions are often quite large, producing shifts in aggregate expenditure patterns. For both of these reasons, incorporating quantitative restrictions at levels existing in the 1980s is likely to result in highly non-linear models.

A related difficulty with models involving quantitative restrictions involves checking that all constraints bind at all times, so that the linearised version of the model one starts with remains valid after an increase in the quantitative restriction. While this problem can be handled straightforwardly in a levels model, within the Johansen framework it is necessary that all model equations hold with strict equality for all possible parameter variations. Thus, for example in a simple demand-supply framework illustrated in Figure 2, a quantitative restriction on total supply is binding only if it is to the left of the unrestricted equilibrium point E. Strictly speaking, the Johansen method requires two sets of equations; one describing the model in the region A and another in the region B. While this could be done, much of the simplicity of the Johansen method would be lost.

Quantitative restrictions are not the only source of potentially troublesome and important non-linearities. Most AGE models employ the assumption of constant returns to scale in production. This assumption has the consequence that there is a close correspondence between prices and industry costs. It is now recognised that in many sectors increasing returns to scale are potentially important. Introducing economies of scale into an AGE model is possible, as in Harris (1984), but doing so greatly complicates the supply side of the model and introduces additional non-linearities. It is my guess that these non-linearities may render linear comparative static results inaccurate, for all but the smallest of changes in policy variables. For these reasons it seems unlikely that one would want to pursue solution methods based solely on linearised methods if increasing returns were a significant empirical feature of the model.

An important practical consideration in the choice of methodology in AGE analysis is the cost of coding and code verification. In this respect I think the Johansen method is at a decided disadvantage over other methods. The reasons for this are three-fold. In most cases the $J^*$ and $B^*$ matrices must be derived analytically. While this is possible, as ORANI demonstrates, the possibility of making an error in the derivation of a large number of analytical derivatives is quite substantial. For any AGE model one must start with algebraic expressions for the $F_i$; the Johansen method requires that in addition the derivatives of $F_i$ be evaluated. On the other hand for models in level form using most non-linear solution methods, it is only necessary to code the algebraic expressions for the $F_i$ functions (12).

A related problem with the Johansen method is that code verification relies on testing the model on a data set with known results. For the most part a data set with known results is not available a priori. A great virtue of the benchmarking procedure of AGE models in level form is that the model must reproduce a given data set as an equilibrium of the model. While this is used in the calibration process, it also serves as a check on the coding of the basic model. In my own experience, and others who I have talked to about such exercises, this particular check on the model coding and structure has been the single most fruitful way of eliminating coding errors.

In many cases the results of the benchmark simulation, relative to the actual benchmark, suggest fairly readily where the coding errors are likely to be found. It should also be noted that basic data errors make themselves evident at this point. In large-scale data sets the probability of errors existing somewhere, either in the original data or as entered into data files in the computer, is virtually certain. In any large-scale model the elimination of such errors is absolutely critical, and some systematic method for identifying data errors must be used.

The Johansen method in principal could be subjected to the same tests of coding and data. However, because it is a method which is designed to produce only changes in equilibrium values there is no natural check one can perform on the model analogous to the simulated benchmark check available in the levels form of AGE analysis. As a result coding and data check procedures must be introduced, other than those associated with benchmarking.

## III. AGE MODELS IN LEVEL FORM

Beginning with the work at the World Bank (13) and the work of Shoven and Whalley (14) on taxation and trade, the solution of AGE models in level form spread rapidly, and is probably the most commonly used methodology today. As discussed in the previous section, this methodology has a number of strengths relative to linearised methods, but at the same time there are some problems with this type of solution method. Briefly these are as follows:

1. Sparse knowledge of global functional forms: While these models are capable of analysing large-scale changes, they require knowledge of relevant production and demand structures globally, not just local elasticity knowledge used in a typical comparative statics exercise. Unfortunately, while econometricians often estimate global functional forms, there is little assurance these represent the true state of affairs; at best they provide a rationalisation of the data in the region where most observations occur. The rather sad experience on estimating energy demand equations in the 1970s suggests how cautious one should be in the interpretation of results requiring global knowledge of functional forms.

2. Multiple equilibria and comparative static results: This is a problem which has been much discussed (15) but about which our ignorance remains. If there are multiple equilibrium in any given model, in doing policy counterfactuals one if always in danger of swinging from one equilibrium to another. At the moment most modellers simply ignore this potential problem.

3. Dimensionality restrictions on model size: Full non-linear models are still quite costly in terms of computing time relative to a linearised comparative statics model. These costs are exponential in the dimensionality of the model. While progress is rapid in this area, I am aware of no serious large-scale fully non-linear models involving in excess of 100 functionally independent commodities.

4. Costs of theoretical consistency: In an applied policy environment the economist is often called upon to use the model for purposes for which it was not originally intended. This in turn puts demands upon the builders and maintainers of the model to make changes in an attempt to accommodate heretofore unconsidered policy experiments. To do this correctly, and to preserve the theoretical consistency of the existing coded AGE model, can often be very expensive, involving extensive changes to code, and additional benchmarking. As a result modellers facing serious time constraints are often forced to make various <u>ad hoc</u> changes to the model, which sacrifice its theoretical consistency, but make it more suitable or relevant for the immediate task.

There is considerable danger in proceeding in this manner unless the modellers are extremely skilled. Sometimes an apparently minor change can have a rather dramatic effect on the overall properties of the model, or worse produce incorrect, but not readily detectable, results. Linearised models can often be changed both more easily, and in a manner that the changes produce fairly transparent results. Again, however, it is important that those making the changes have a thorough understanding of the linkages between different parts of the model.

## IV. CLOSURE PROBLEMS IN AGE ANALYSIS (16)

All policy-relevant AGE models must ultimately come to grips with the issue of "macroeconomic closure" as the problem has come to be called, somewhat erroneously, since it has really little to do with macroeconomics <u>per se</u> . Rather the problem is simply that any economic model must be fully specified and in a manner that is relevant for the problem at hand. In practice, closure issues focus on those problems which are most commonly addressed in the open-economy macroeconomic literature. These include:

1. The specification of real or nominal rigidity in prices and the attendant non-market clearing.

2. Specification of the asset side of the model, and in particular the specification of money demand, and price level determination.

3. In open-economy models with money, the specification of equations determining the exchange rate (real or nominal or both).

4. In open-economy models, the issue of capital flows must be treated.

5. The specification of government expenditure and tax policies, and the nature of resolution of the government budget constraint.

6. The specification of savings and investment in otherwise static AGE models.

These are typically the set of problems which are addressed under the general rubric of "closure" problems. These are not problems about the appropriate solution method; they are important issues about relevant model structure. In the early days of AGE modelling there seemed to be an impression among some modellers that because they were concerned about resource allocation issues, the problems raised above could be ignored, or more reasonably were thought to be unimportant for the problems being addressed. This impression reflected an unjustified sense of security. In reality every model makes some implicit or explicit assumption about each of the above factors. These assumptions in turn affect the results of the model. Models with rigid prices and unemployment, for example, are going to give quite different results than flexible-price models. Models with assets will incorporate wealth effects not present in models without an asset side.

In choosing a theoretical structure for a given policy problem, experience and good judgement are highly desirable. Simplicity is of course highly desirable, provided the simplifying assumptions do not eliminate the most interesting possible effects of the policy being considered. In general terms, the shorter the time horizon of the model, the more important many of these issues become in departing from classical flexible price theory of the real economy.

When these departures are thought to be important, a simple and practical way of proceeding is as follows. Write down a highly aggregated version of the model, in the form that one would find in an advanced macroeconomic textbook. Be certain how the various aggregates in the model are determined, and how they react to exogenous changes. After this is done, one should make certain that the disaggregate version of the model is consistent with the aggregate version. While actual policy simulations will be done using the large disaggregate model, I have found it useful to keep a working version of the small aggregate model on the computer. In this case if it turns out that a particular closure condition is the possible source of either most of the results, or the source of apparently odd results, it is possible to check these out by some modification of the small-scale model relatively quickly.

In my experience proceeding this way is invaluable. It eliminates mistakes in model design, improves intuition as to how the model works, and gives greater insight as to how alternative closures might affect the results. Furthermore this methodology is appropriate using either the Johansen or levels form of solution method.

## V. CONCLUSION

This concludes my review of alternative solution methods for large-scale AGE modelling. The paper has focused on the comparison between linearised or Johansen methods, and full non-linear general equilibrium models. Both methods have strengths and weaknesses. The principal strength of the linearised method is dealing with large numbers of commodities and sectors, and a consequent saving in computational costs.

The principal strength of the AGE model solved in level form by non-linear methods is the ability to produce more accurate results for large-scale policy changes in a highly non-linear model, together with an automatic procedure for checking code and data via the benchmarking procedure used.

From the practical point of research organisation it should be recognised that building and debugging of large scale AGE models is as much an art as it is science. Researchers generally develop their own methods for data and code verification. Learning how to do this takes considerable time and expertise, in the areas of economic accounting, economic theory, and computer programming. Having supervised a number of Ph.D. dissertations in the area, I do not think it would be unreasonable to say that it takes about two years for someone to achieve this type of competency, provided they have the right background. Once this is realised, on any given project it is most sensible for a researcher to proceed using methods they have applied in the past. Therefore, in judging the appropriateness of any particular methodology for a project, the background of the personnel involved should be an important consideration.

Ultimately, the appropriate method depends upon the problem at hand, and the use to which the model will be put. In principle the full non-linear AGE models solved in level form have to be the "right" ones since they are closer to the basic economic theory from which any analysis starts. Because of this, and because of the overwhelming practical importance of debugging both code and data, it is the method I would on balance recommend for those beginning on a large-scale modelling project. The substantive issues will remain those regarding the theoretical structure of the model, and quantifying the relevant policies and institutions.

NOTES

1. For readers interested in fixed-point methods and their variants there is an excellent review by Scarf (1984). Newton-type methods are reviewed in a number of texts on numerical methods.

2. Models with fixed prices of one sort or another now abound. For representative examples see Dervis et al . (1982) and Deardorff and Stern (1981).

3. $\underline{0}_n$ denotes the zero vector of dimension n.

4. $\hat{x}_i = \Delta x_i / x_i^0$, the relative change in $x_i$. Likewise for $\hat{\alpha}_i$.

5. Jones (1965) is the standard reference on the ^-calculus approach to the comparative statics methodology used in general equilibrium analysis.

6. This is only true if all factors are mobile across all sectors. While true in most models, a number of models now incorporate factors of production which are specific to each sector. In this case no reduction in model dimension is usually feasible.

7. One obvious problem is that the optimisation problem involves choice over individual production and consumption allocations. Thus, with H consumers, F production units, and N goods, the optimisation problem in allocation space involves optimising over N(H+F) variables. In a supply equals demand framework, such as in the other methods discussed, much of the detail on individual allocations is subsumed in the supply and demand equations. Solution of the model involves finding only N-1 prices which are market clearing.

8. Diewert (1984) and Fuss and McFadden (1978) discuss flexible functional forms and their uses. Thus far, there has been little use of these functional forms in the AGE literature. In many cases this has simply been because the parameter estimates needed for a flexible functional form have not been available on anywhere near the basis of coverage necessary for implementation in an AGE model.

9. For an excellent discussion of this problem, see Ethier (1983).

10. Dixon et al. (1984) use just such a method in attempting to evaluate the impact of a large tariff change.

11. See Dixon et al. (1984), Table 8, page 528.

12. From a computational point of view it would be possible to use numerical derivatives of the $F_i$ to get evaluations for the $J^*$ and $B^*$ matrices. In general this does not seem to be the method used in Johansen models to date.

13. See Dervis et al. (1982) for a summary of some of this work.

14. See Shoven and Whalley (1984) for a survey of this work.

15. See Kehoe and Whalley (1982) for a discussion of the non-uniqueness problem and for one simple model a demonstration that the equilibrium is in fact unique. Unfortunately no general numerical methods exist for checking uniqueness. In some recent work with Lawrence McDonough (unpublished) we found a model which yielded numerous equilibria. This suggests the problem may be more important than hitherto suspected.

16. Closure rules are discussed in Dervis et al. (1982), Chapters 6-7, and Taylor (1979). The general problem of closure will be discussed in Section 5.

Figure 1

ERRORS DUE TO LINEARISATION

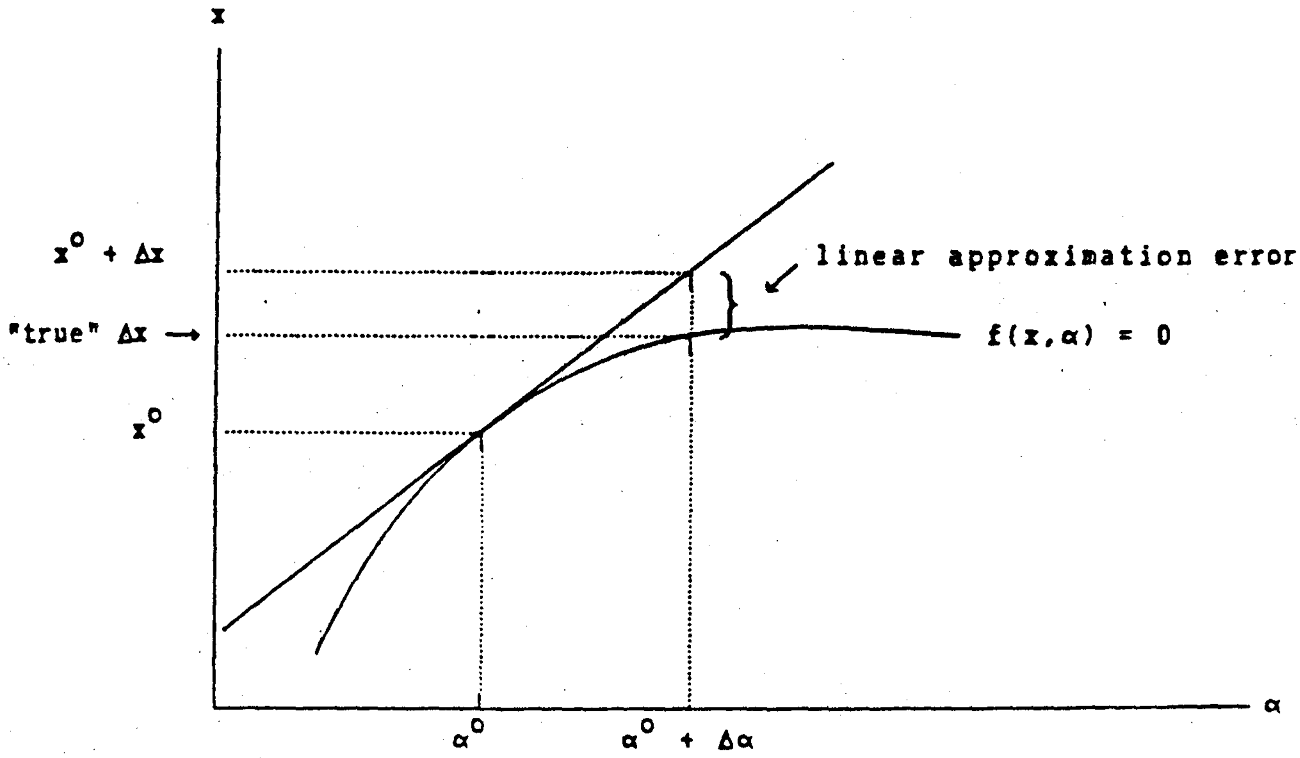
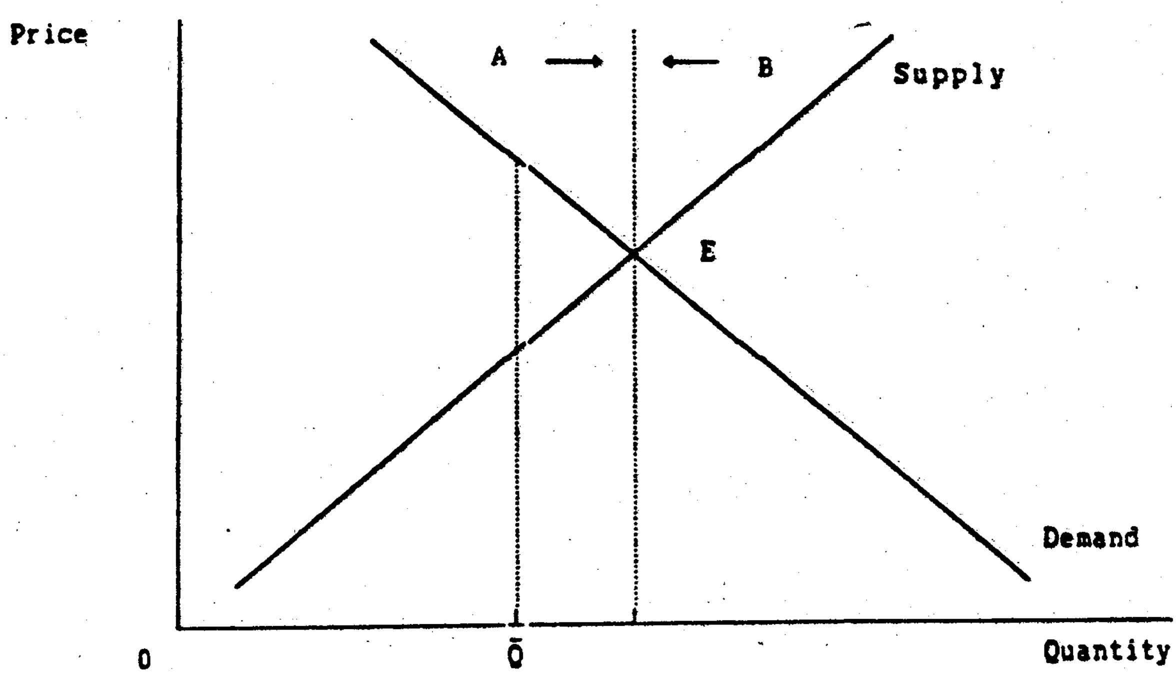


Figure 2

EQUILIBRIUM WITH SUPPLY QUOTA $\bar{Q}$



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